Optica - Webinar

Ill-defined topologies and energy sinks in photonic systems

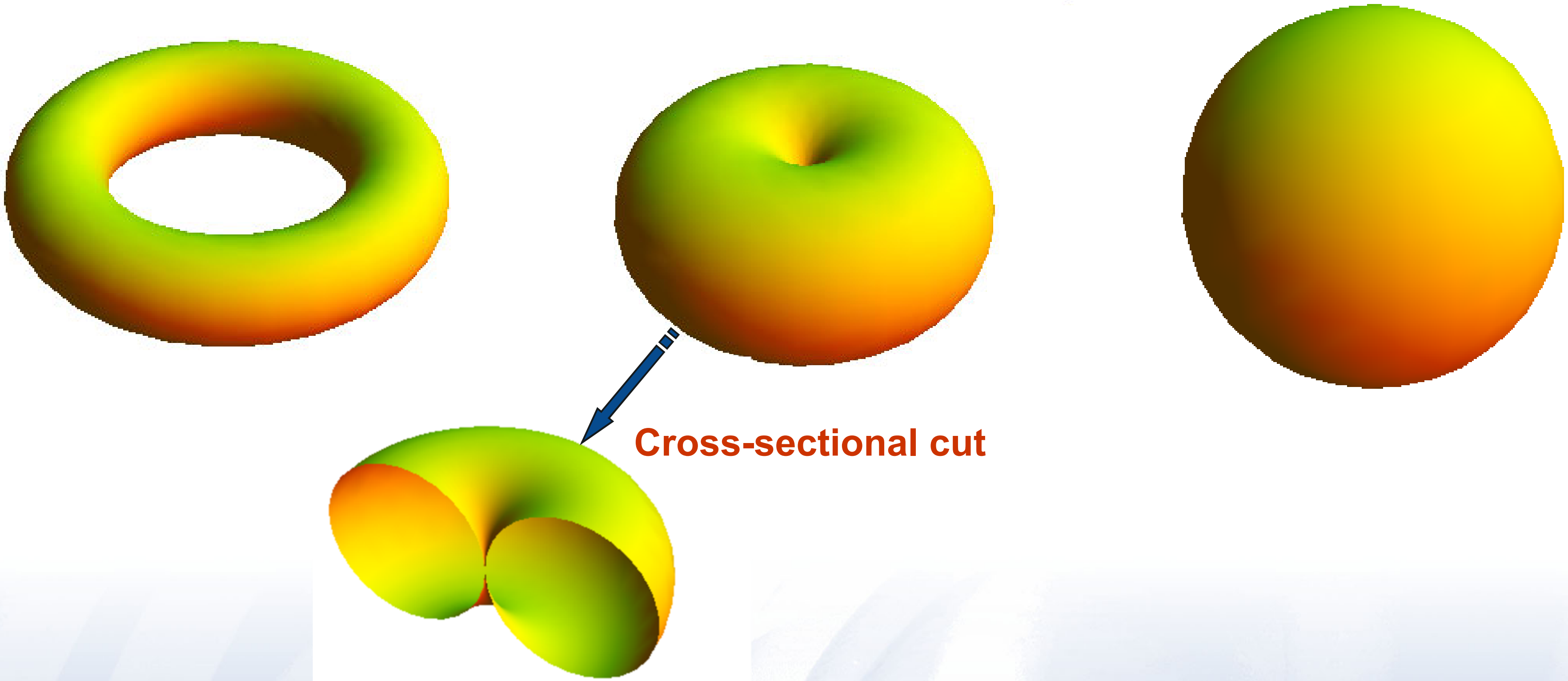
Mário G. Silveirinha

Geometrical illustration of ill-defined topology

Well defined topology, g=1

Ill defined topology

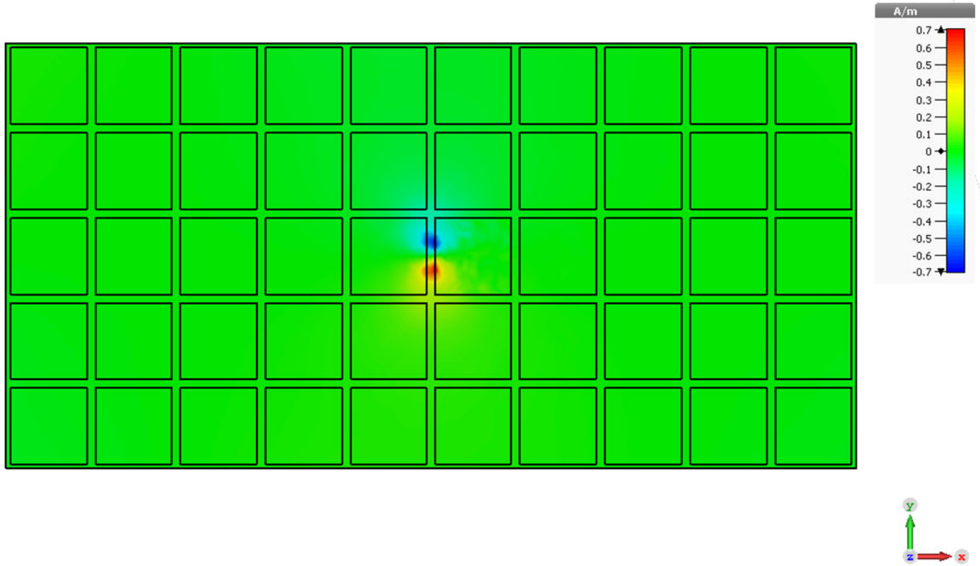
Well defined topology, g=0



Topological Photonics

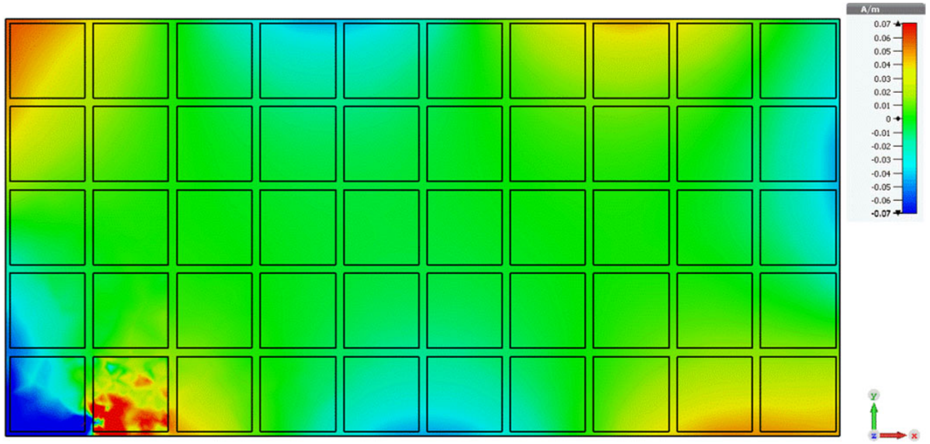
Topological systems

No propagation in the bulk region



Topological systems (contd.)

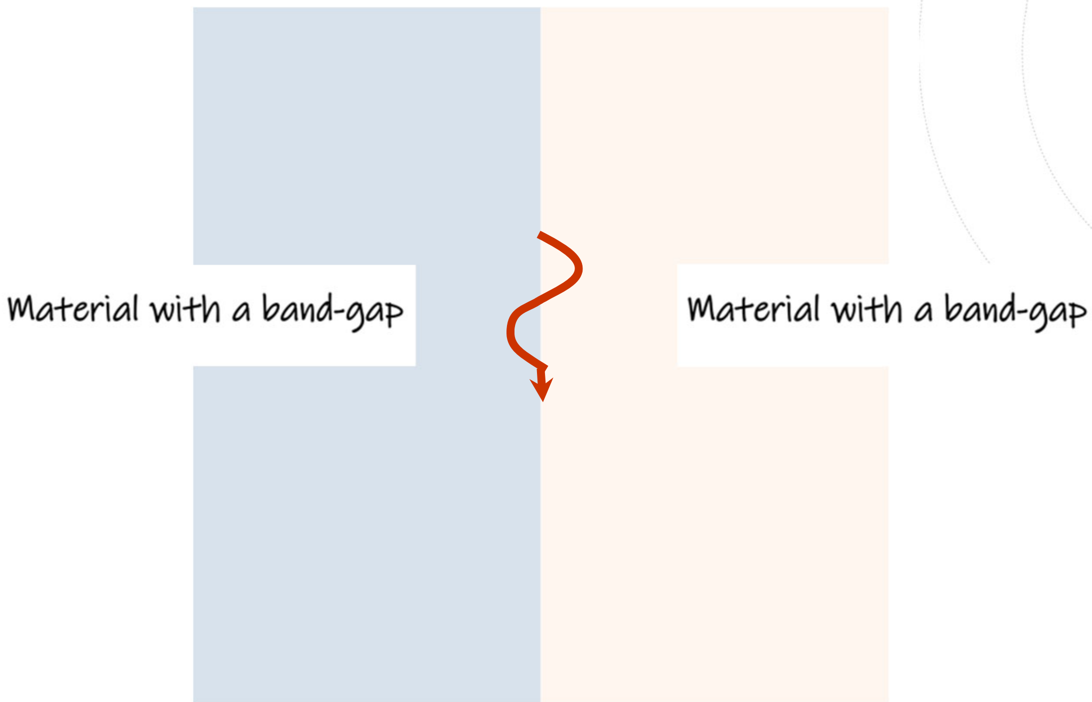
Edge states on the boundary:



Net number of unidirectional of edge states is determined by the gap Chern number

Bulk-edge correspondence and stability

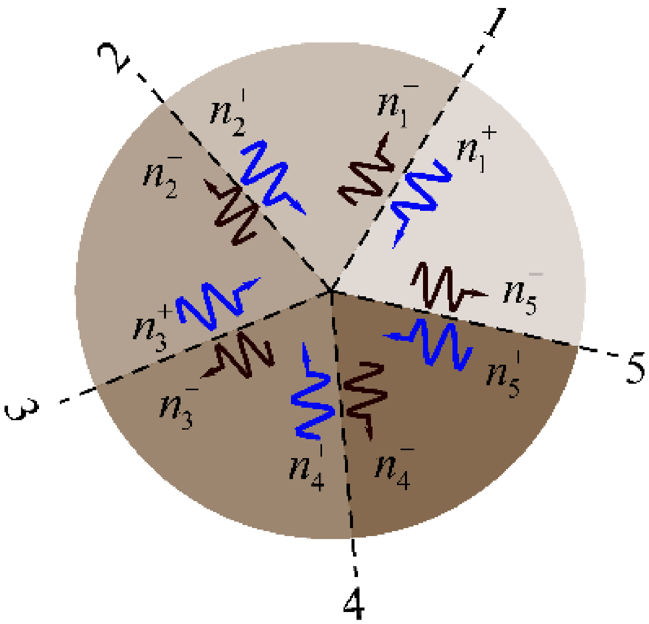
Bulk edge correspondence



Net number of unidirection edge states = difference topological charge

Origin of the topological properties

$$\sum_i n_i^+ = \sum_i n_i^-$$

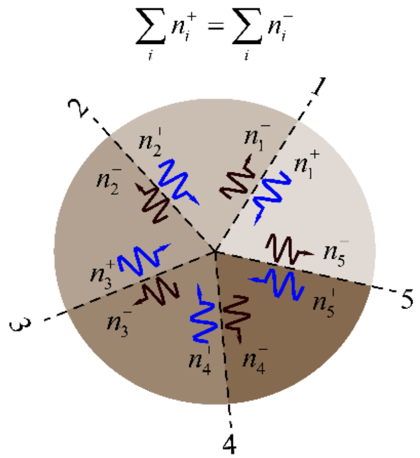


System formed by materials with a common band gap is topological if:

