

The OSA Fiber Modeling and Fabrication Technical Group Welcomes You



EVERYTHING YOU ALWAYS WANTED
TO KNOW ABOUT SUPERCONTINUUM
MODELLING IN OPTICAL FIBERS

26 August 2019 • 8:00 a.m. ET



OSA Fiber Modeling
and Fabrication
Technical Group

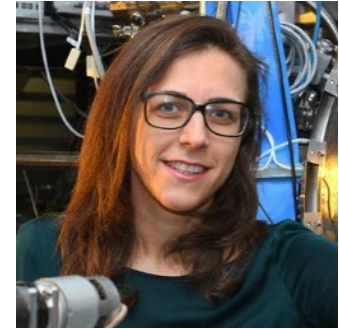
Technical Group Leadership 2019



Giulio Vampa
Chair
Stanford PULSE Institute



Eric Cunningham
SLAC National Accelerator
Laboratory



Hanieh Fattahi
Max-Planck-Institute
of Quantumoptics



Benjamin Webb
Laboratory for Laser Energetics,
University Of Rochester



Zhiyi Wei
Institute of Physics,
Chinese Academy of Sciences

Technical Group at a Glance

- Focus

- Development and application of high intensity lasers as well as novel XUV and x-ray sources
- The physics of high intensity light interactions with matter
- Short wavelength sources including insertion devices for storage rings (undulators and wigglers), plasma X-ray lasers, electron beam based sources and X-ray free electron lasers.

- Mission

- To benefit *YOU* and to strengthen *OUR* community
- Webinars, podcasts, publications, technical events, business events, outreach
- Interested in presenting your research? Have ideas for TG events? Contact us at TGactivities@osa.org.

- Find us here

- Website: www.osa.org/OH
- Facebook: www.facebook.com/OSAShortWavelengthTG
- LinkedIn: www.linkedin.com/groups/8356401

Today's Webinar



Reaching for the brightest light at SLAC's FACET-II

Dr. Sebastian Meuren

Postdoctoral Researcher and PI of a strong-field QED experimental campaign at SLAC's FACET-II
Princeton University, USA

smeuren@pppl.gov

Speaker's Short Bio:

Ph.D. degree from Heidelberg University/Max Planck Institute for Nuclear Physics in 2015. Otto Hahn Medal from the Max Planck Society. Currently a postdoctoral researcher in the department of Astrophysical Sciences at Princeton University.



**PRINCETON
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Everything you always wanted to know about supercontinuum modelling in optical fibers

Alexander Heidt

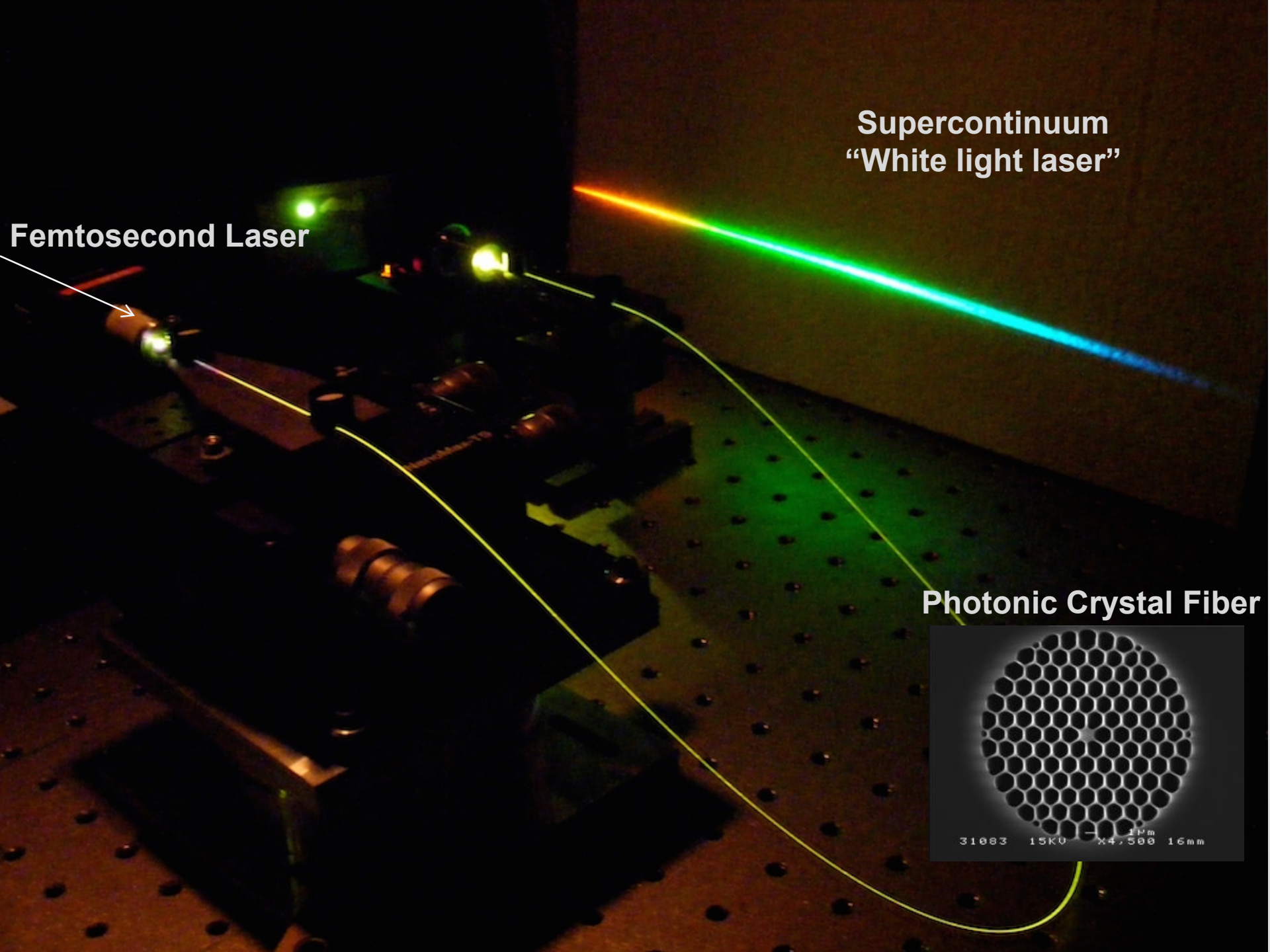
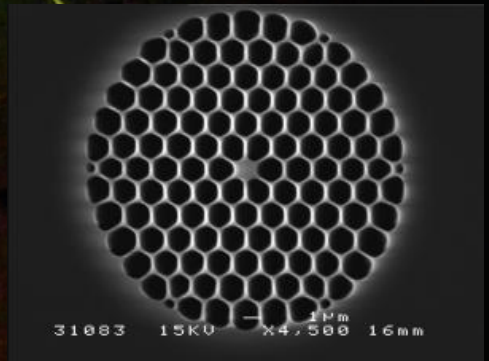
Institute of Applied Physics
University of Bern
Switzerland

26.08.2019, OSA Webinar Series

Femtosecond Laser

Supercontinuum
"White light laser"

Photonic Crystal Fiber



What you will learn

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- **Concepts of Nonlinear fiber optics**
 - the power of dispersion engineering
- A short overview of the most important **nonlinear effects** occurring during supercontinuum generation in optical fibers
- Describing the physical effects mathematically in the **Generalized Nonlinear Schrödinger equation (GNLSE)**
- **Solving the GNLSE** in an efficient way
- **Noise effects in supercontinuum generation** and how to include them into your simulation

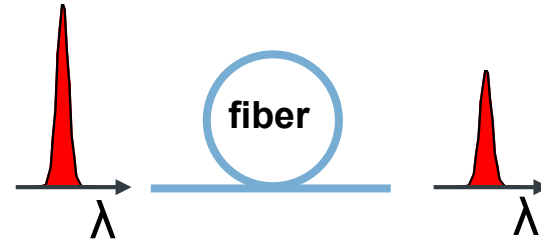
Nonlinear optics – the tailor shop for light

Propagation of electromagnetic fields through fibers

linear



- Dispersion
- Absorption

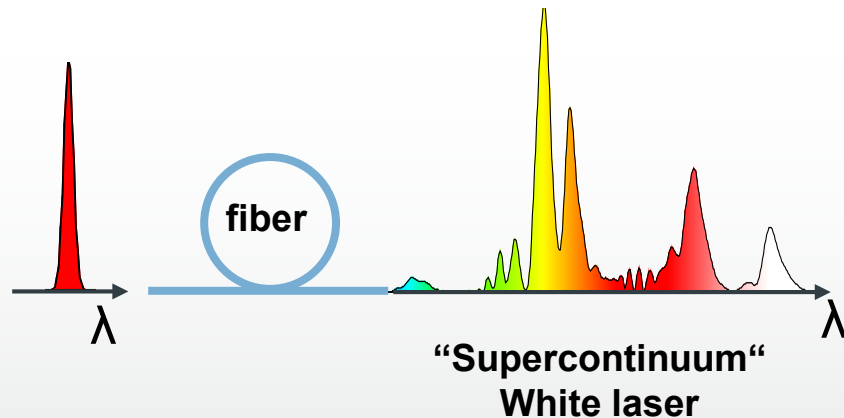


nonlinear



- Four wave mixing
- Self phase modulation (SPM)
- Cross phase modulation (XPM)
- Modulation instability
- Raman scattering
- Brillouin scattering
- ...

intensity dependent!



Understand & Control
Nonlinearity



Transform laser light to
application demands

What intensities are we talking about?



“Everyday life intensities”

Intensity of sunlight on earth:

$$1000 \text{ W/m}^2 = 0.1 \text{ W/cm}^2$$

When focused with a magnifying glass:

$$\sim 100 \text{ W/cm}^2 \text{ (max)}$$

Intensities in nonlinear fiber optics:

Femtosecond laser pulse (100 fs, 10 nJ)
focused into 2 μm core diam. fiber:

$$\sim 1\,000\,000\,000\,000 \text{ W/cm}^2$$

$$= 1 \text{ TW/cm}^2$$

**10 orders of magnitude higher
than everyday life intensities!**

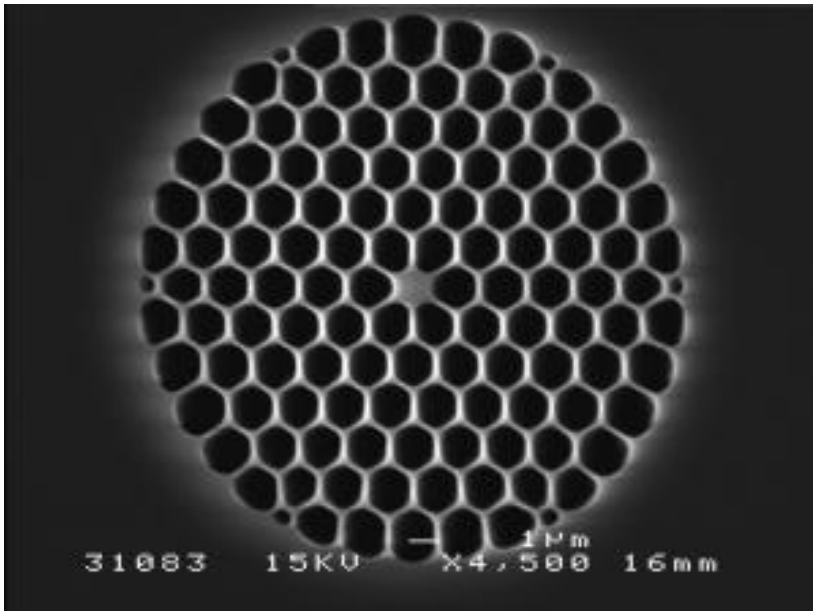
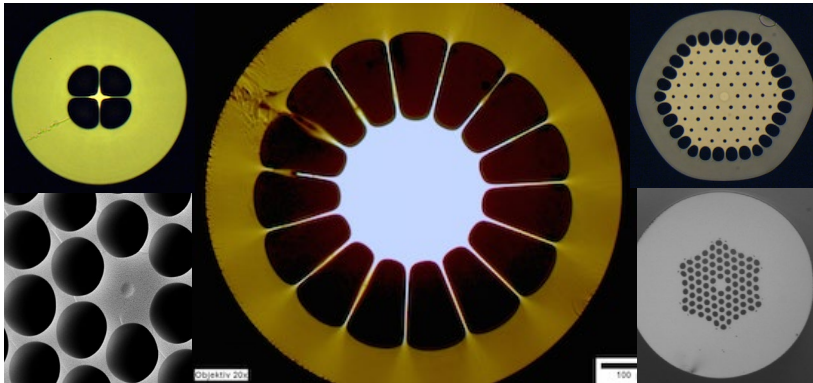


Fibers for nonlinear optics

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Revolution: photonic crystal fibers (PCF)

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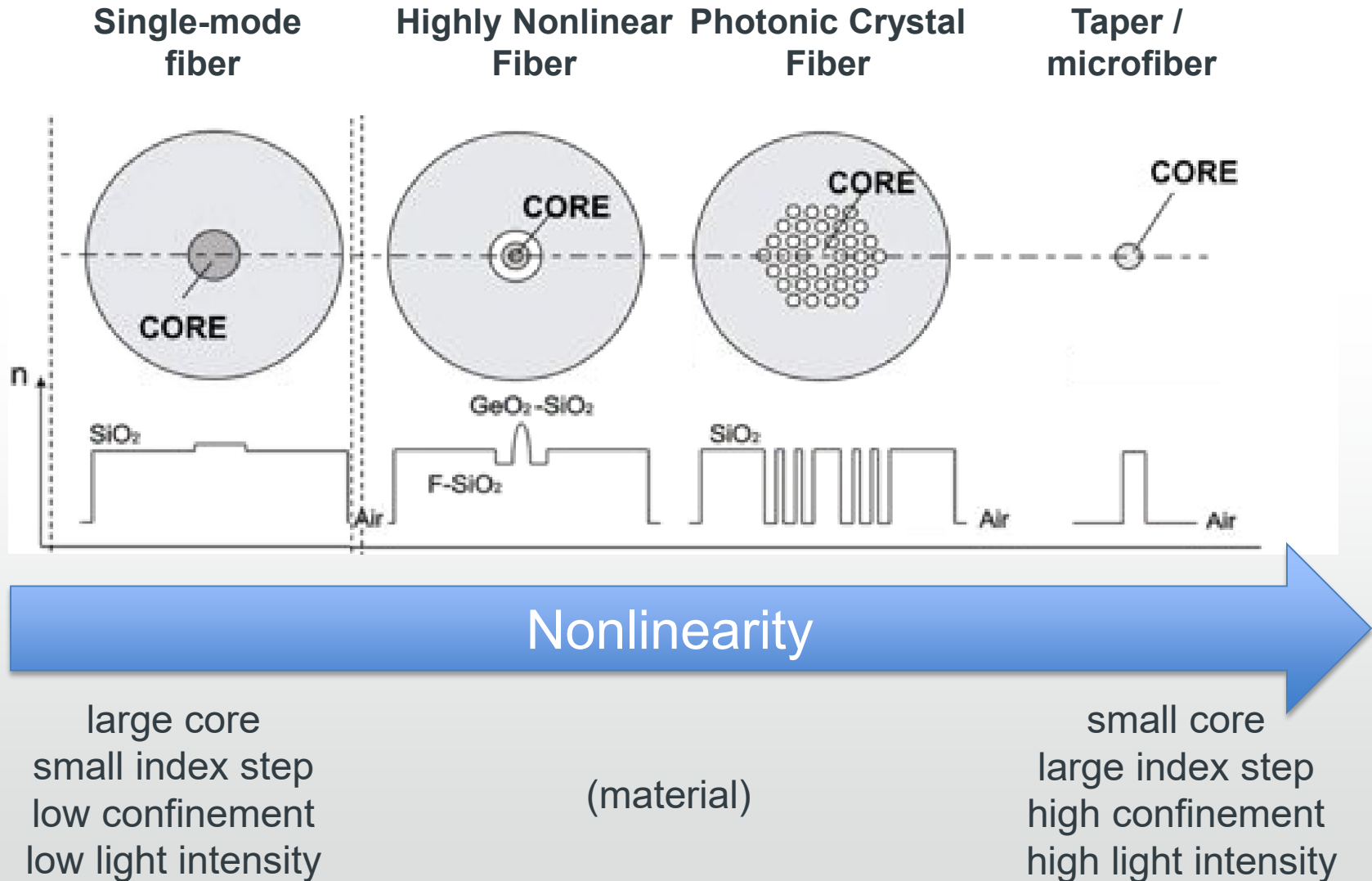
- Silica core
- Cladding of air-holes
- Design allows to “squeeze” the light into tiny cores ($\sim 1 \mu\text{m}$)
 - ➔ very high intensities!
 - ➔ very high nonlinearity!
- Most important:

We can control nonlinear effects with the geometry of the fiber

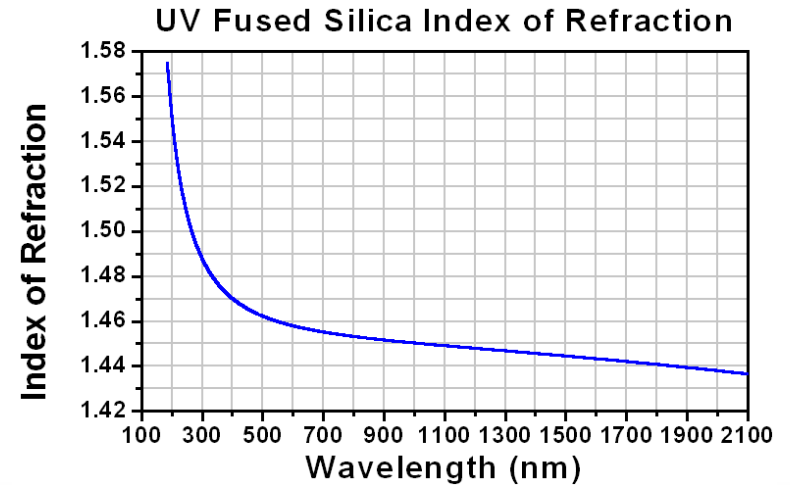
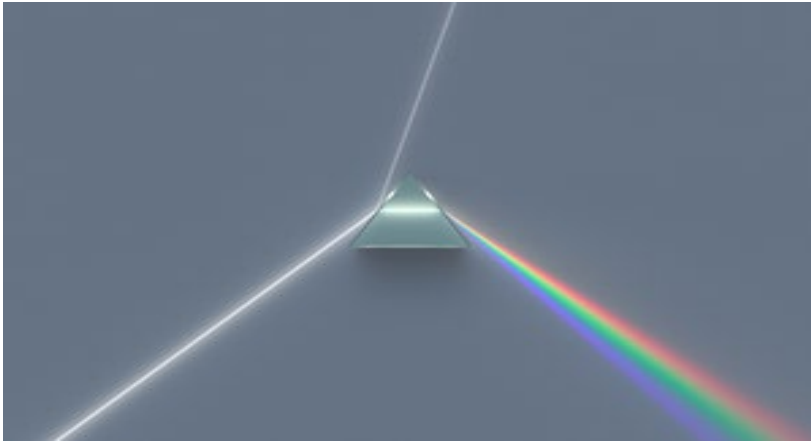
This design flexibility only exists in fibers and waveguides!

Engineering the nonlinearity

Refractive index profile of specialty fibers



Dispersion



- Refractive index of materials depends on the wavelength
- A laser pulse propagates in a medium with the group velocity

$$v_g = c \left(n - \lambda \frac{dn}{d\lambda} \right)^{-1}$$

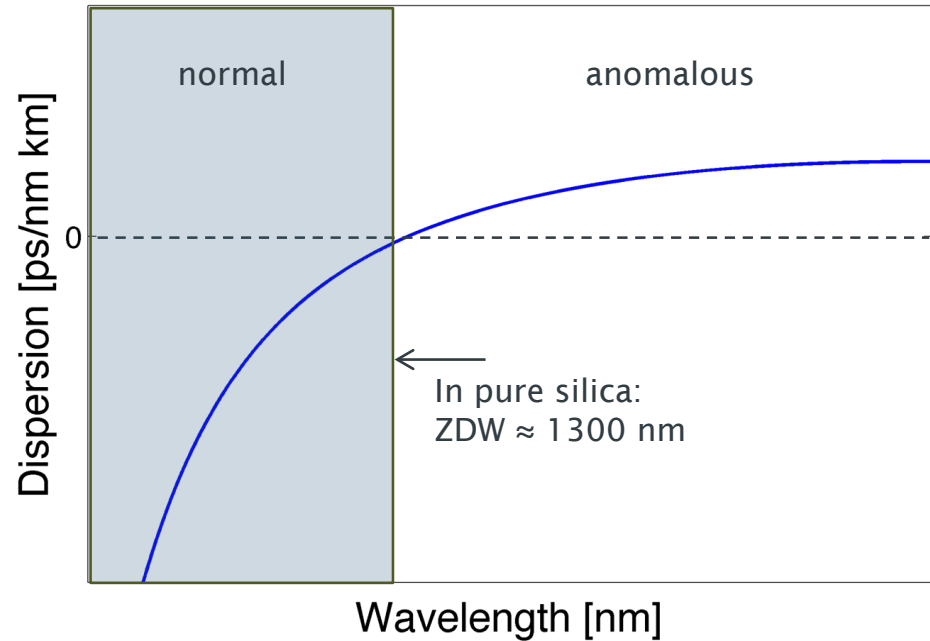
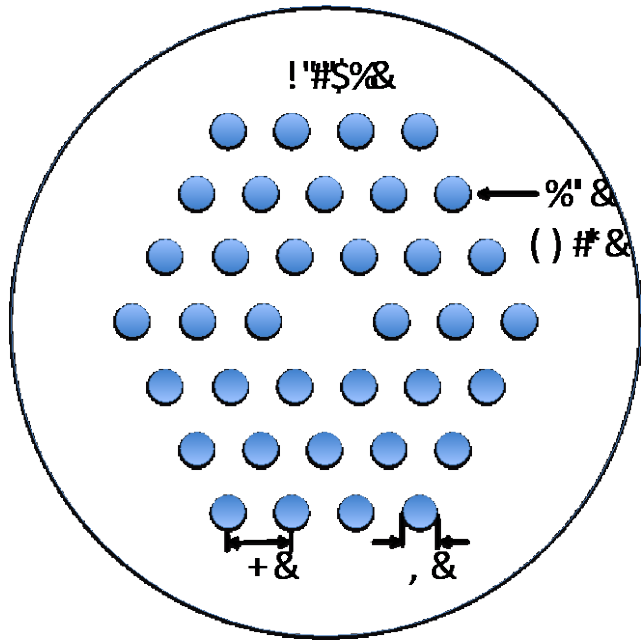
- The group velocity itself depends on the wavelength, i.e. there exists a Group Velocity Dispersion (GVD)

$$D = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2}$$

D < 0: “Normal” dispersion: red faster than blue
D > 0: “Anomalous” dispersion: blue faster than red