 # edge states propagate inward = # edge states propagate outward

= system has a "stable" (nonsingular) response

Stability



Suppose that more waves go "inward" than "outward":

$$\begin{pmatrix} E_1^- \\ \ldots \\ E_M^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \ldots & \ldots & S_{N1} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ S_{M1} & \ldots & \ldots & \ldots & S_{MN} \end{pmatrix} \begin{pmatrix} E_1^+ \\ E_2^+ \\ \ldots \\ \ldots \\ E_N^+ \end{pmatrix}_{N>M}$$

Underdetermined system (null space nontrivial):

It is always possible to pick an inward excitation that does not lead to outgoing waves

Energy is concentrated at the junction point: topological sink

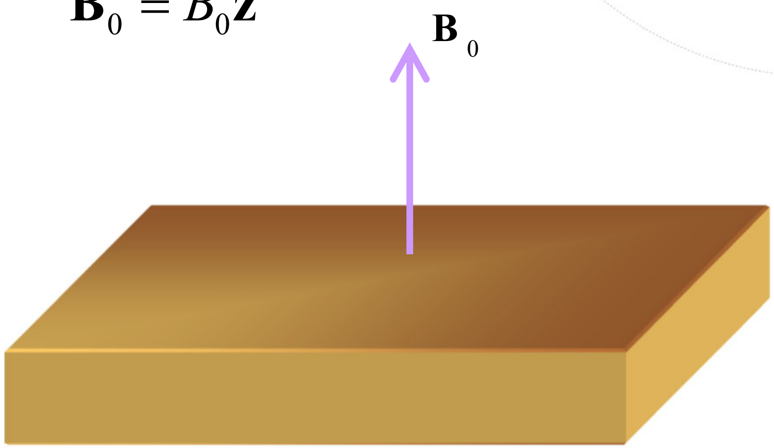
Ill-defined topology due to a continuous translational symmetry

M. G. Silveirinha, "Chern Invariants for Continuous Media", Phys. Rev. B, 92, 125153, 2015.

Electromagnetic continua: invariant under continuous translations

$$\overline{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$$

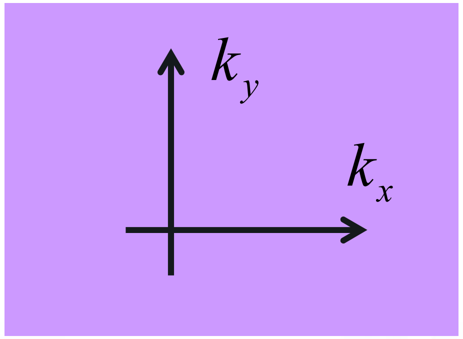


Topological band theory

$$\mathcal{C} = \frac{1}{2\pi} \iint dk_x dk_y \, \mathcal{F}_{\mathbf{k}}$$

$$\mathcal{F}_{\mathbf{k}} = \frac{\partial \mathcal{A}_y}{\partial k_x} - \frac{\partial \mathcal{A}_x}{\partial k_y}$$

Electromagnetic continuum



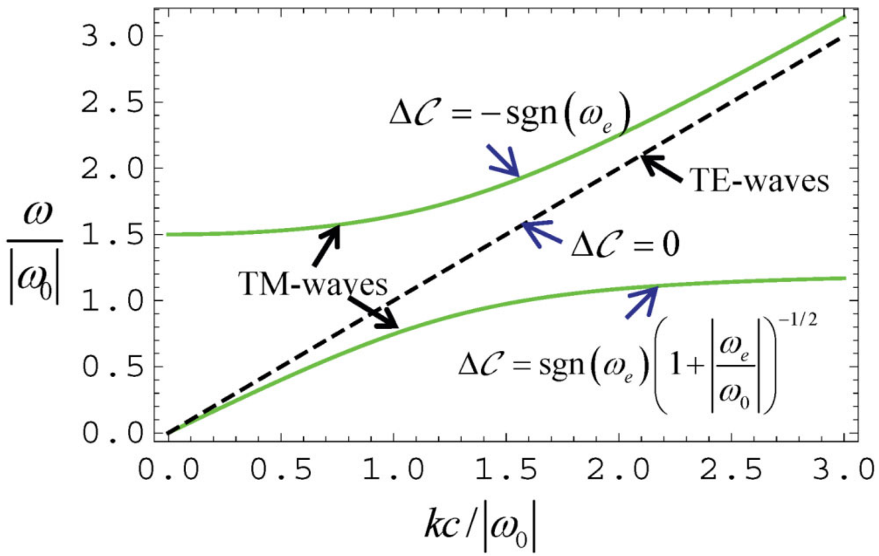
Magneto-optical material

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\varepsilon_t = 1 + \frac{\omega_0 \omega_e}{\omega_0^2 - \omega^2} \qquad \varepsilon_g = \frac{\omega_e \omega}{\omega_0^2 - \omega^2} \qquad \varepsilon_a = 1$$

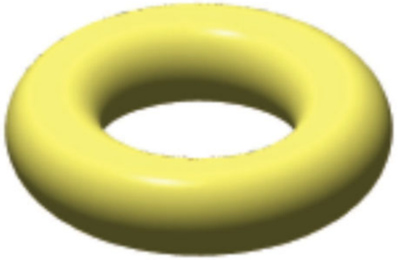
TM waves: $$k^2 = \frac{\varepsilon_t^2 - \varepsilon_g^2}{\varepsilon_t} \left(\frac{\omega}{c} \right)^2$$

Band structure and Chern numbers

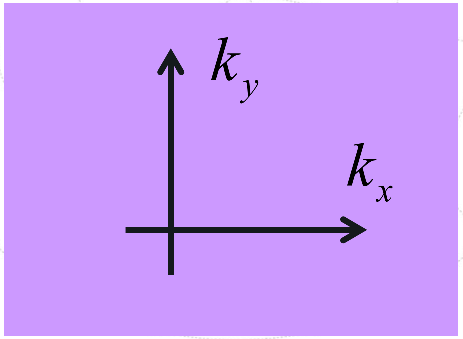


Origin of the ill-defined topology

Photonic crystals (BZ is a torus):

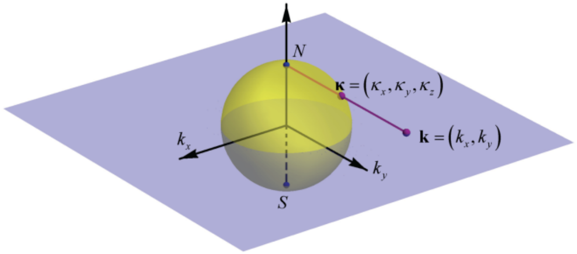


Electromagnetic continuum



The continuous translational symmetry leads to a ill-defined topology (the "Euclidean plane" is not a compact set)

High Frequency Spatial Cut-off

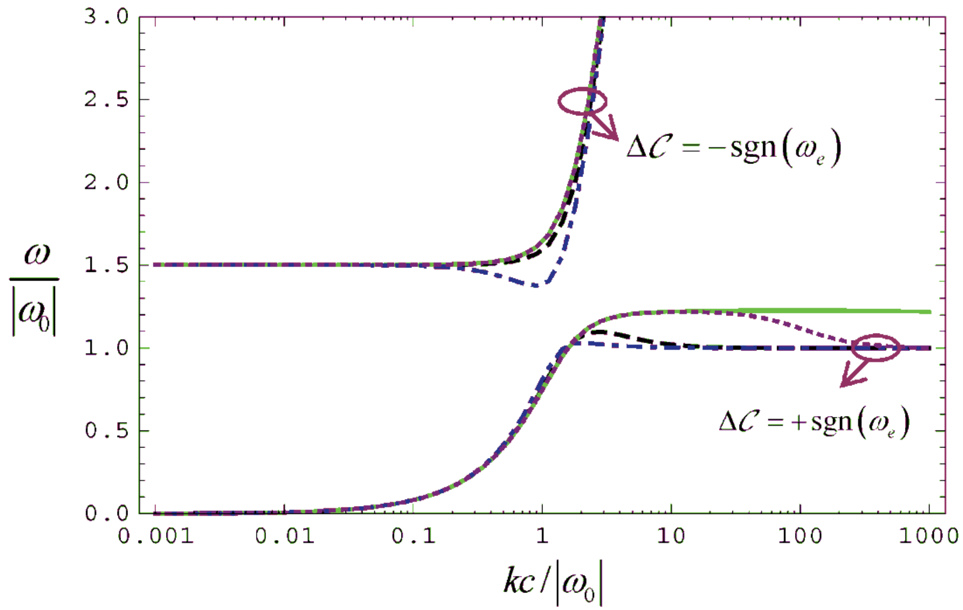


$$\bar{\varepsilon} \to \varepsilon_0 \mathbf{1} + \frac{1}{1 + k^2 / k_{\max}^2} \left[\bar{\varepsilon}(\omega) - \varepsilon_0 \mathbf{1} \right]$$

$$k_{\max} = \text{spatial cut-off}$$

For large wave vectors the material response becomes asymptotically the same as the vacuum!

Effect of the spatial cut-off



$$k_{\max} = 100|\omega_0|/c, \quad 3|\omega_0|/c, \quad |\omega_0|/c$$

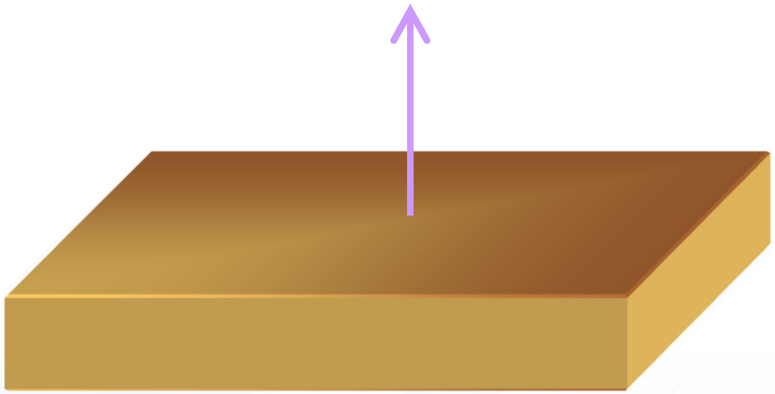
$$\omega_e = 0.5\omega_0$$

Do other types of cut-off lead to the same topology?

Another example: magnetized plasma

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\mathbf{B}_0 = B_0\hat{\mathbf{z}}$$



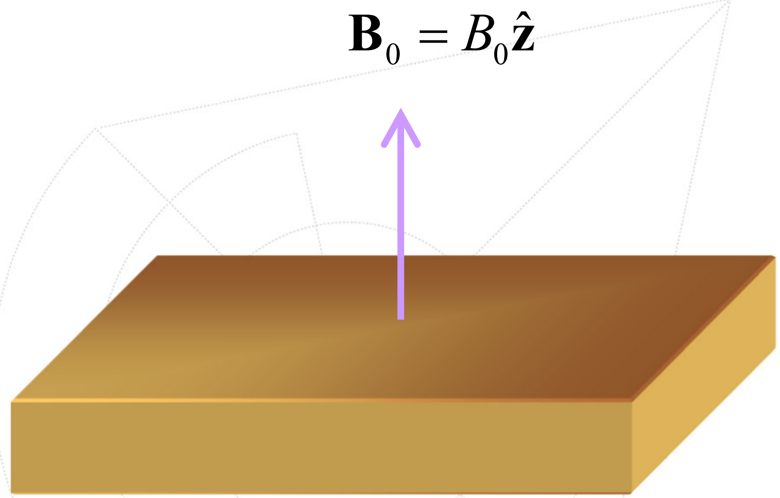
$$\varepsilon_t = 1 - \frac{\omega_p^2\left(1 + i\Gamma/\omega\right)}{\left(\omega + i\Gamma\right)^2 - \omega_0^2}$$

$$\varepsilon_g = \frac{1}{\omega}\frac{\omega_p^2\omega_0}{\omega_0^2 - \left(\omega + i\Gamma\right)^2}$$

$$\varepsilon_a = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\Gamma\right)}$$

$$\omega_0 = -qB_0/m$$

Different models



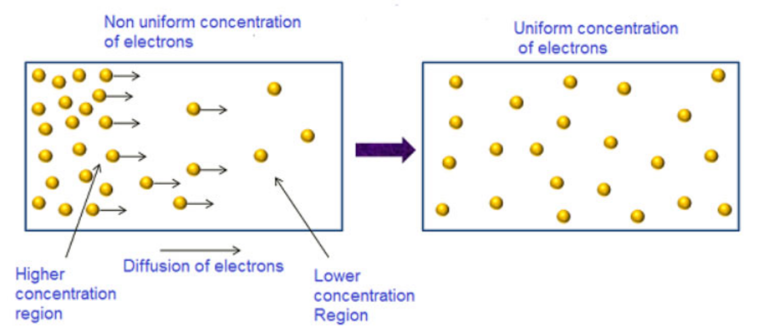
- **Local description** $\overline{\varepsilon}_{\mathrm{loc}}$

- **Full spatial cut-off**

$$\overline{\varepsilon} = \varepsilon_0\mathbf{1} + \frac{1}{1 + k^2 / k_{\mathrm{max}}^2}\left[\overline{\varepsilon}_{\mathrm{loc}}(\omega) - \varepsilon_0\mathbf{1}\right]$$

- **Hydrodynamic model**

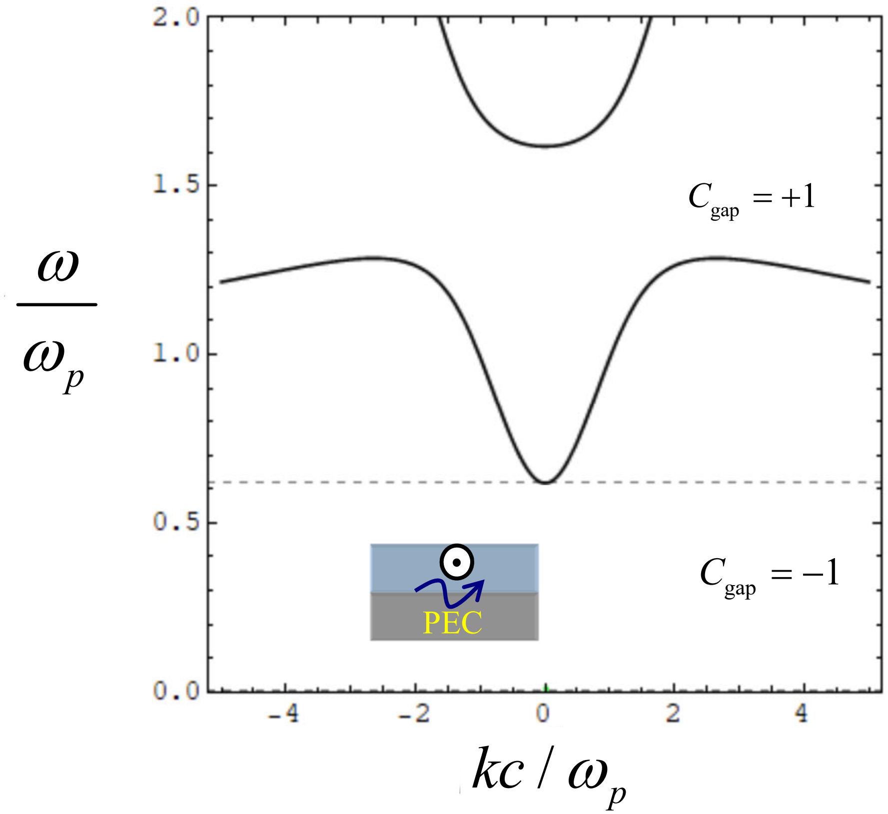
$$\beta = \text{"diffusion" velocity}$$



tinyurl.com/a2mnsxuj

Topology of a magnetized plasma calculated with a <u>full wave vector cut-off</u>

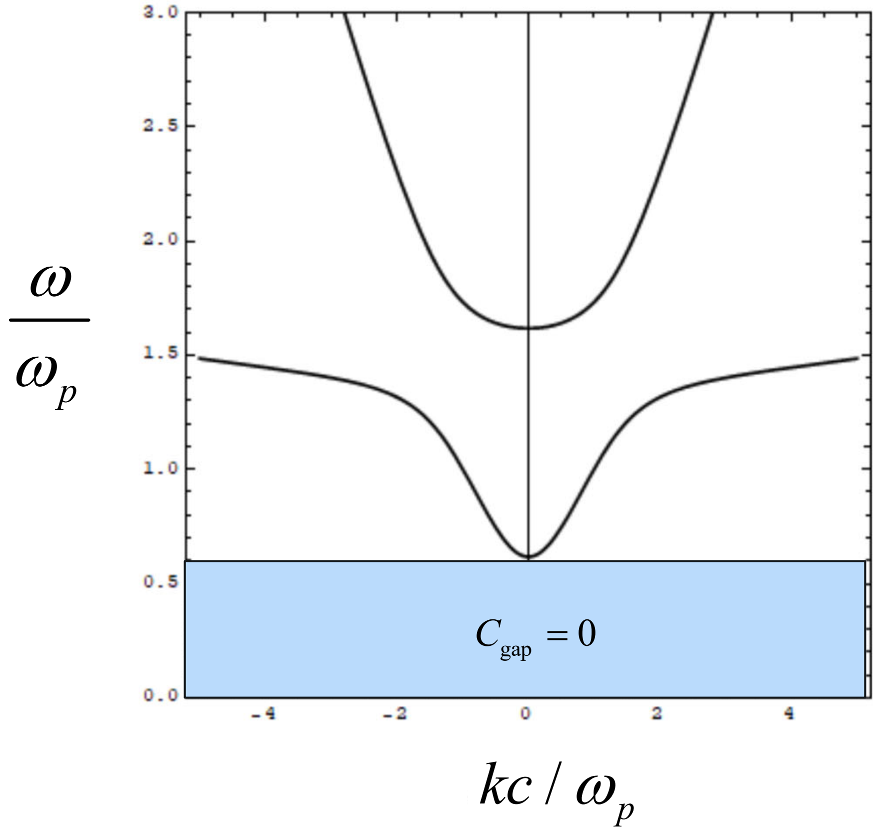
$$\bar{\varepsilon} = \varepsilon_0 \mathbf{1} + \frac{1}{1 + k^2 / k_{\max}^2}\left[\bar{\varepsilon}_{\mathrm{loc}}(\omega) - \varepsilon_0 \mathbf{1}\right]$$



$\omega_0 = \omega_p$

$k_{\max} = 5.0\omega_p / c$

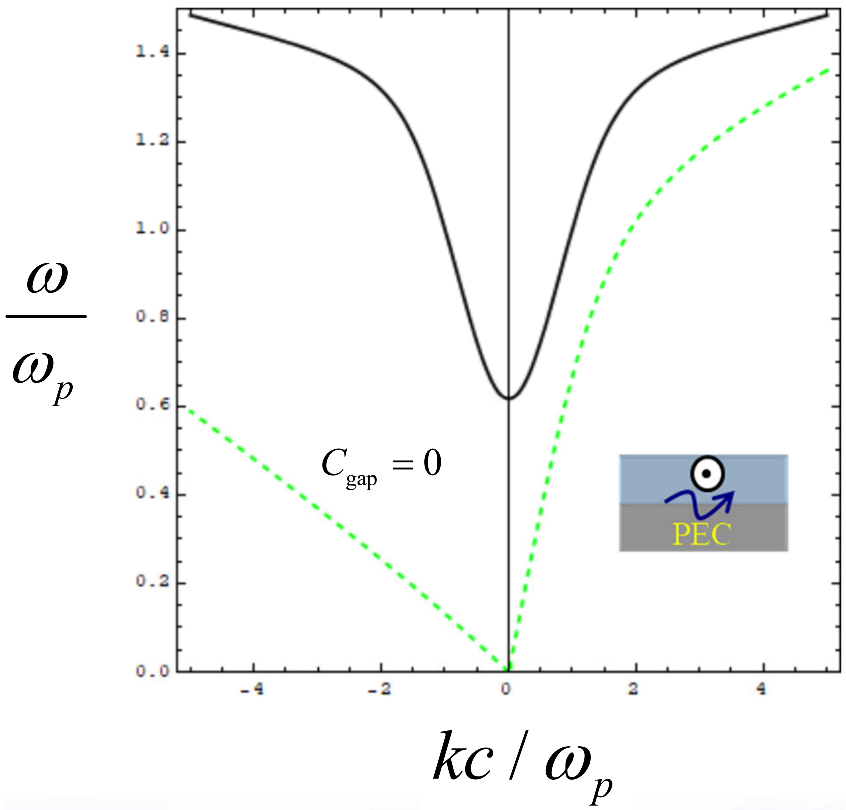
Topology of a magnetized plasma calculated with the <u>hydrodynamic model</u>