Dispersion engineering

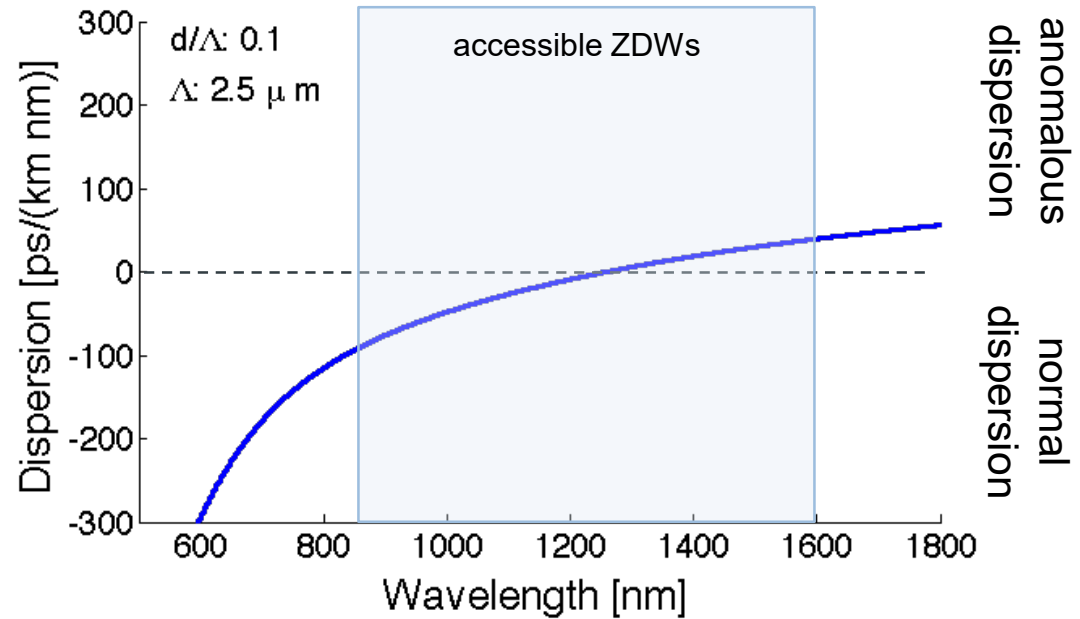
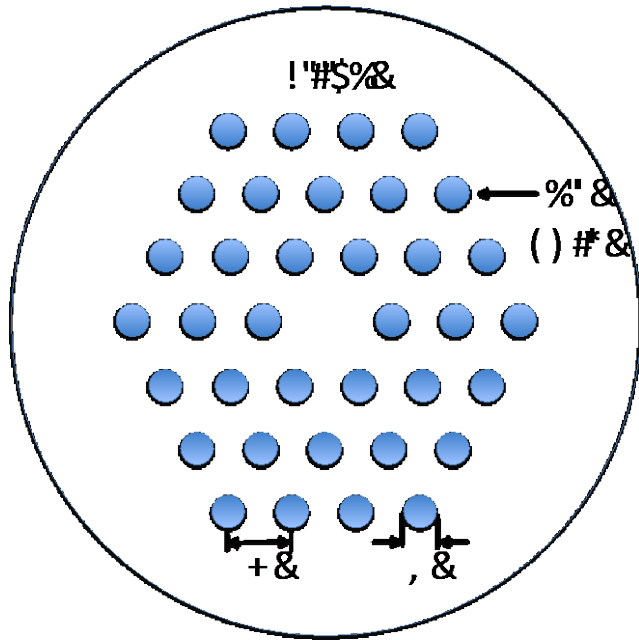
Most powerful tool in fiber optics



- > Dispersion is highly sensitive to the geometry of the air hole cladding

Dispersion engineering

Most powerful tool in fiber optics



> Dispersion is highly sensitive to the geometry of the air hole cladding

Control nonlinear effects

by tailoring the **dispersion profile**

by designing the **geometry of the fiber**

Nonlinear effects and dispersion

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Most powerful tool in fiber optics

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Occurrence of nonlinear effects

	Anomalous dispersion	Normal dispersion
Self-phase modulation	✓	✓
Soliton dynamics	✓	✗
Raman effect	✓	✓
Modulation instability/ Phase-matched 4-wave- mixing	✓	✗

**Control of nonlinear effects and their interaction
by tailoring the dispersion!**

Nonlinear effects in optical fibers

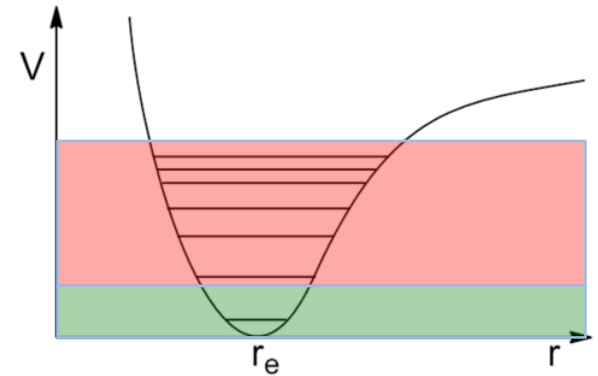
- Pulse propagation obeys

$$\nabla \times \nabla \times E(r,t) + \frac{1}{c^2} \frac{\partial^2 E(r,t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P(r,t)}{\partial t^2}$$

with

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{E}\mathbf{E} + \chi^{(3)} : \mathbf{E}\mathbf{E}\mathbf{E} + \dots \right)$$

Dispersion
Absorption **Nonlinearities**



- $\chi^{(2)} = 0$ in silica! (centro-symmetric material)
(no second-harmonic generation, sum frequency generation, etc)
- Only THIRD ORDER nonlinear effects in optical fibers!

Constructing a pulse propagation equation

- General Maxwell's equations are 4-dimensional
 - computationally expensive to solve
 - How can we adapt them to nonlinear fiber optics?

Assumptions:

Input pulse

- **linearly polarized** along x-axis,
- carrier frequency ω_0

Fiber

- **single mode**
- **polarization maintaining**
- propagation along z-axis

Ansatz:

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{\hat{x}F(x, y, \omega)}_{\text{fiber mode profile}} \underbrace{\tilde{A}(0, \omega)}_{\text{spectral envelope}} \underbrace{e^{i[\beta(\omega)z - \omega t]}}_{\text{phase shift}} d\omega$$

Constructing a pulse propagation equation

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{\hat{x}F(x, y, \omega)}_{\text{fiber mode profile}} \underbrace{\tilde{A}(0, \omega)}_{\text{spectral envelope}} \underbrace{e^{i[\beta(\omega)z - \omega t]}}_{\text{phase shift}} d\omega$$

Some simplifications:

- **mode profile / effective mode area independent of frequency:**

$$F(x, y, \omega) \rightarrow F(x, y, \omega_0) \quad A_{\text{eff}}(\omega_0) = \frac{\left(\iint_{-\infty}^{\infty} |F(x, y, \omega_0)|^2 dx dy \right)^2}{\iint_{-\infty}^{\infty} |F(x, y, \omega_0)|^4 dx dy}$$

- **Taylor expansion of propagation constant:**

$$\beta(\omega) = \beta_0 + \underbrace{(\omega - \omega_0)\beta_1}_{\beta_1 = v_g^{-1}} + \frac{1}{2} \underbrace{(\omega - \omega_0)^2 \beta_2}_{\beta_2 > 0 : \text{normal GVD}} + \dots \quad \beta_m = (d^m \beta / d\omega^m)_{\omega=\omega_0}$$

$\beta_2 < 0 : \text{anomalous GVD}$

- **Transform to reference frame co-moving with the input pulse**

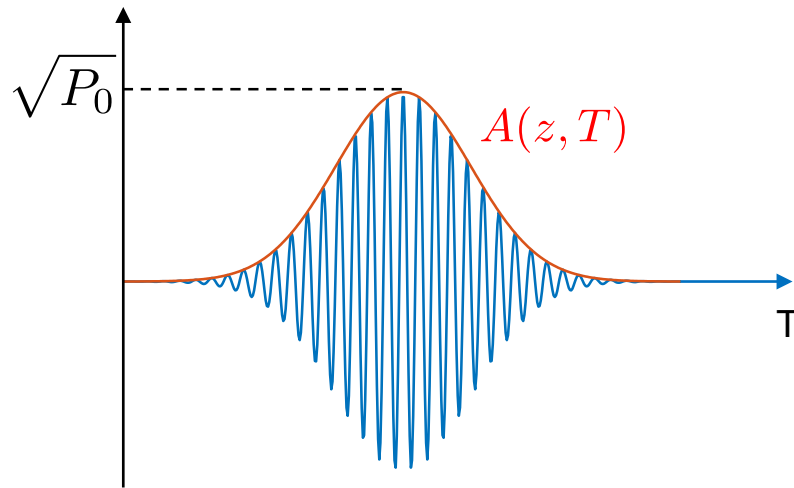
$$T = t - \beta_1 z$$

→ in practice we set $\beta_0 = \beta_1 = 0$

Constructing a pulse propagation equation

$$\mathbf{E}(\mathbf{r}, T) = \hat{x} F(x, y, \omega_0) \underbrace{A(z, T)}_{\text{slowly varying envelope}} \underbrace{e^{i\omega_0 T}}_{\text{rapid oscillations}}$$

slowly varying envelope rapid oscillations



A is normalized such that $|A(z, T)|^2$ yields instantaneous power in Watts.

Example for Gaussian input pulse:

$$A(z = 0, T) = \sqrt{P_0} e^{-(T^2/T_0^2)}$$

Now: plug ansatz into Maxwell's equations to derive equation for $A(z, T)$

→ details: e.g. Agrawal, Nonlinear Fiber Optics (Academic Press, 2013)

Generalized Nonlinear Schrödinger Equation

$$\frac{\partial A(z, T)}{\partial z} = (\hat{D} + \hat{N}) A(z, T)$$

Remarkably simple 1+1-dimensional partial differential equation

Dispersive operator:

$$\hat{D} = \underbrace{-\frac{\alpha}{2}}_{\text{absorption}} - \underbrace{\sum_{n \geq 2} \beta_n \frac{i^{n-1}}{n!} \frac{\partial^n}{\partial T^n}}_{\text{dispersion}}$$

Nonlinear operator:

$$\hat{N}A(z, T) = i\gamma \left(\underbrace{1 + i\tau_{\text{shock}} \frac{\partial}{\partial T}}_{\text{dispersion of nonlinearity}} \right) \times \underbrace{\left[A(z, T) \int_{-\infty}^{\infty} R(t') |A(z, T - t')|^2 dt' \right]}_{\text{SPM, FWM, Raman}}$$

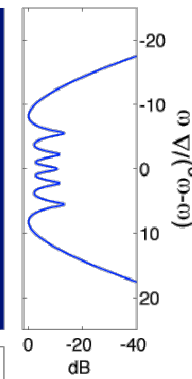
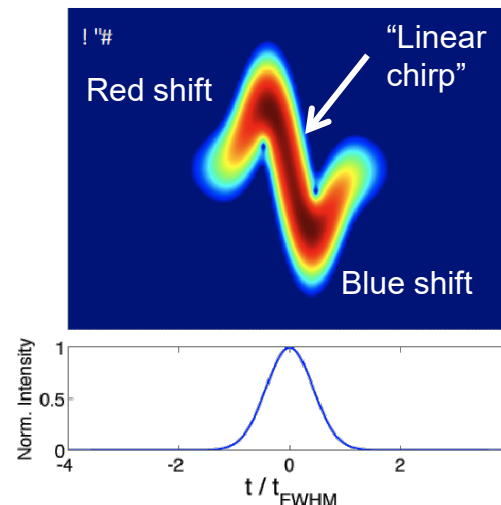
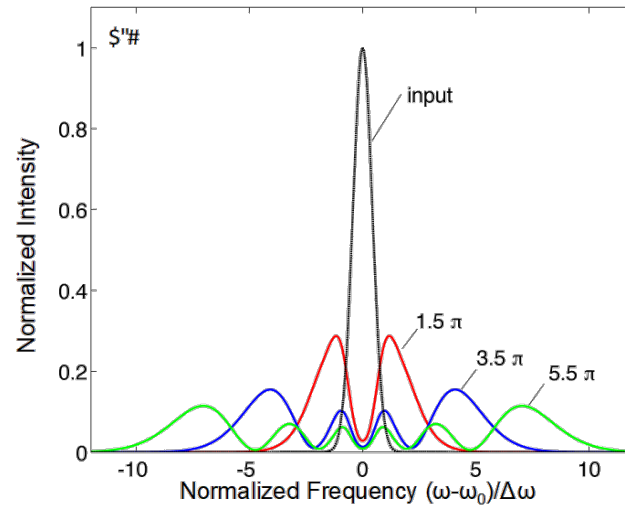
Material response function: $R(t) = \underbrace{(1 - f_R)\delta(t)}_{\text{instantaneous (SPM, FWM)}} + \underbrace{f_R h_R(t)}_{\text{delayed (Raman)}}$ $f_R \sim 0.2$ (silica)
fractional contribution of delayed response