$\omega_0 = \omega_p$

$\beta = 0.1c$

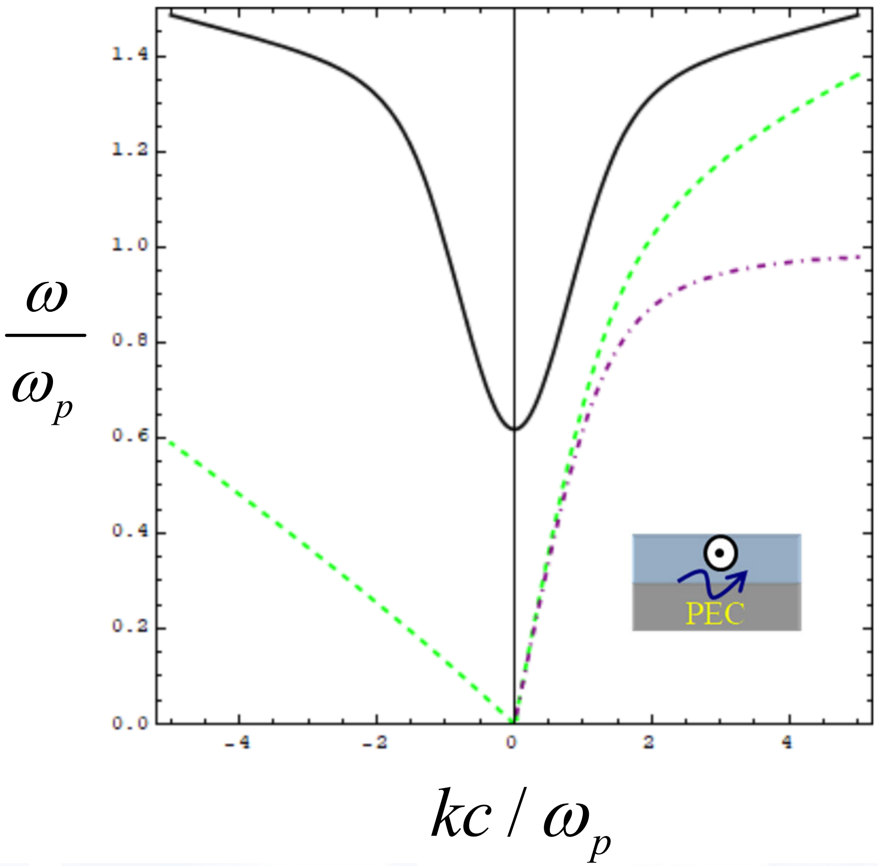
Edge states with the hydrodynamic model



$\omega_0 = \omega_p$

$\beta = 0.1c$

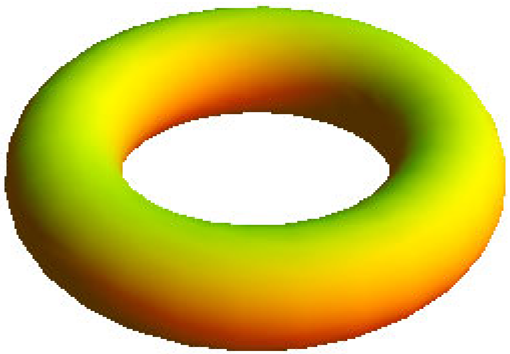
Comparison with the “local” formulation (without a cut-off)



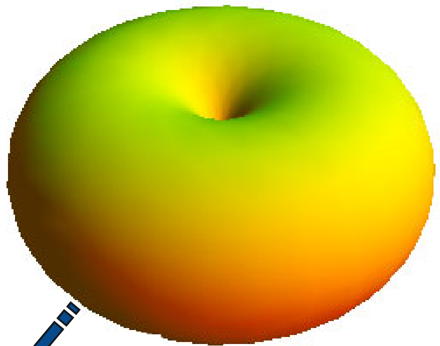
$\omega_0 = \omega_p$

Material with an ill-defined topology

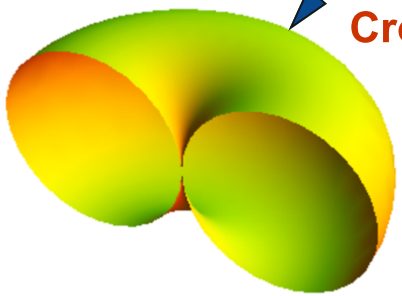
Full wave vector cut-off: well defined topology



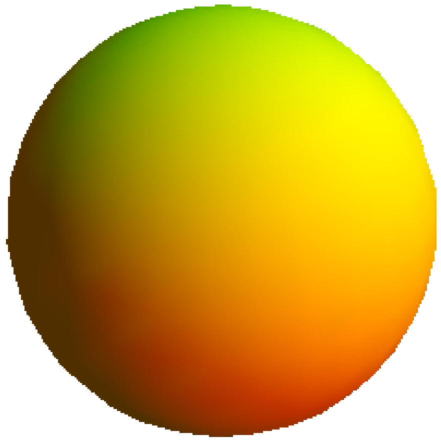
Magnetized plasma with no cut-off: ill-defined topology



Cross-sectional cut

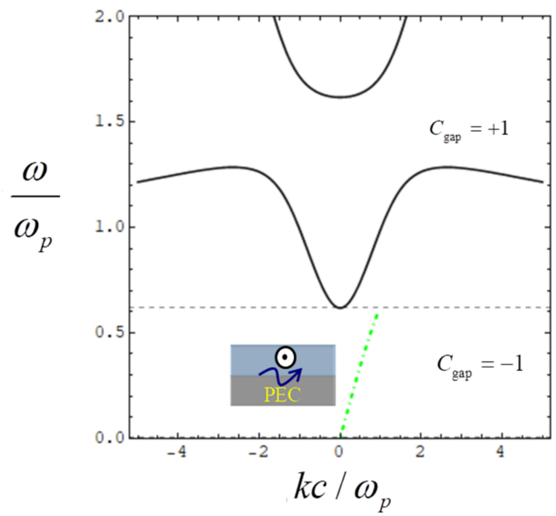


Hydrodynamic model: well-defined topology

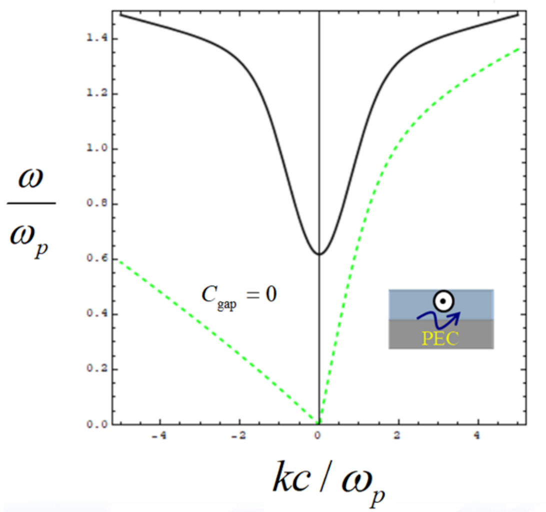


Can the "correct" model be determined with an experiment?

Full cut-off



Hydrodynamic

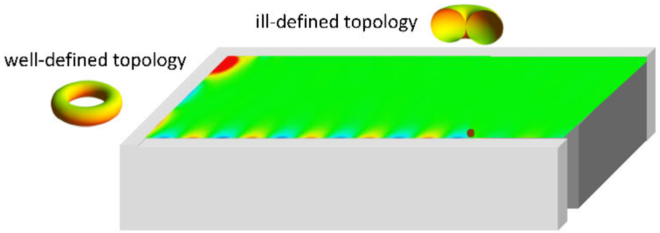


In practice no, because the additional edge state is impossible to measure as it is extremely confined to the interface!

The topology depends on the unknown high-k response:

The topology can change with the complexity of the model that describes the system!!!

Topological energy sinks

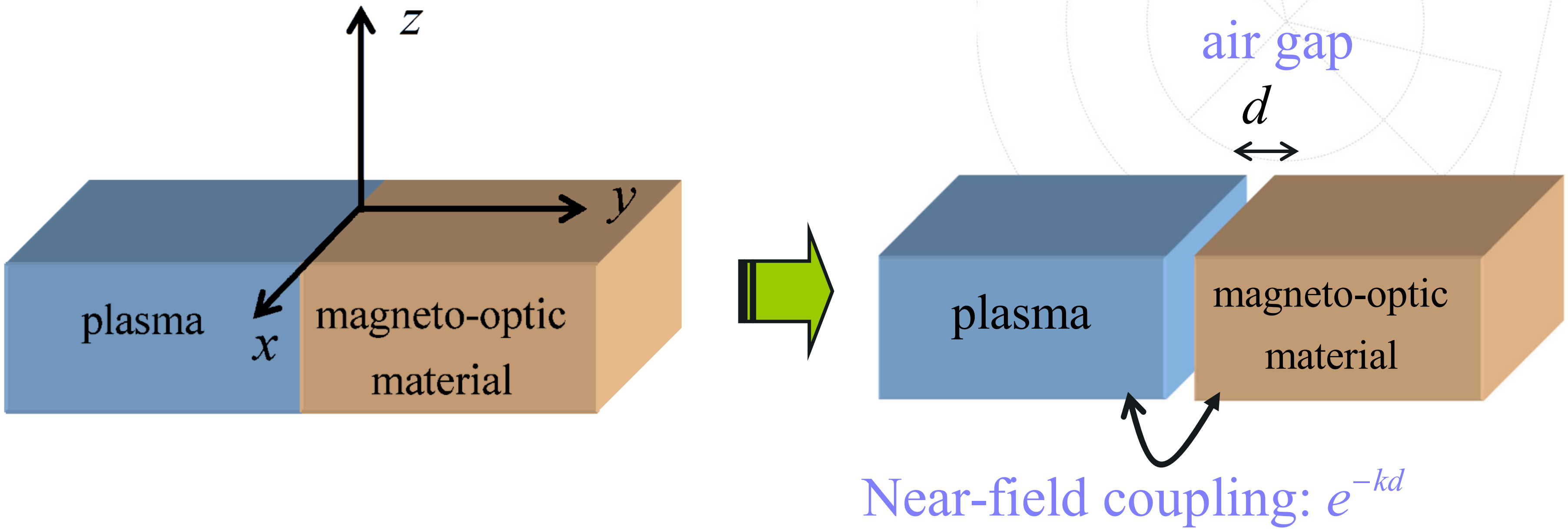


D. E. Fernandes, M. G. Silveirinha, “Topological origin of electromagnetic energy sinks”, Phys. Rev. Appl., 12, 014021, 2019.

D. E. Fernandes et al., “Experimental verification of ill-defined topologies and energy sinks in electromagnetic continua,” Adv. Photon. 4(3) 036002 (2022), doi 10.1117/1.AP.4.3.036002.

The cut-off can be imitated with an air gap

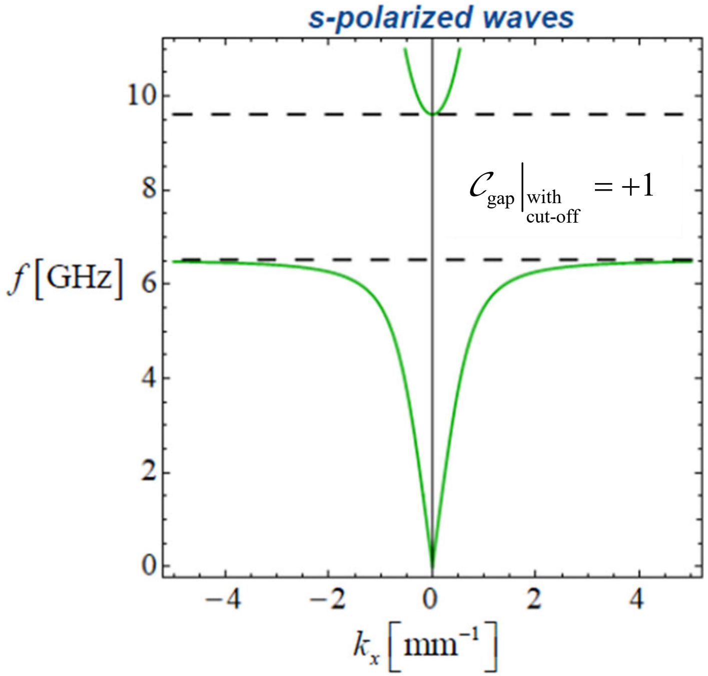
$$\bar{\varepsilon} \to \varepsilon_0 \mathbf{1} + \frac{1}{1+k^2/k_{\max}^2}\left[\bar{\varepsilon}(\omega) - \varepsilon_0 \mathbf{1}\right]$$



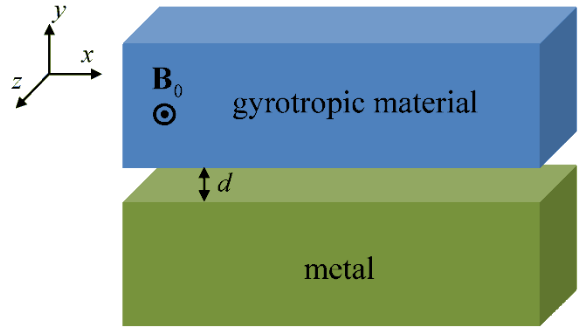
"Synthetic" cut-off : $k_{\max} = 1/d$

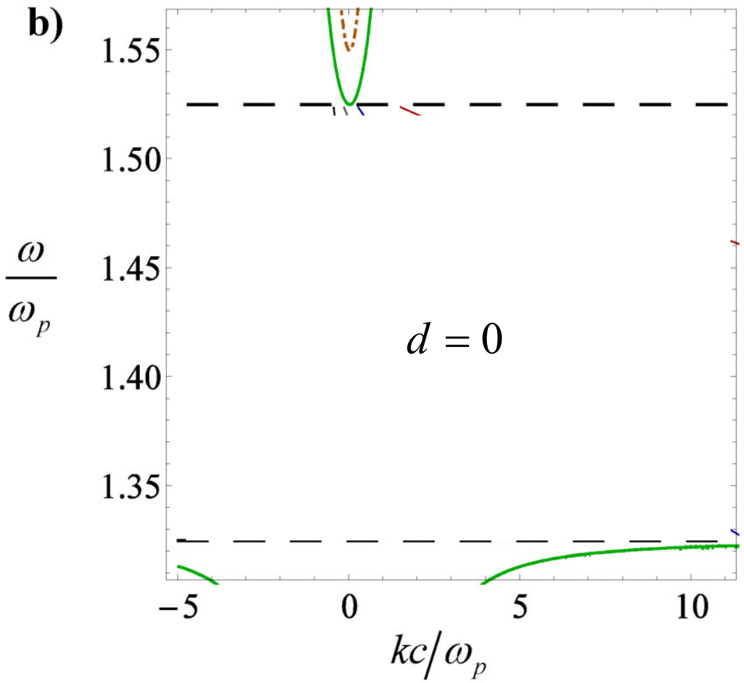
Biased microwave ferrite

$$\overline{\mu} = \begin{pmatrix} \mu_t & -i\mu_g & 0 \\ i\mu_g & \mu_t & 0 \\ 0 & 0 & \mu_a \end{pmatrix}$$