Nonlinear parameter: $\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}(\omega_0)}$ $n_2 \sim 3 \times 10^{-20} \text{ m}^2/\text{W}$ (silica)
nonlinear refractive index

Self-phase modulation (SPM)

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$$\begin{aligned} \hat{D} &= 0 \\ \hat{N} &= i\gamma|A|^2 \\ (f_R &= 0) \end{aligned}$$

- Pulse itself changes the refractive index of the medium (Kerr effect):

$$n = n_0 + n_2 I(t) = n_0 + n_2 (P(t)/A_{eff})$$

- New frequency components are created with a time correlation:

$$\omega(z, t) = \omega_0 - \gamma z \partial P(t) / \partial t$$

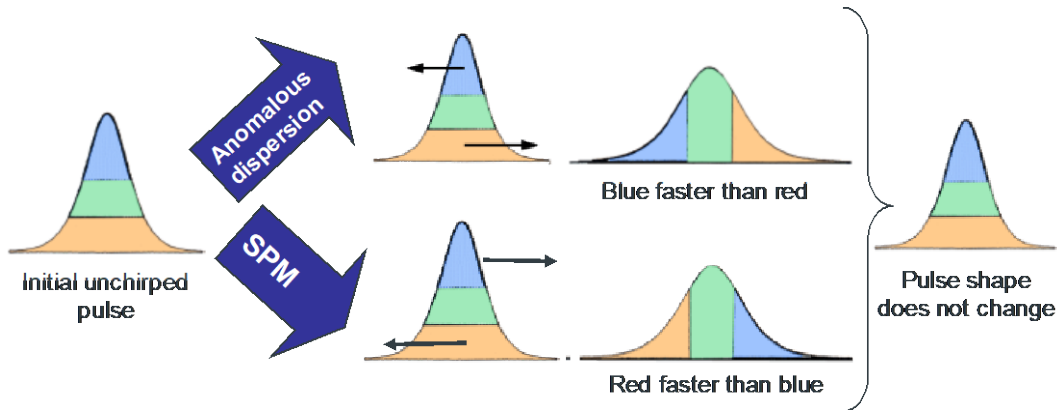
Spectral broadening
Multi-peak structure in the spectrum
“Chirped” pulse

Soliton dynamics

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Balance of SPM and anomalous dispersion

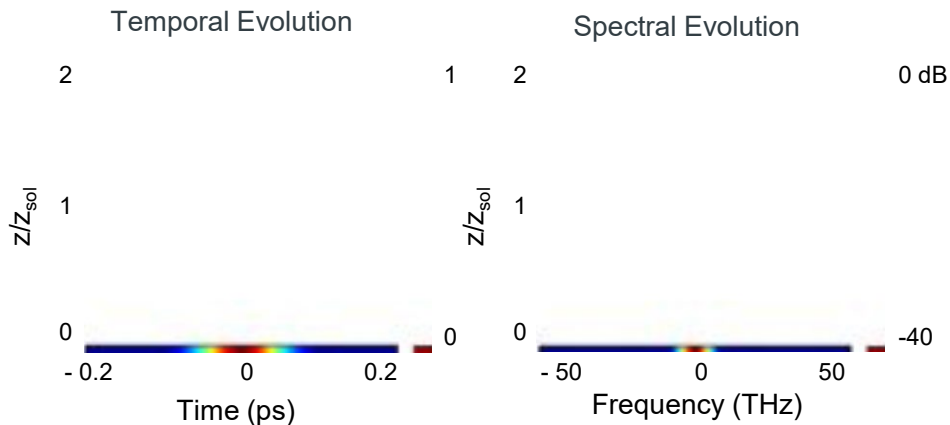


$$\hat{D} = -i \frac{\beta_2}{2} \frac{d^2}{dT^2}$$

$$\beta_2 < 0$$

$$\hat{N} = i\gamma |A|^2$$

- Fundamental soliton: invariant upon propagation (except constant nonlinear phase shift)



Requirements:

$$A(0, T) = \sqrt{P_0} \operatorname{sech}(T/T_0); \quad N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} = 1$$

soliton number

Solution:

$$A(z, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) e^{ik_{\text{sol}}z}; \quad k_{\text{sol}} = \gamma P_0 / 2;$$

Soliton dynamics

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Higher order solitons

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- Higher order solitons are periodic upon propagation:

$$A(z + z_{\text{sol}}, T) = A(z, T)$$

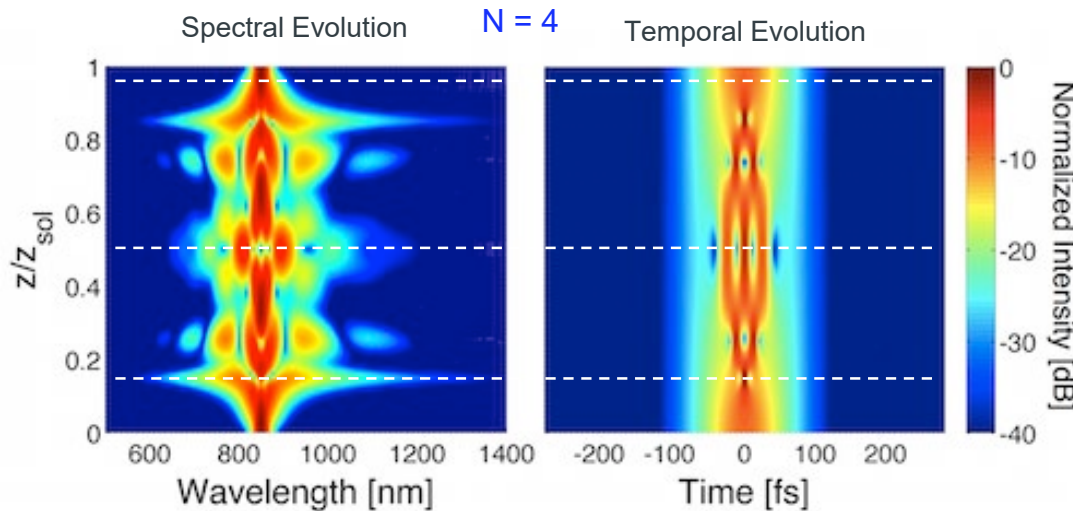
$$\hat{D} = -i \frac{\beta_2}{2} \frac{d^2}{dT^2}$$

$$\beta_2 < 0$$

$$\hat{N} = i\gamma|A|^2$$

Requirements:

$$A(0, T) = \sqrt{P_0} \operatorname{sech}(T/T_0); \quad N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} = 2, 3, 4, \dots \text{ quantized!} \quad z_{\text{sol}} = \frac{\pi T_0^2}{2 \beta_2}$$

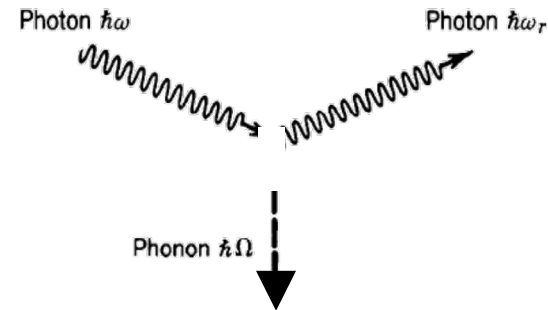
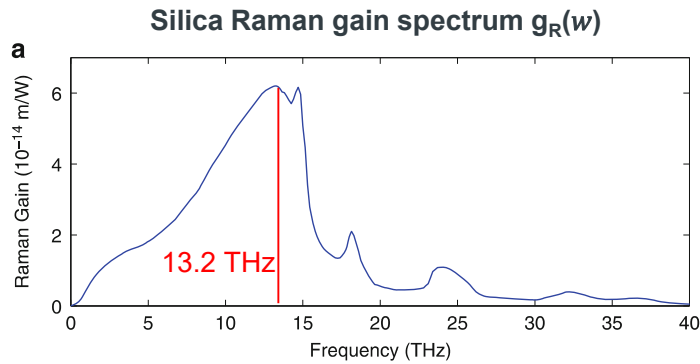


Back to original state

Multiple pulses

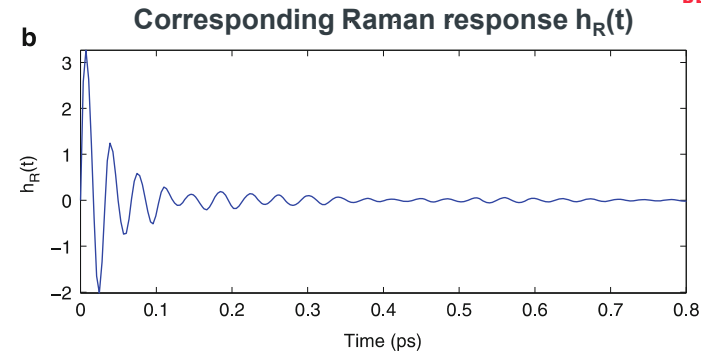
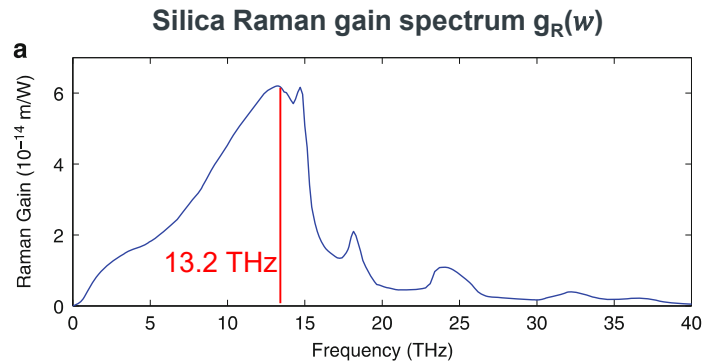
Extreme compression and spectral broadening

Stimulated Raman scattering



- Quantum mechanical picture: photon loses energy to phonons excited in the material
- Classical picture: amplification of “Stokes” wave red-shifted from the pump

Stimulated Raman scattering



- Quantum mechanical picture: photon loses energy to phonons excited in the material
- Classical picture: amplification of “Stokes” wave red-shifted from the pump
- GNLSE includes material response:

$$R(t) = (1 - f_R)\delta(t) + \underline{f_R h_R(t)}?$$

- use analytical approximation for $h_R(t)$ developed in literature

→ e.g. Lin & Agrawal, Opt. Lett. 31, 3086 (2006)

- alternative: use measured Raman spectrum $g_R(\omega) \propto \text{Im}(\tilde{h}_R(\omega))$ and Kramers-Kronig to determine real part

Soliton self-frequency shift and dispersive waves

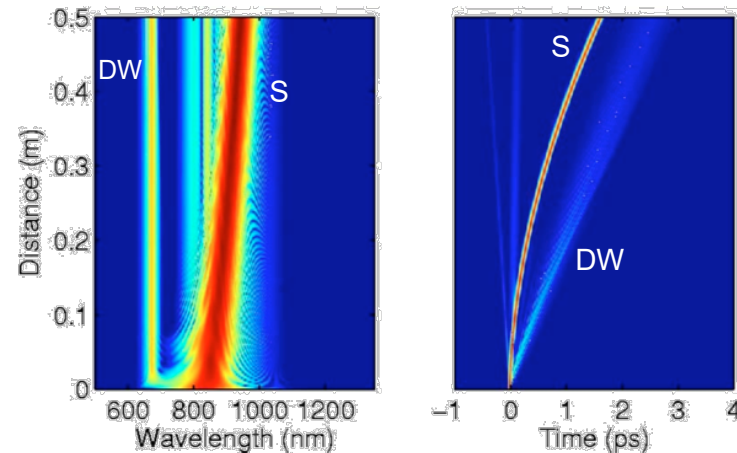
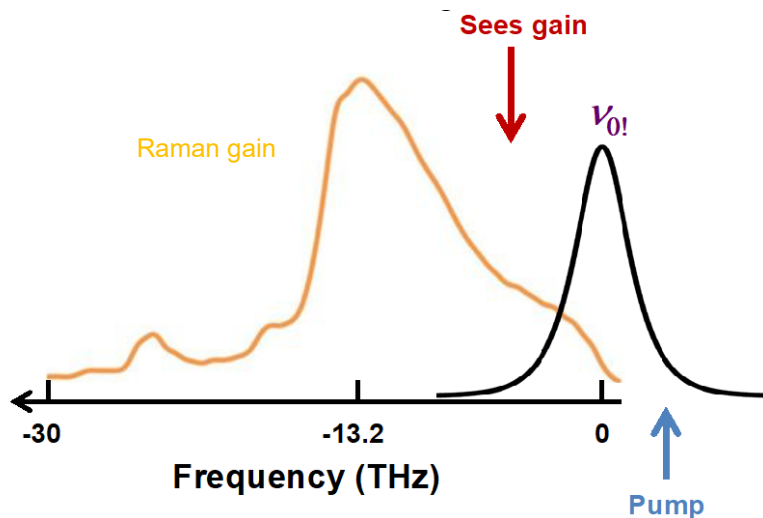
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Perturbations of $N = 1$ solitons