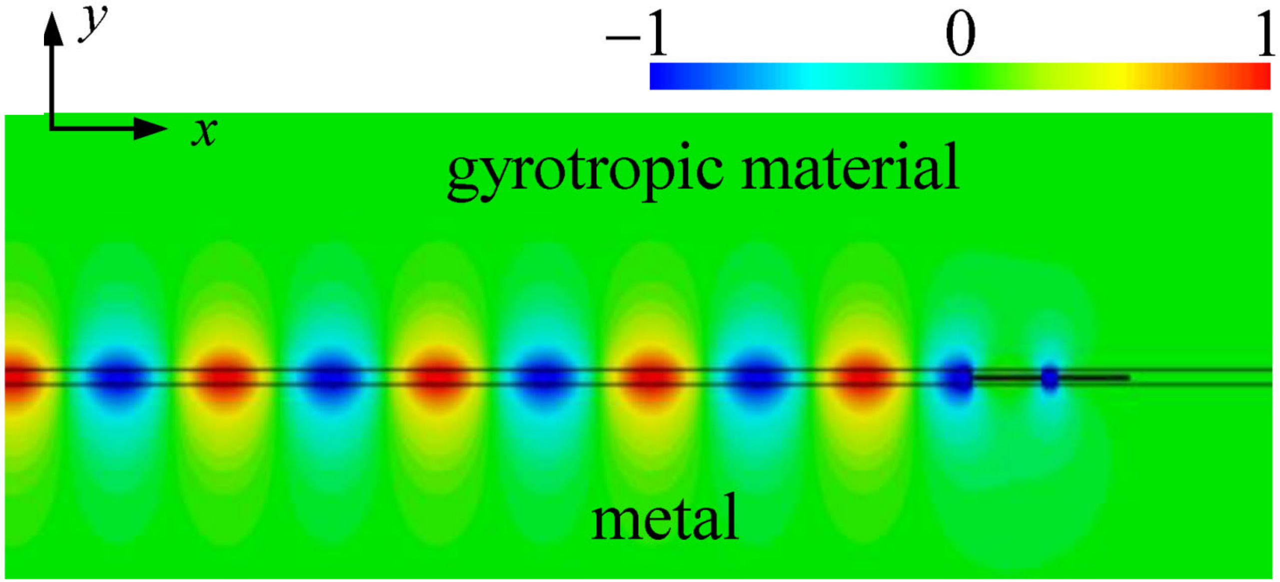
Edge modes



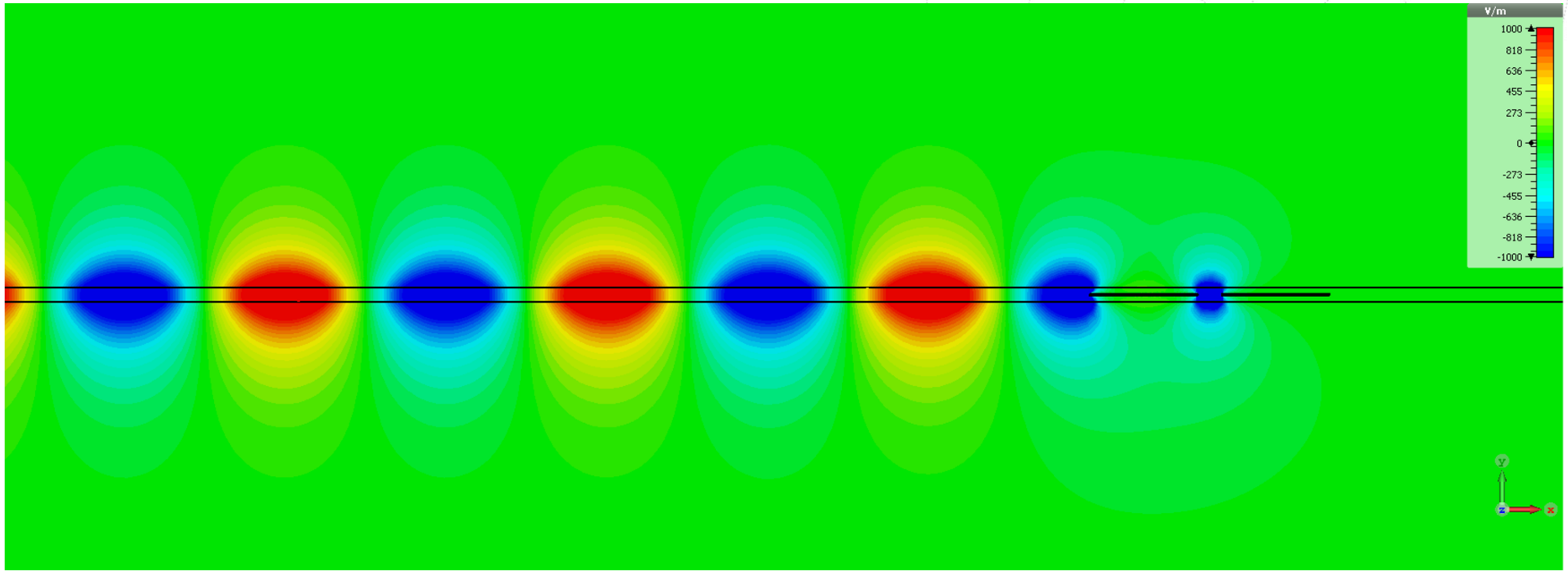


When the air gap is closed the edge mode is supressed

Unidirectional (backward) edge mode

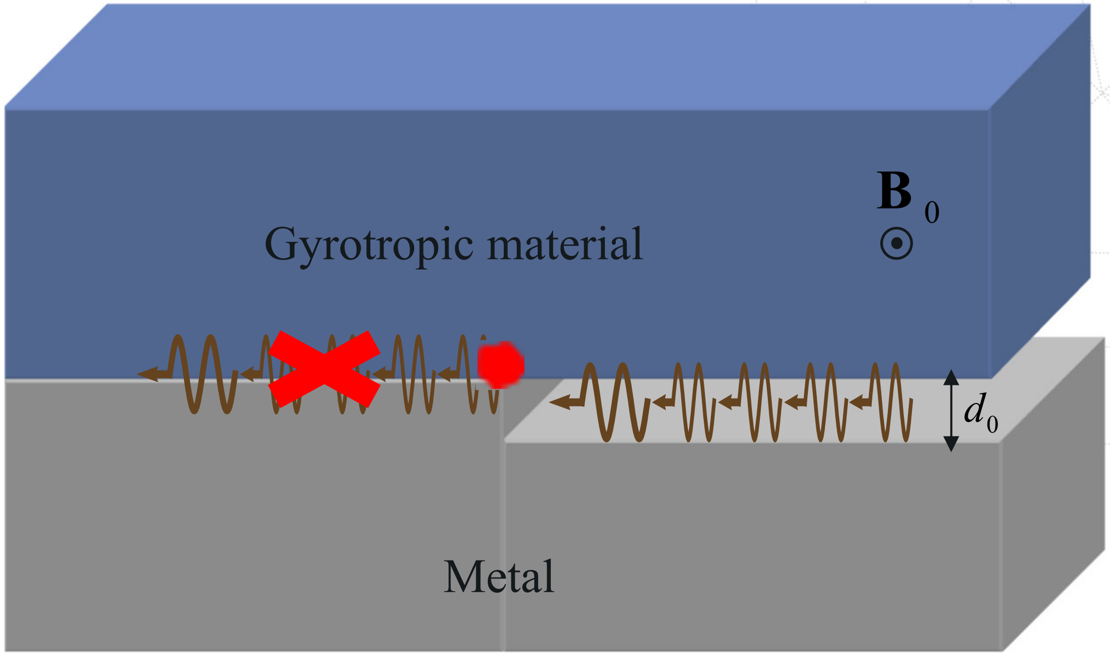


Unidirectional (backward) edge mode



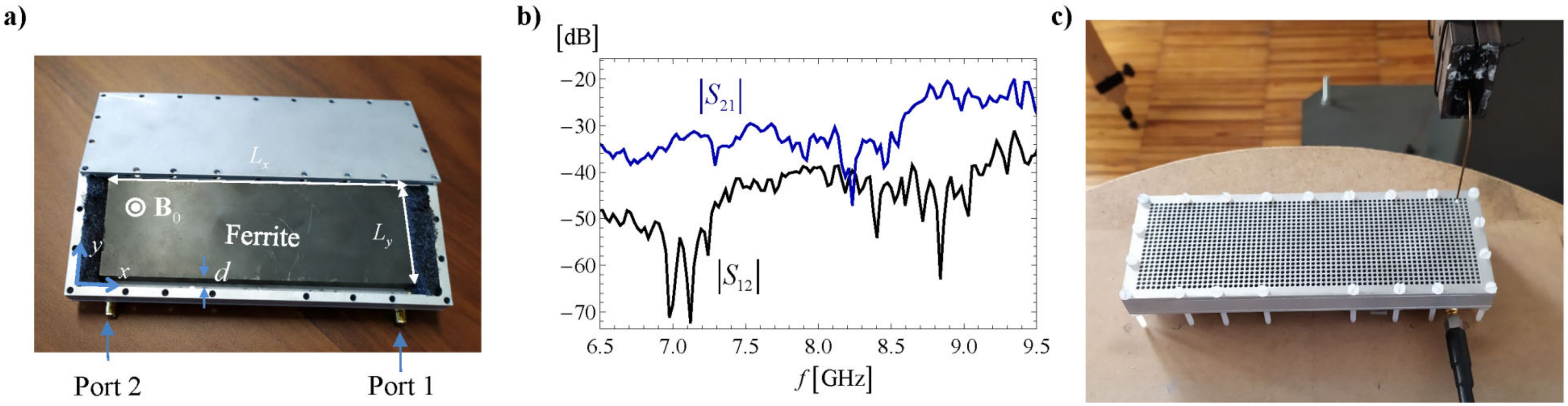
Topological sink

See also:
A. Ishimaru, Tech. Rep. (Washington Univ. Seattle, 1962).
U. K. Chettiar, A. R. Davoyan, and N. Engheta, Opt. Lett. 39, 1760 (2014).



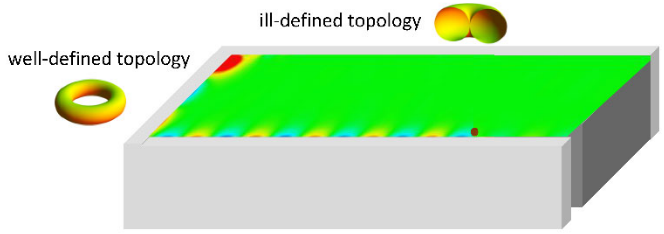
The wave is halted in its tracks at the "topological" singularity!

Experimental verification of the topological sink



D. E. Fernandes et al., “Experimental verification of ill-defined topologies and energy sinks in electromagnetic continua,” Adv. Photon. 4(3) 036002 (2022), doi 10.1117/1.AP.4.3.036002.

Topological singularity



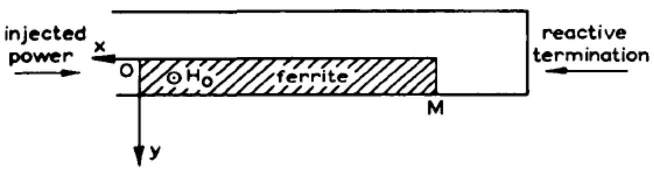
A beautiful paper from the 60’s

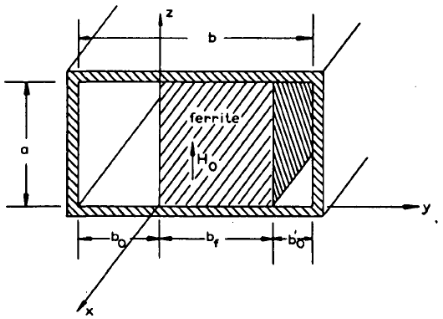
Rectangular waveguides loaded with magnetised ferrite, and the so-called thermodynamic paradox

Prof. G. Barzilai and G. Gerosa

PROC. IEE, Vol. 113, No. 2, FEBRUARY 1966

One-way guide:





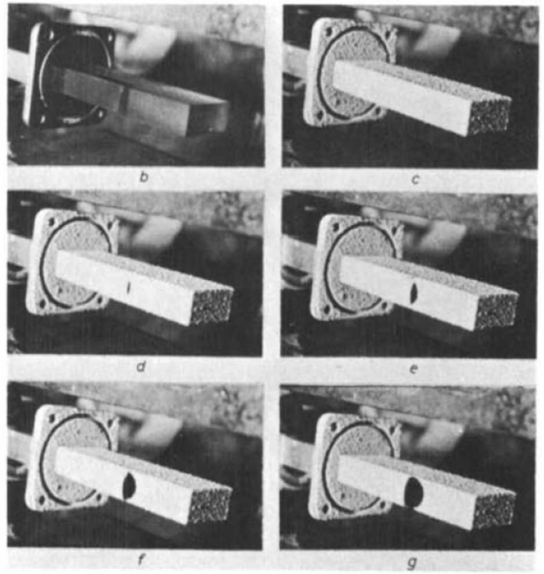
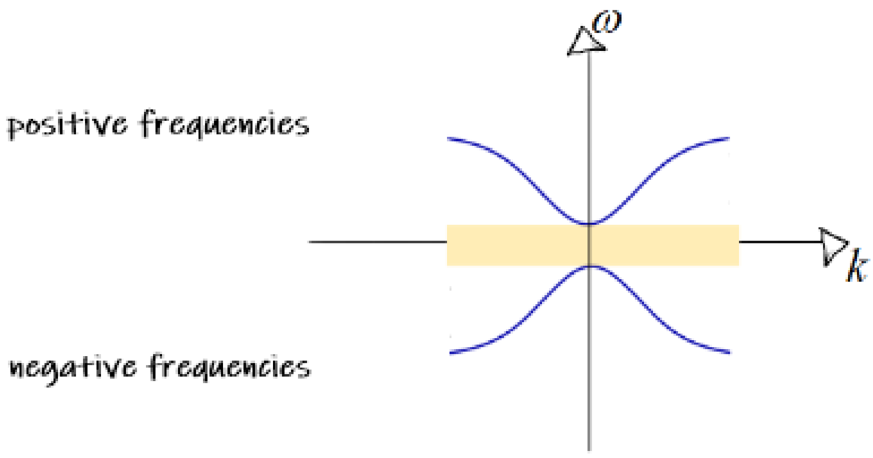


Fig. 7
Structure realised for the experiment
a Line drawing (dimensions in millimeters)
b Piece uncovered
c Piece before sending r.f. energy
d, e, f, g Appearance of the piece after sending r.f. energy, at about 10 s intervals
The ferrite used was Ferramic R4, magnetised by a d.c. external magnetic field of $1{\cdot}9 \times 10^6/4\pi$ A/m, and the total thickness of the metallic walls (silver and copper) was about 0·05 mm

Ill-defined topologies of dispersive photonic crystals

F. R. Prudêncio, and M. G. Silveirinha, *Phys. Rev. Lett.* 129, 133903, 2022

Photonic systems are different from electronic systems: No ground state!

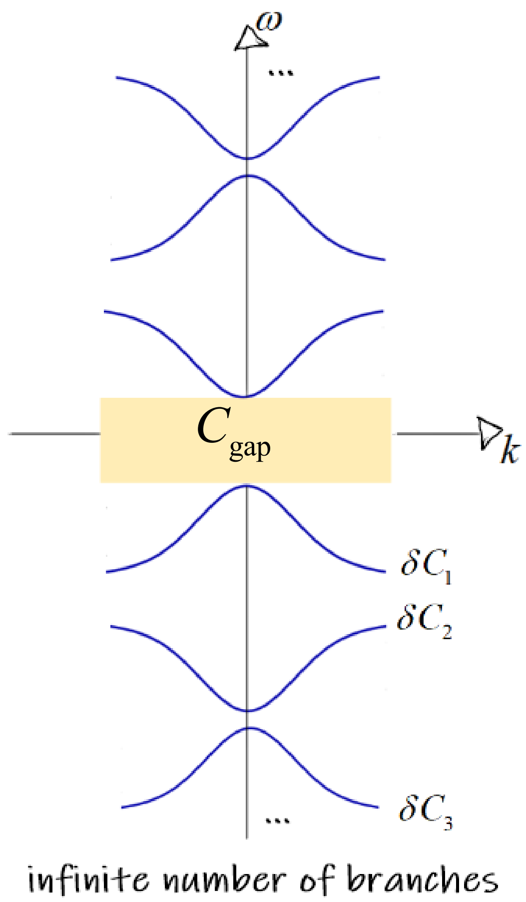


particle-hole symmetry

The spectrum is symmetric with respect to the line ω=0. In particular, this implies that in a photonic crystal there are infinite number of bands below the gap.

No ground state (contd.)