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Ideal soliton propagation is disturbed by presence of Raman scattering and higher order dispersion:

Full GNLSSE



Soliton self-frequency shift:

- continuous **spectral red-shift**
- **Soliton slows down** due to lower group velocity at longer wavelengths

Higher order dispersion:

- soliton sheds energy to a **dispersive wave** in the normal dispersion regime
- position determined by phase matching condition

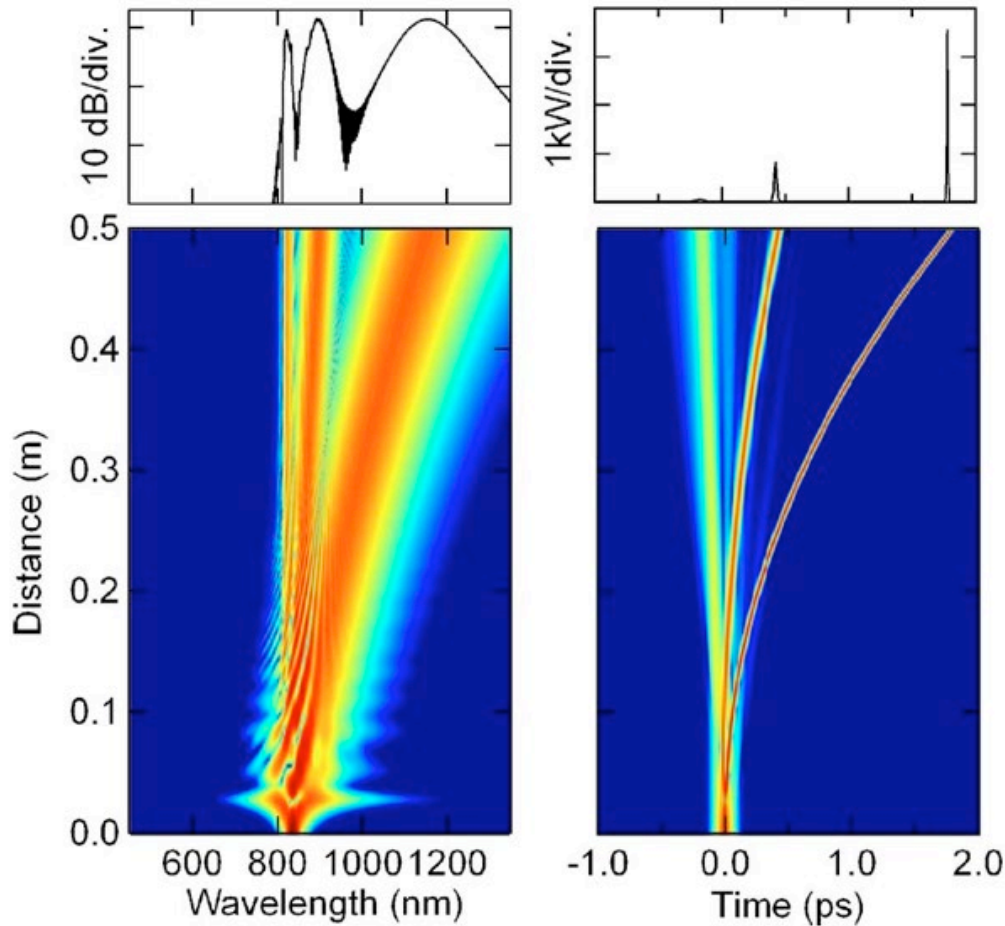
Soliton fission

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Perturbations of higher order solitons

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Full GNLSSE

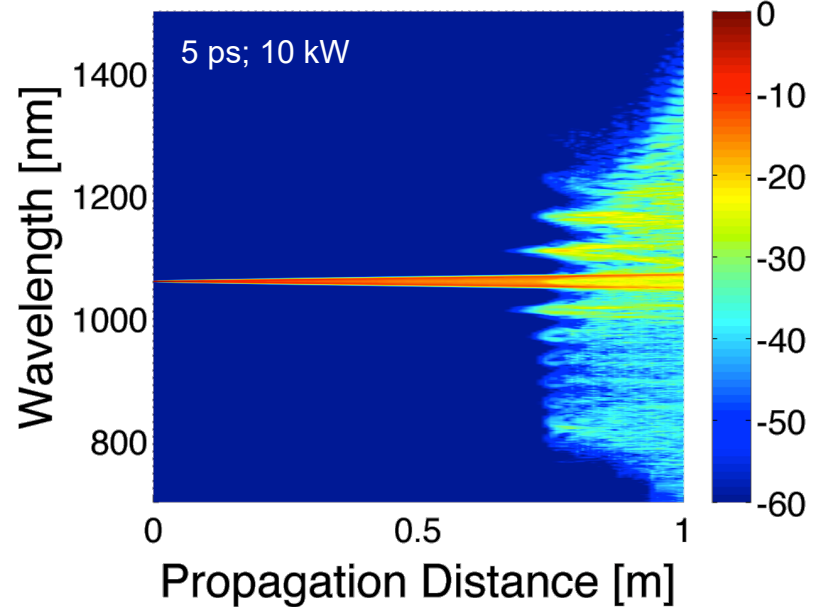
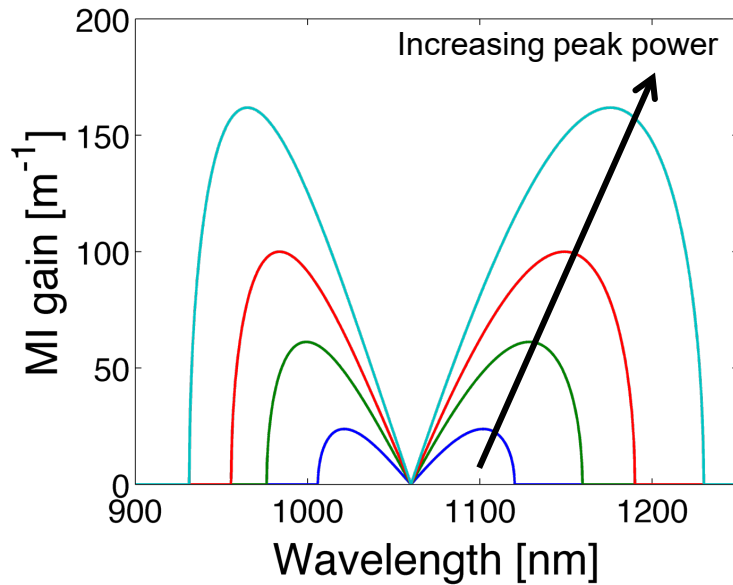


- Higher order soliton propagation disturbed by Raman scattering and higher order dispersion
- Break up into fundamental solitons (here: $N = 3$)
- Break up at the point of strongest temporal compression
- Pulses separate in time and spectrum due to Raman scattering

Modulational instability

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Full GNLSSE

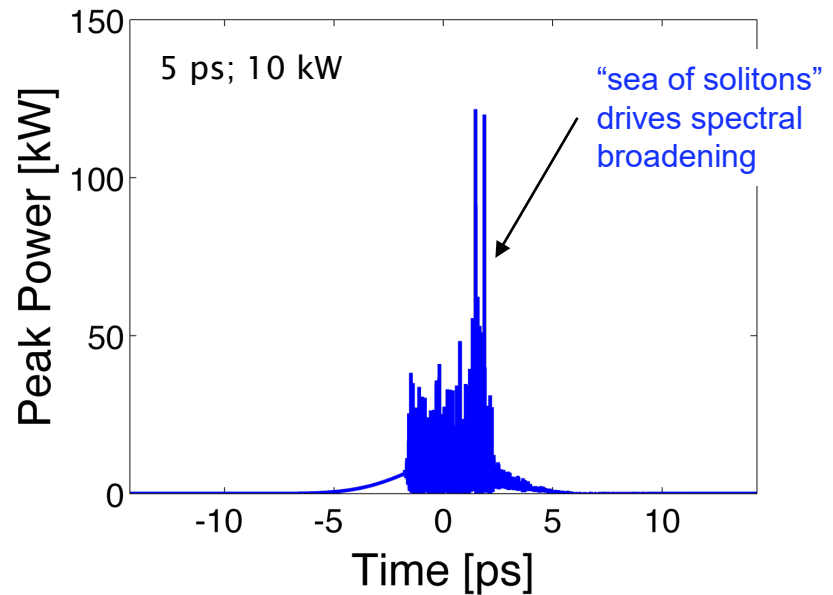
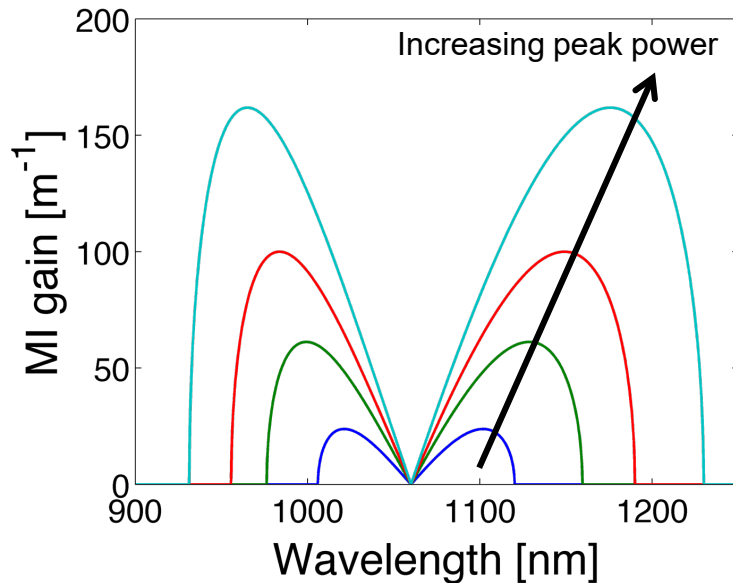


- Occurs mainly in anomalous dispersion regime
- 2 pump photons annihilated, create 1 photon in each side band
- Side band position: energy / momentum conservation \rightarrow dispersion!
- If unseeded: shot noise amplification!

Modulational instability

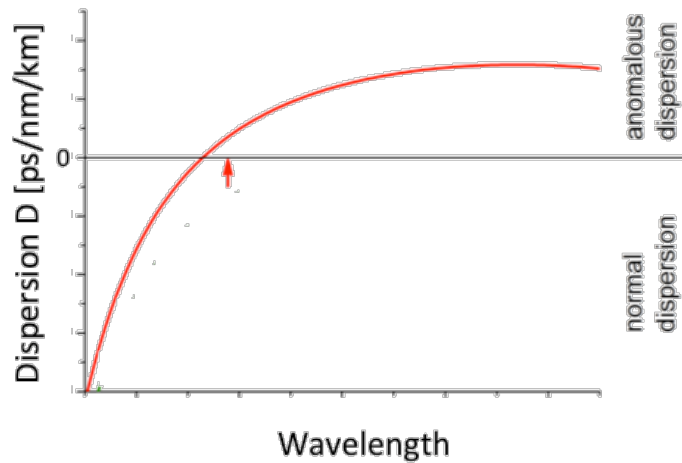
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Full GNLSSE



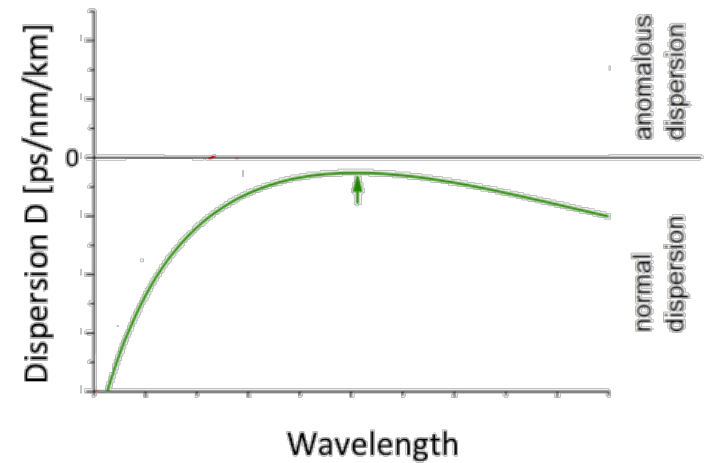
- Occurs mainly in anomalous dispersion regime
- 2 pump photons annihilated, create 1 photon in each side band
- Side band position: energy / momentum conservation \rightarrow dispersion!
- If unseeded: shot noise amplification!

Conventional design



- > both normal and anomalous dispersion regions
- > pumping in **anomalous dispersion** close to zero dispersion wavelength (ZDW)
- > designed to maximize spectral bandwidth
- > soliton dynamics and phase-matched 4-wave mixing play dominant role

All-normal dispersion (ANDi) design



- > **normal dispersion** at all wavelengths
- > pumping close to the minimum dispersion wavelength (MDW)
- > designed for low-noise performance
- > soliton dynamics and phase-matched 4-wave mixing completely suppressed
- > SPM and “optical wave-breaking” play dominant role

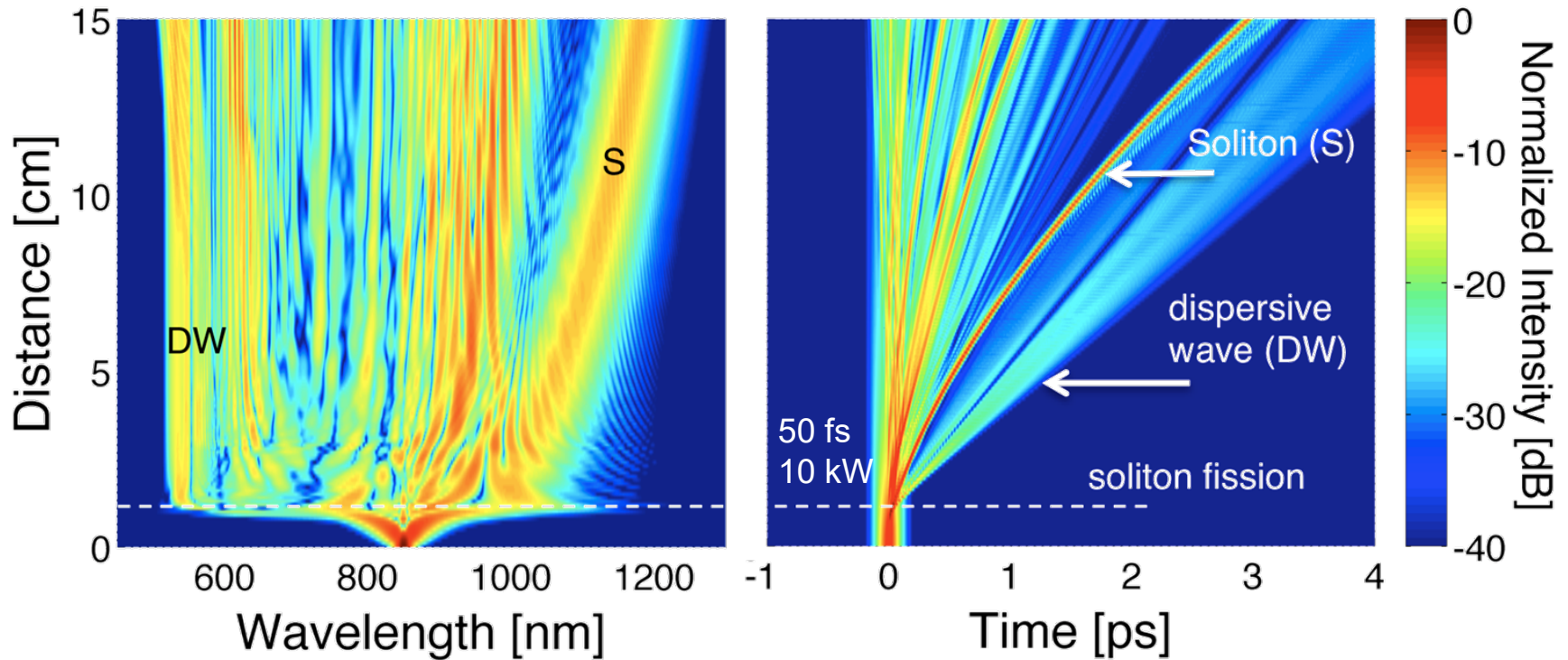
“Conventional” supercontinuum generation

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Nonlinear dynamics

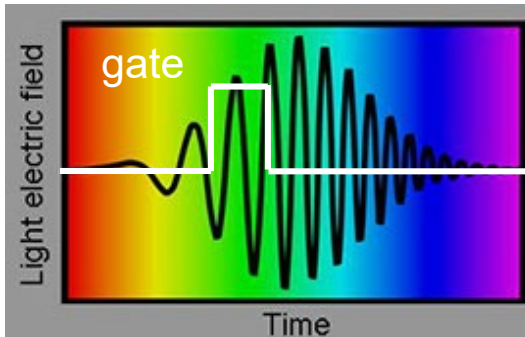
Pump pulse: soliton number $N \approx 6$

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Analysis of simulation results

Time-frequency analysis

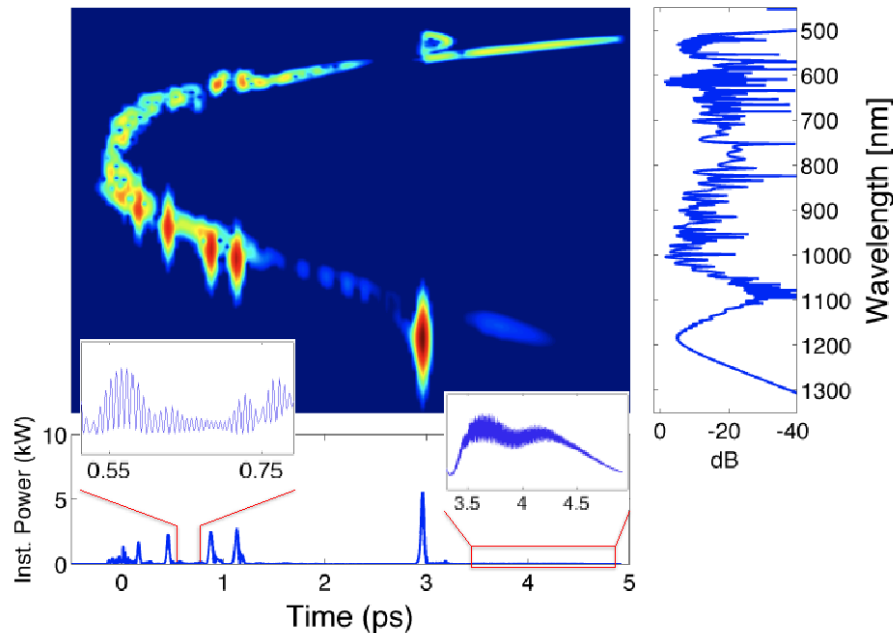


$$\Sigma_g^A(\omega, \tau) = \left| \int_{-\infty}^{\infty} A(t) g(t - \tau) \exp(i\omega t) dt \right|^2$$

simulated field

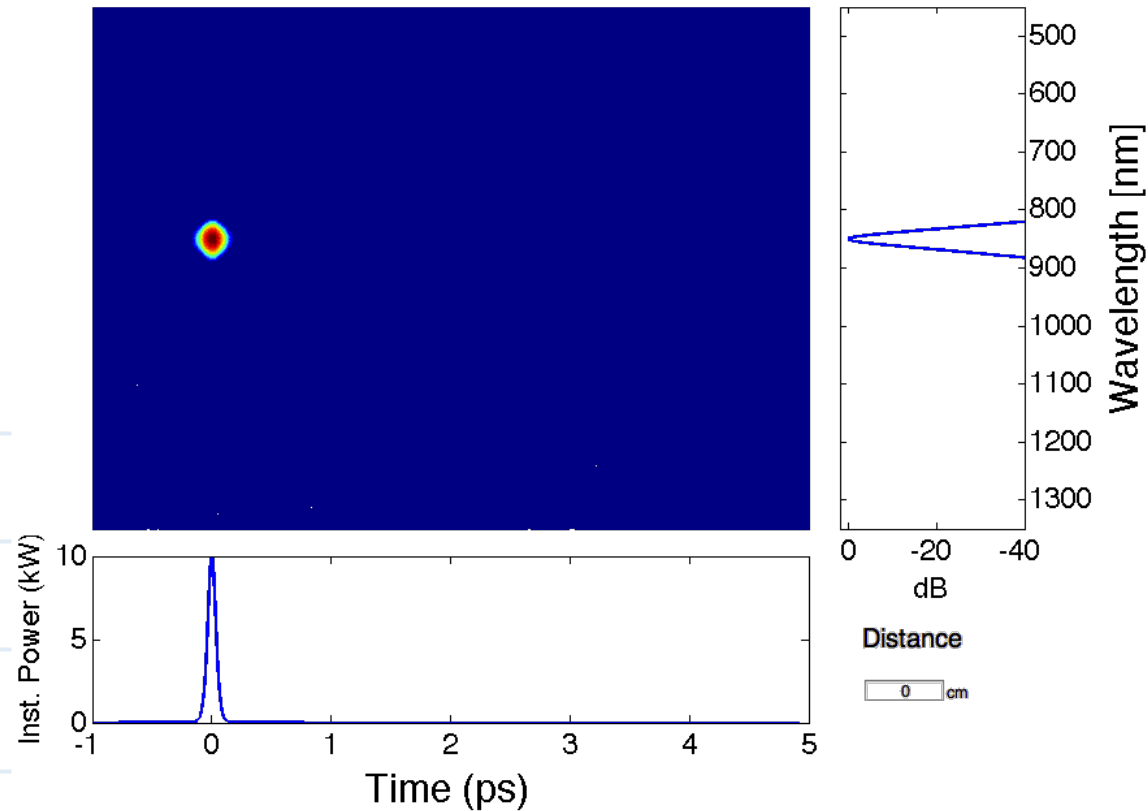
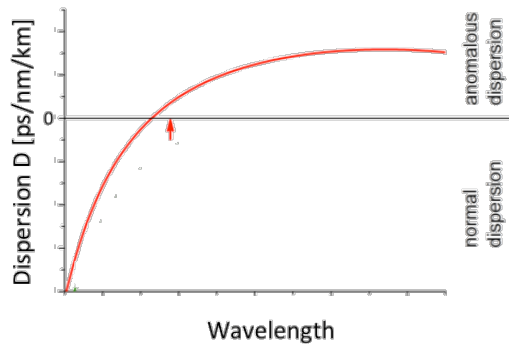
gate pulse

(e.g. input pulse for your simulation)



Full dynamics of continuum generation

Anomalous dispersion pumping



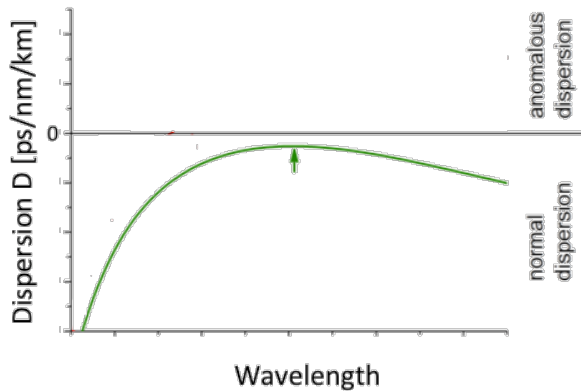
- + low required peak power
- pulse break-up
- complex profiles, fine structure
- can be affected by quantum noise (modulation instability)