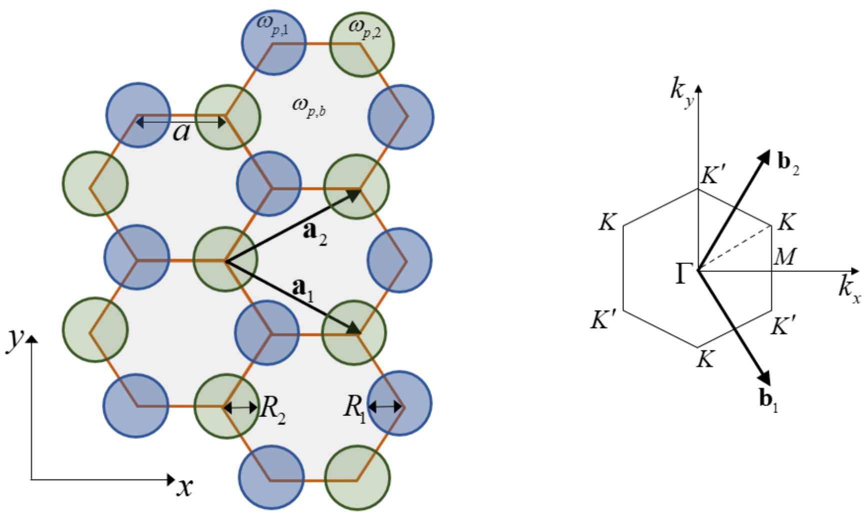
$$C_{\text{gap}} = \delta C_1 + \delta C_2 + \delta C_3 \ldots$$

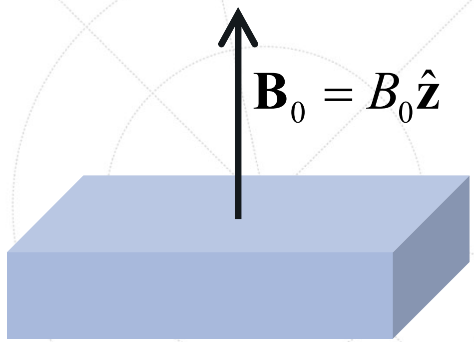


An infinite number of terms may contribute to the gap Chern number

Magnetized electric plasma photonic crystal

Hexagonal array of air rods in a magnetized plasma

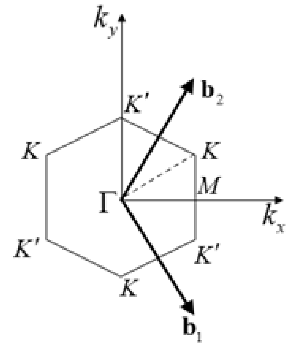


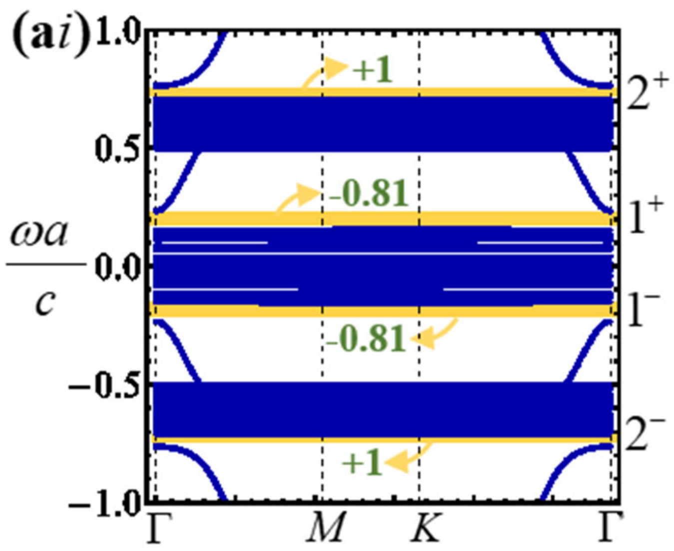


$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_t & -i\varepsilon_g & 0 \\ i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}$$

$$\varepsilon_t = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} \qquad \varepsilon_g = \frac{1}{\omega}\frac{\omega_p^2\omega_c}{\omega_c^2 - \omega^2}$$

Band structure (hexagonal array of air rods in a magnetized plasma)



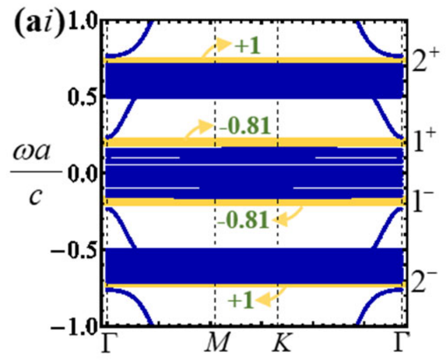


$\omega_p = 0.5c/a$

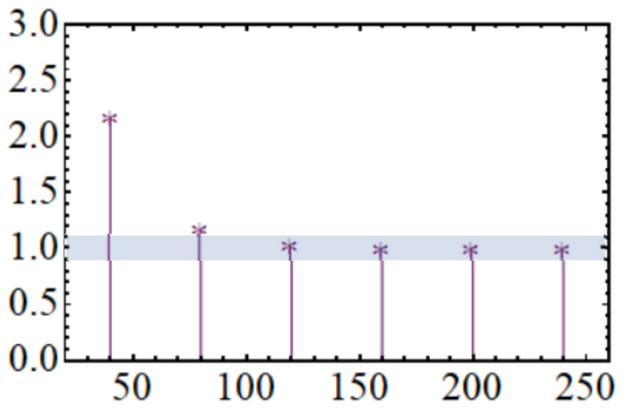
$\omega_c = \omega_p$

Bands pile up near the plasma frequency and at low frequencies.

Gap Chern numbers

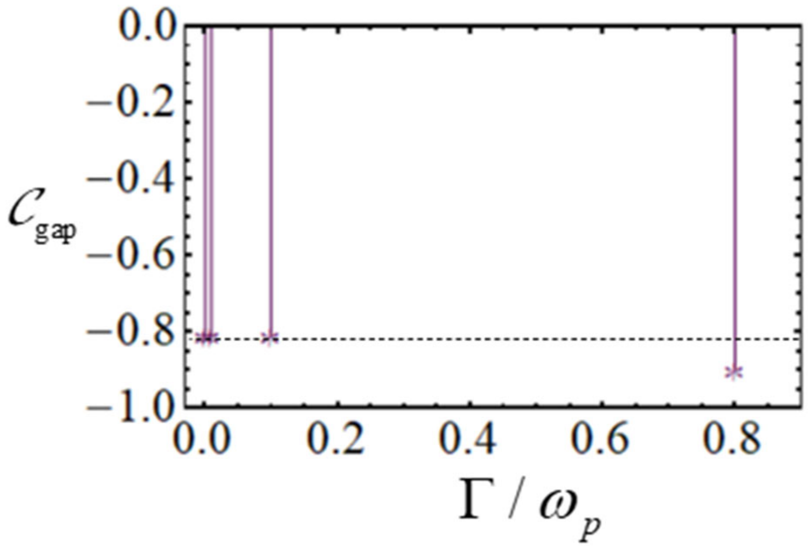


Gap 2+



Chern number of low frequency gap is not an integer!

Effect of loss (low frequency gap)



$\omega_p = 0.5c/a$

$\omega_c = \omega_p$

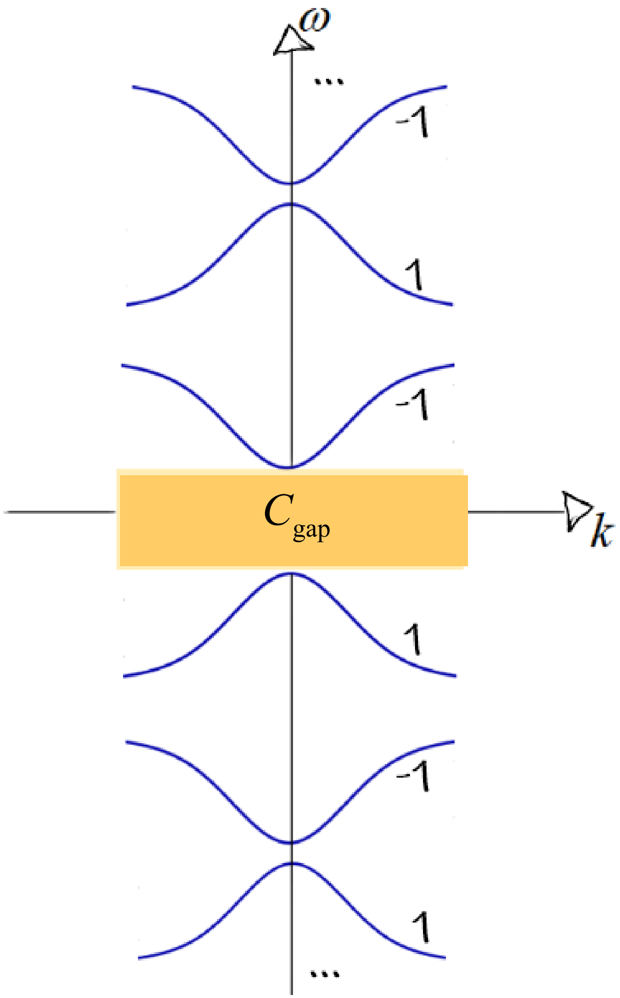
$N = 20$

$n_{\max} = 3$

$N_w = 120$

What is wrong? Why the Chern theorem is not valid?

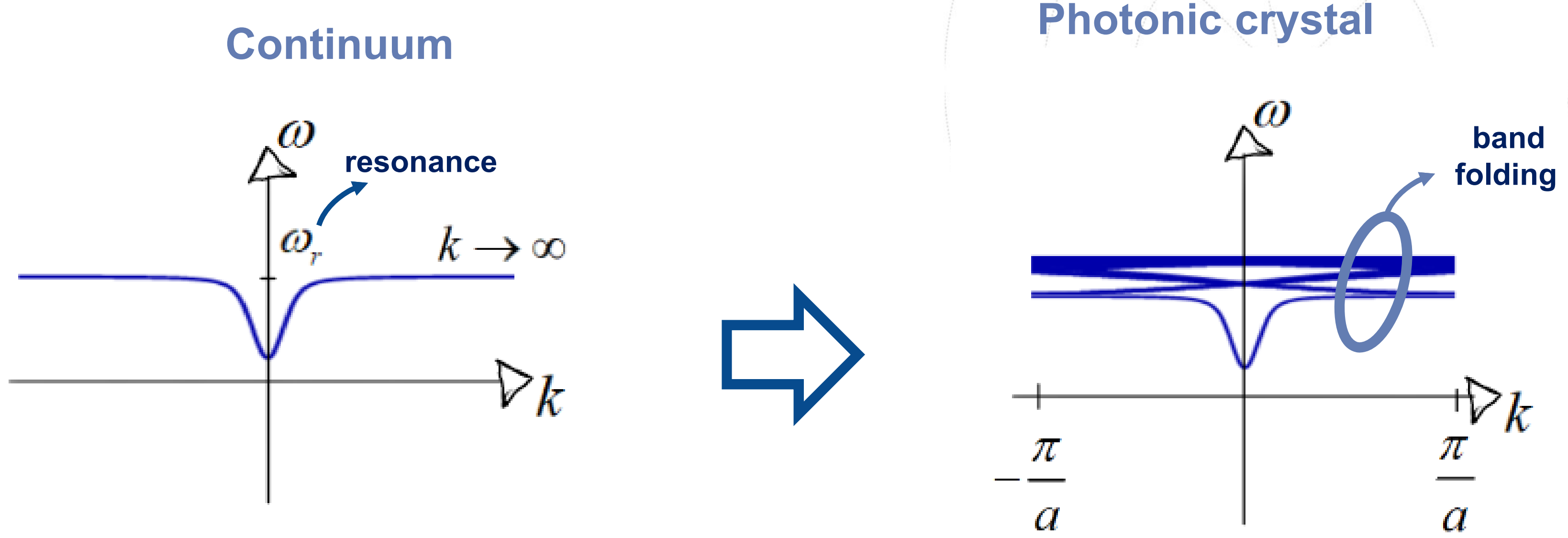
Origin of the ill-defined topology (1/3)