Full dynamics of continuum generation

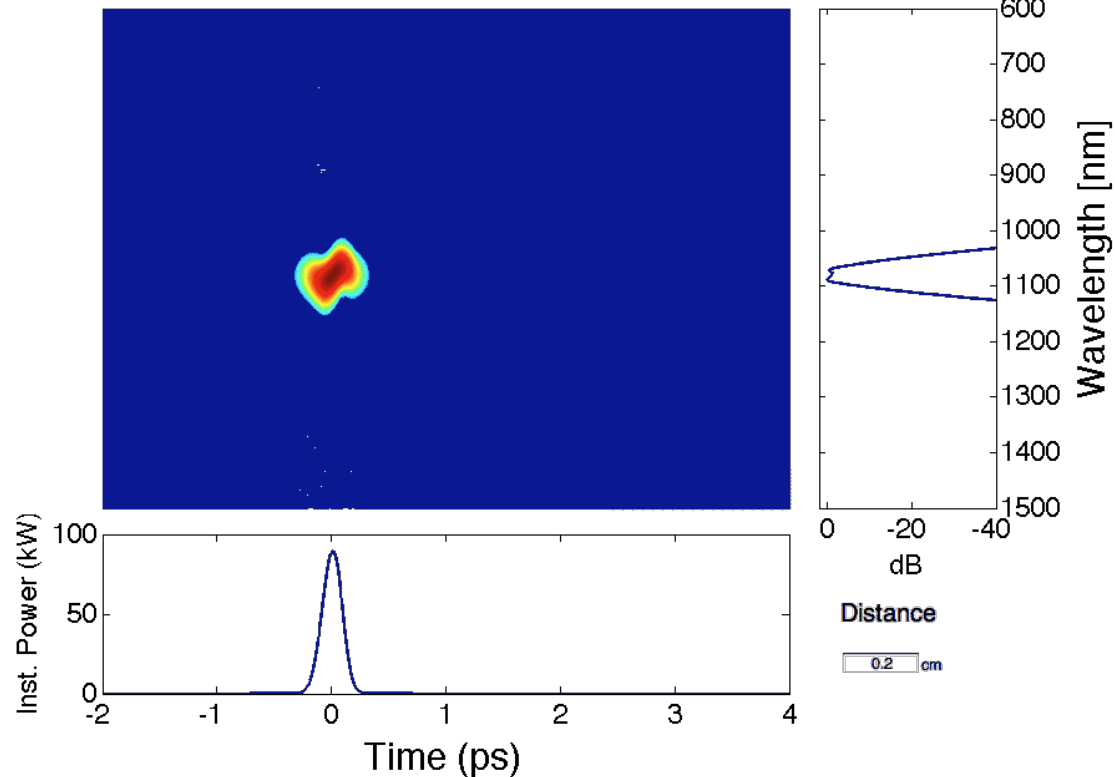
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All-normal dispersion supercontinuum

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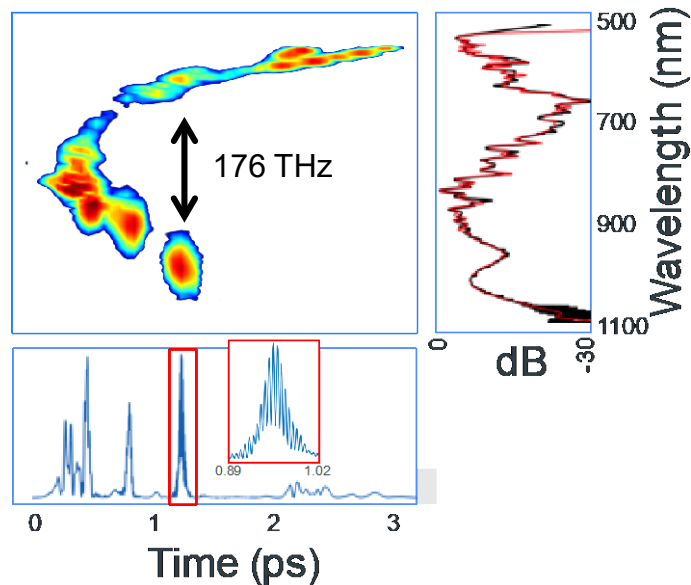
- no pulse-breakup
- minimum fine structure
- Unaffected by noise (up to ~1 ps pump pulses)



Dispersion engineering for SC generation

Conventional vs. ANDi SC (Femtosecond pumping)

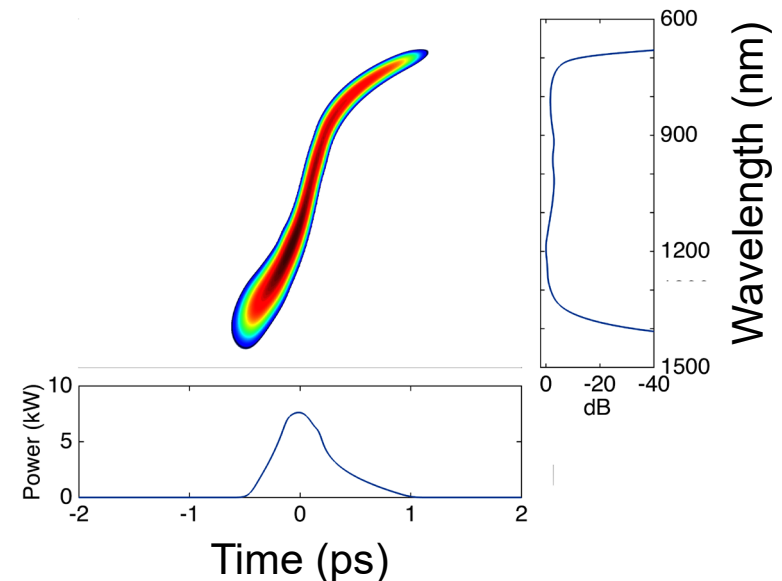
Conventional supercontinuum



Focus: spectral bandwidth

- > low pump power, very broad spectra
- > highly structured and complex spectral profiles
- > pump pulse breaks up into multiple solitons
- > temporal and spectral interference effects
- > susceptible to noisy pulse-to-pulse fluctuations

ANDi supercontinuum



Focus: ultrafast and low-noise applications

- > single ultrashort pulse maintained
- > smooth, flat spectra, steep edges
- > excellent pulse-to-pulse stability
- > higher pump peak power required to obtain bandwidth comparable to conventional SC

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + i\tau_{\text{shock}} \frac{\partial}{\partial T} \right) \times \left(A(z, T) \int_{-\infty}^{\infty} R(T') |A(z, T - T')|^2 dT' \right)$$

- Numerics: continuous problem is solved approximately on a discrete grid of points
 - Idea:
 - represent input field on a discrete temporal grid of size n_t with resolution ΔT
 - Fourier relations then define a frequency grid with resolution $\Delta\nu = 1/(n_t \Delta T)$
 - propagate stepwise through z (fiber length)
 - Time-domain formulation contains a few difficulties:
 - Temporal derivatives in dispersive operator and shock term can only be approximated in discrete case \rightarrow errors
 - Convolution integral difficult to compute
- \rightarrow Solution: transfer into frequency domain!**

Advantages of frequency domain formulation

- Time derivatives vanish:

$$\mathcal{F}\left(\frac{\partial}{\partial T}\right) = -i(\omega - \omega_0)$$

- dispersive and nonlinear operators can be applied in approximation-free manner
- frequency domain formulation is fundamentally more accurate

- Convolution integral vanishes:

$$\mathcal{F}\left(\int_{-\infty}^{\infty} A(\tau)B(t-\tau)d\tau\right) = \tilde{A}(\omega)\tilde{B}(\omega)$$

Explicitly:

$$\begin{aligned}\int_{-\infty}^{\infty} R(T')|A(z, T - T')|^2 dT' &= \int_{-\infty}^{\infty} [(1 - f_R)\delta(T') + f_R h_R(T')] |A(z, T - T')|^2 dT' \\ &= \underbrace{(1 - f_R)|A(z, T)|^2}_{\text{Kerr effect}} + \underbrace{f_R \int_{-\infty}^{\infty} h_R(T') |A(z, T - T')|^2 dT'}_{\text{stimulated Raman scattering}}\end{aligned}$$

$$\mathcal{F}\left(\int_{-\infty}^{\infty} h_R(T') |A(z, T - T')|^2 dT'\right) = \mathcal{F}(h_R(T))\mathcal{F}(|A(z, T)|^2)$$

Frequency domain version of the GNLSE

$$\frac{\partial \tilde{A}(z, \omega)}{\partial z} = \left(\hat{D}(\omega) + \hat{N}(z, \omega) \right) \tilde{A}(z, \omega)$$

$$\hat{D}(\omega) = -\frac{\alpha(\omega)}{2} + i \underbrace{(\beta(\omega) - \beta_1(\omega - \omega_0) - \beta(\omega_0))}_{\text{co-moving reference frame}}$$

$$\hat{N}(z, \omega) \tilde{A}(z, \omega) = i\gamma \left(1 + \frac{\omega - \omega_0}{\omega_0} \right) \text{frequency dependence of the nonlinearity}$$

$$\times \mathcal{F} \left\{ \underbrace{(1 - f_R) |A(z, T)|^2 A(z, T)}_{\text{Kerr effect}} + \underbrace{f_R A(z, T) \mathcal{F}^{-1} \left(\mathcal{F}(h_R(T)) \mathcal{F}(|A(z, T)|^2) \right)}_{\text{Stimulated Raman scattering}} \right\}$$

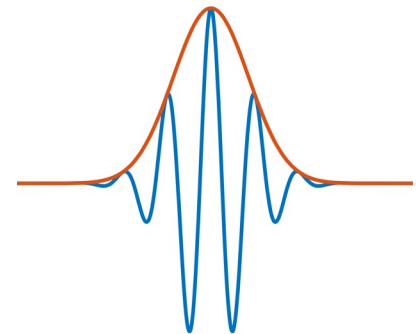
Kerr effect

Stimulated Raman scattering

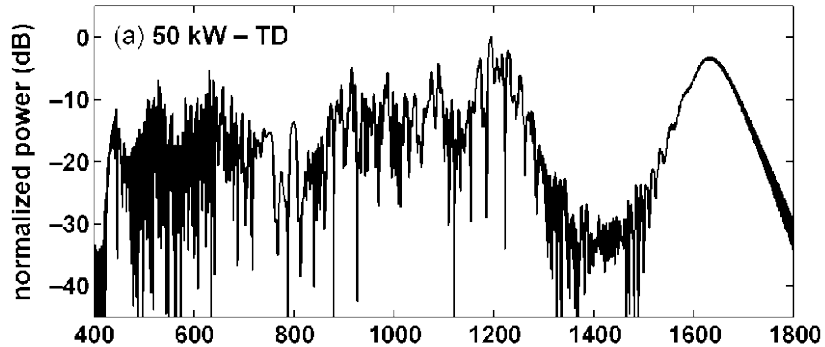
Validity: does the envelope approximation break down for very short pulses?

→ No, still valid even for single cycle and sub-cycle pulses!