infinite number of branches

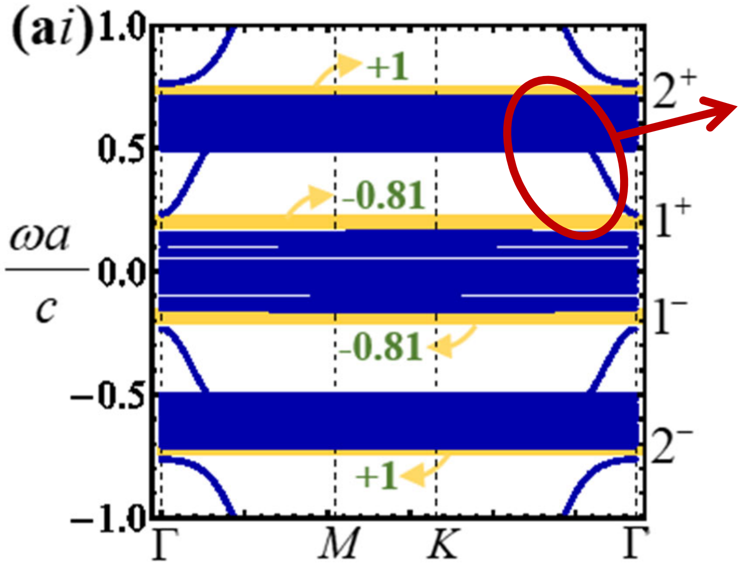
$$C_{\text{gap}} = 1 + (-1) + 1 + \ldots = ???$$

Origin of the ill-defined topology (2/3)



$$C_{\text{gap}} = \delta C_1 + \delta C_2 + \ldots = ???$$

Origin of the ill-defined topology (conclusion)



$$\delta C = C_{\text{gap},2+} - C_{\text{gap},1+} = 1.81$$

Regularization of the topology

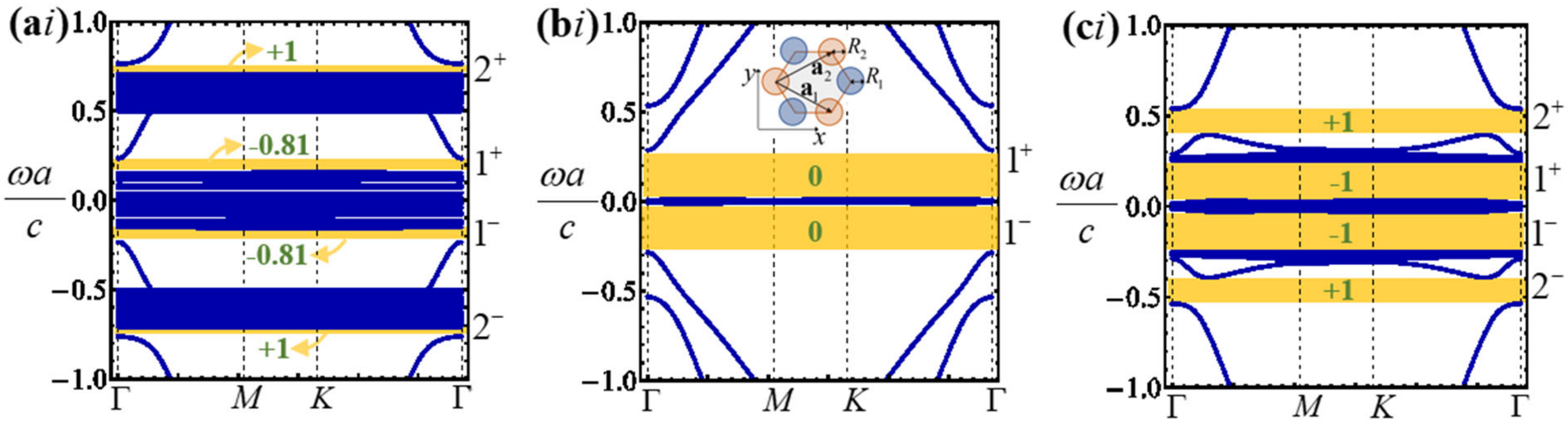
Spatial dispersion effects

I) Hydrodynamic model (takes into account the effects of diffusion which prevents localization)

II) Full cutoff model: high-spatial frequency material response is uniformly suppressed.

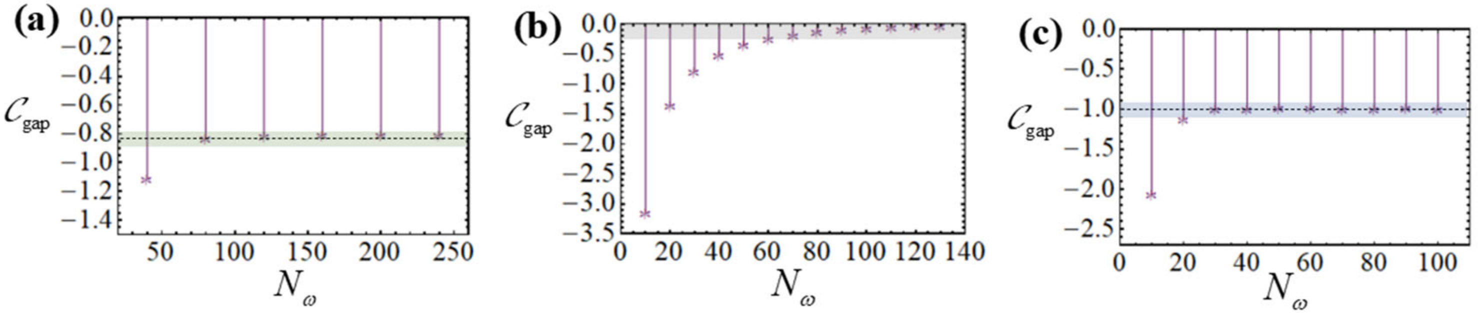
Band diagrams and topology

Local | **Hydrodynamic** | **Full cut-off**



Convergence of low-frequency gap Chern number

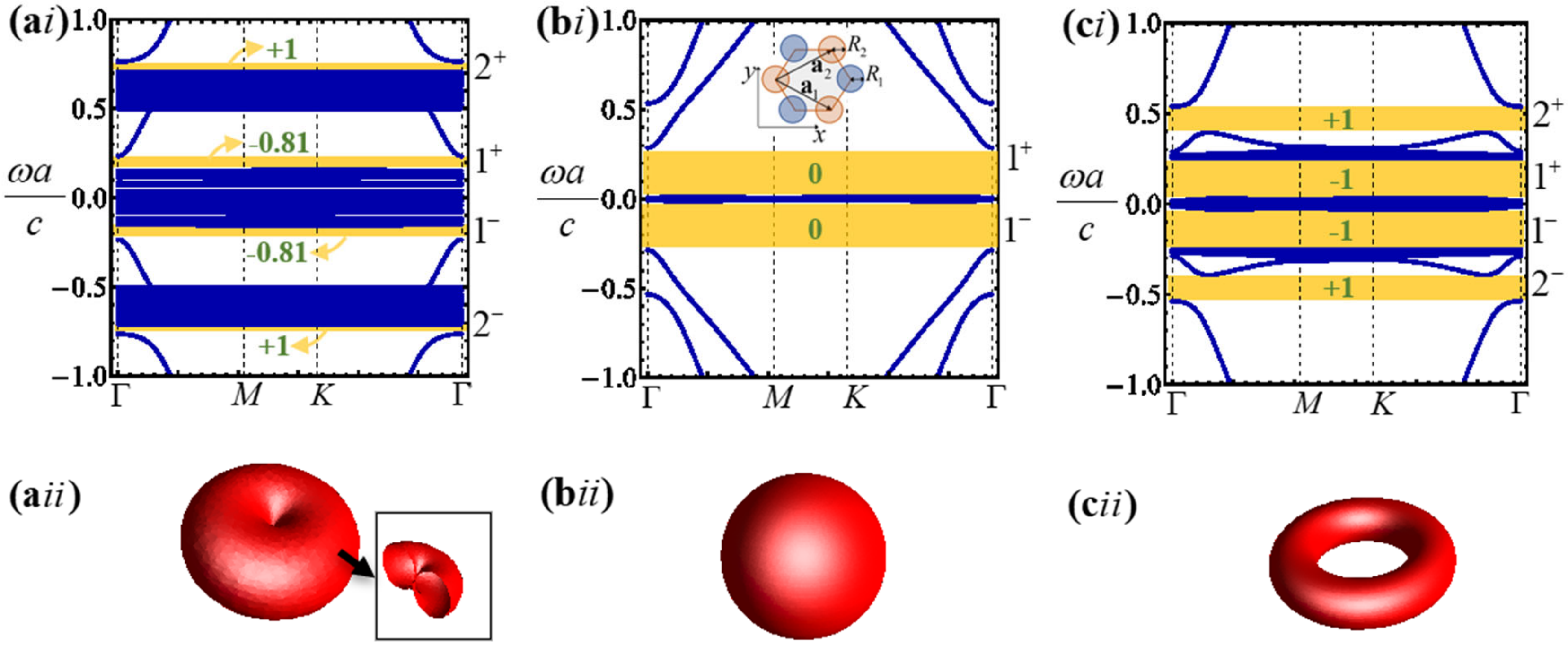
Local **Hydrodynamic** **Full cut-off**



Regularized topology depends on the considered cut-off!

Geometrical analogue

Local | **Hydrodynamic** | **Full cut-off**



Ill-defined topology

Well-defined topology with g=0

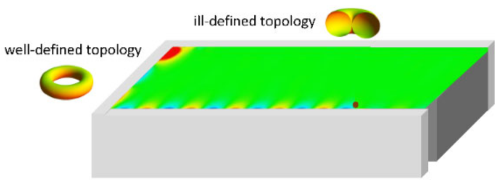
Well-defined topology with g=1

<u>**The crystal periodicity is insufficient to guarantee a well defined topology**</u>

Summary

- The topology of electromagnetic continua is typically ill-defined due to the continuous translational symmetry.

- The bulk-edge correspondence breaks down in systems with an ill-defined topology. This creates the opportunity to abruptly halt a wave and generate a topological singularity that dissipates all the incoming energy essentially at a single point of space.



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D. E. Fernandes et al., "Experimental verification of ill-defined topologies and energy sinks in electromagnetic continua," Adv. Photon. 4(3) 036002 (2022), doi 10.1117/1.AP.4.3.036002.