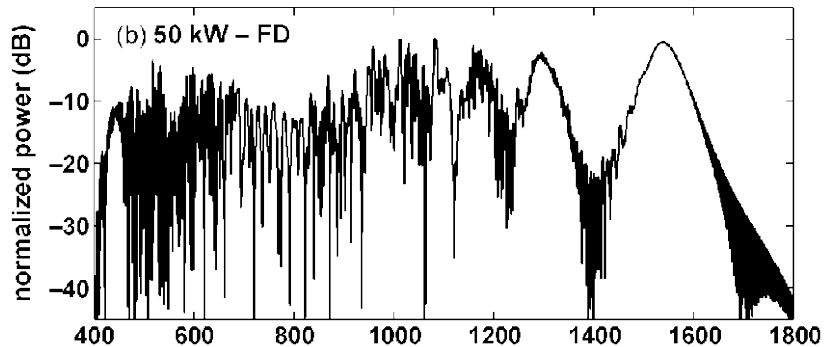
→ e.g. Genty et al., Opt. Express 15, 5382 (2007)



Time vs. frequency domain formulation



Errors caused by approximate treatment of derivatives in time-domain



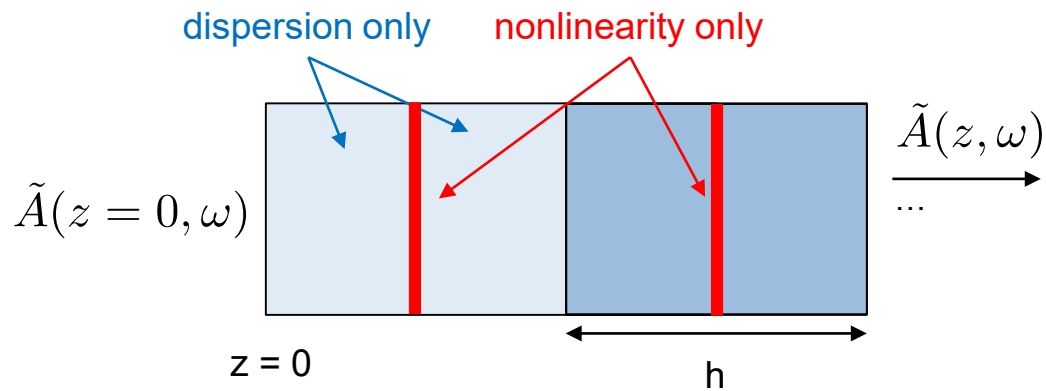
"Correct" frequency domain simulation

Numerical solution of the GNLSE

Split-step Fourier method

$$\frac{\partial \tilde{A}(z, \omega)}{\partial z} = \left(\hat{D}(\omega) + \hat{N}(z, \omega) \right) \tilde{A}(z, \omega)$$

Idea: dispersion and nonlinear operator act independently over small propagation step h



$$\tilde{A}(z+h, \omega) \approx \exp\left(\frac{h}{2}\hat{D}\right) \exp\left(\int_z^{z+h} \hat{N}(z') dz'\right) \exp\left(\frac{h}{2}\hat{D}\right) \tilde{A}(z, \omega)$$

Error: in reality, dispersion and nonlinearity act together

the bigger, the better!

$$\tilde{A}_{\text{calc}}(z+h, \omega) = \tilde{A}_{\text{true}}(z+h, \omega) + \mathcal{O}(h^3)$$

Numerical solution of the GNLSE

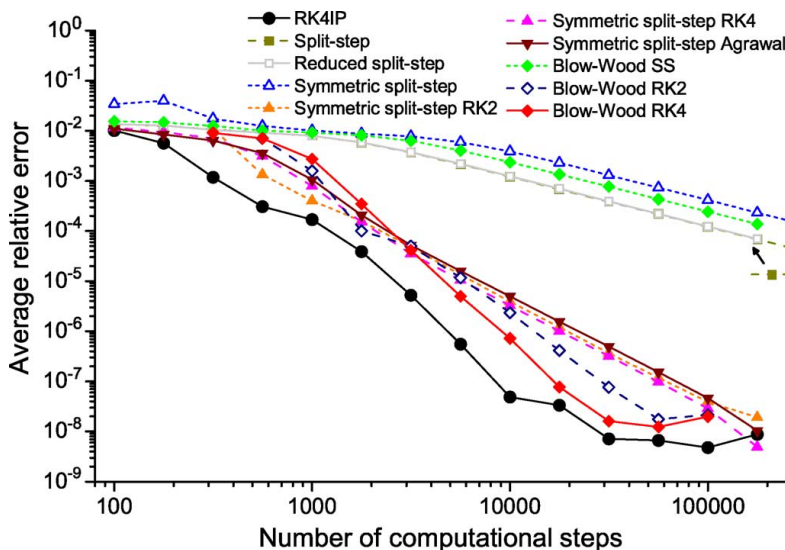
Runge-Kutta in the Interaction Picture (RK4IP)

$$\frac{\partial \tilde{A}(z, \omega)}{\partial z} = \left(\hat{D}(\omega) + \hat{N}(z, \omega) \right) \tilde{A}(z, \omega)$$

Idea: intelligent combination of the split-step Fourier method and an efficient integration of the nonlinear step using a Runge-Kutta algorithm

Explicit algorithm to propagate $\tilde{A}(z, \omega) \rightarrow \tilde{A}(z + h, \omega)$

→ see Hult, J. Lightwave Technol. 25, 3770 (2007)



Error of RK4IP method:

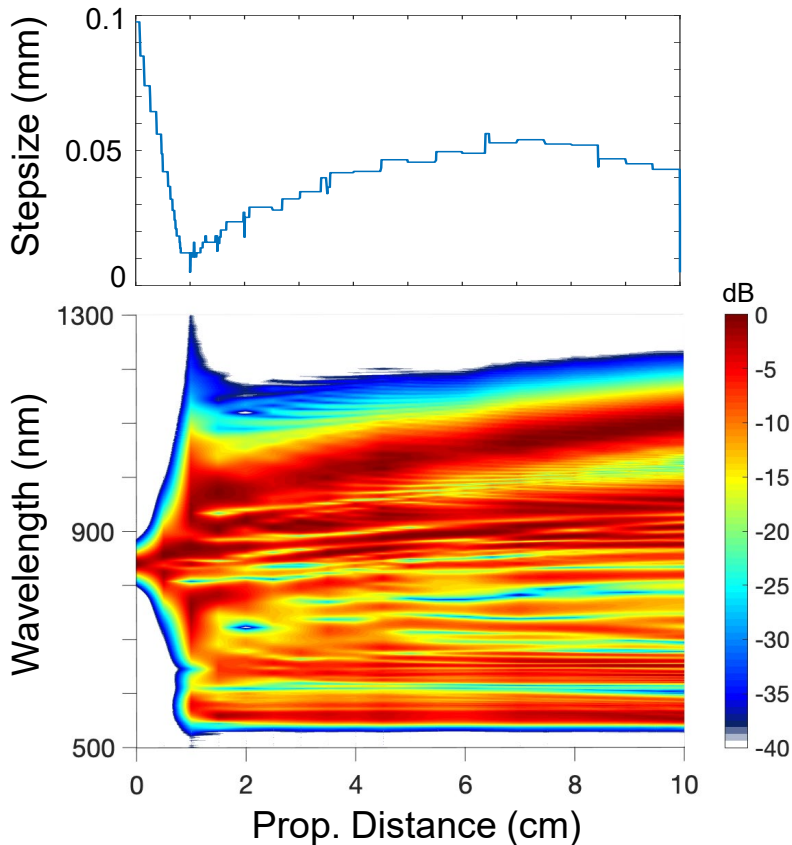
$$\tilde{A}_{\text{calc}}(z + h, \omega) = \tilde{A}_{\text{true}}(z + h, \omega) + \mathcal{O}(h^5)$$

Adaptive step size algorithms

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Efficient and fast calculations

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Concept:

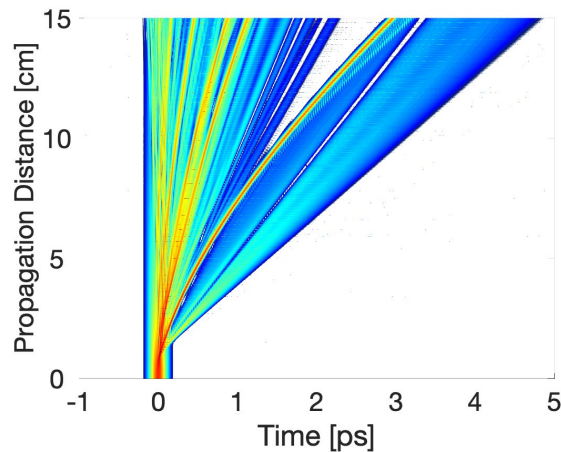
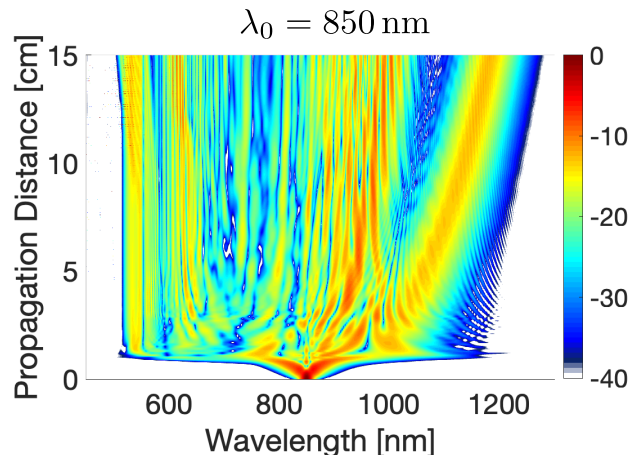
- Estimate current error during calculation
 - Increase or decrease step size in order to maintain a given level of accuracy
- ➔ Makes the calculation significantly faster
- ➔ Avoids manual search for an appropriate constant step size

Examples of methods:

- Photon number conservation
Heidt, J. Lightwave Technol. 27, 3984 (2009)
- Step size doubling
Sinkin et al., J. Lightwave Technol. 21, 61 (2003)

Implementation details

Determining grid sizes



2 constraints:

- Sampling frequency > maximum frequency of the field (Nyquist)

$$\lambda_{\min} = \frac{c}{\frac{1}{2\Delta t} + \frac{\omega_0}{2\pi}} = \frac{1}{\frac{1}{2c\Delta t} + \frac{1}{\lambda_0}} \quad \lambda_{\min} \sim 500 \text{ nm} \rightarrow \Delta t < 2\text{fs}$$

- Width of the grid > maximum time delay of the field

Maximum delay $\sim 5 \text{ ps} \rightarrow$ Time window $T > 10 \text{ ps}$

Number of grid points $n_p = T/\Delta t = 5000$ (set $n_p = 2^{13}$)

To avoid negative frequencies: $\Delta t > \lambda_0/(2c) \rightarrow \Delta t > 1.41\text{fs}$

Be aware of wrap around effects if your window size is too small!

Extras

- **Frequency dependent nonlinear parameter** $\gamma(\omega_0) \rightarrow \gamma(\omega)$

$$\tilde{C}(z, \omega) = \left[\frac{A_{\text{eff}}(\omega)}{A_{\text{eff}}(\omega_0)} \right]^{-1/4} \tilde{A}(z, \omega) \quad \gamma(\omega) = \frac{n_2 n_0 \omega_0}{c n_{\text{eff}}(\omega) \sqrt{A_{\text{eff}}(\omega) A_{\text{eff}}(\omega_0)}}$$

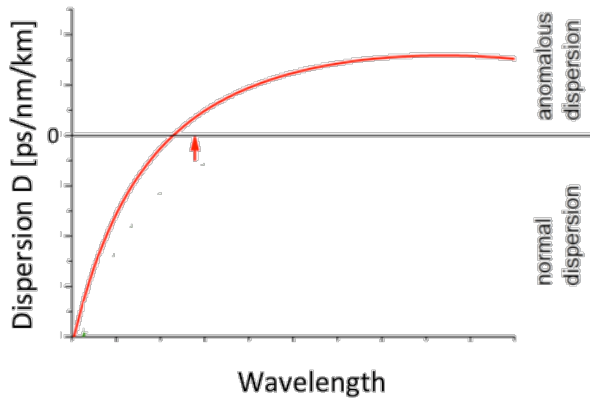
solve GNLSE as usual. Requires knowledge of $A_{\text{eff}}(\omega)$, $n_{\text{eff}}(\omega)$

- **Non-polarization maintaining fiber**
 - 2 coupled GNLSEs, one for each principal polarization axis.
 - implementation / solver identical to "simple" GNLSE
 - e.g. Bravo Gonzalo et al., Sci. Rep. 8, 6579 (2018), "Methods"
- **Multimode fiber**
 - many coupled GNLSEs
 - gets complicated
 - Poletti and Horak, J. Opt. Soc. Am. B 25, 1645 (2008).

Noise properties of SC sources

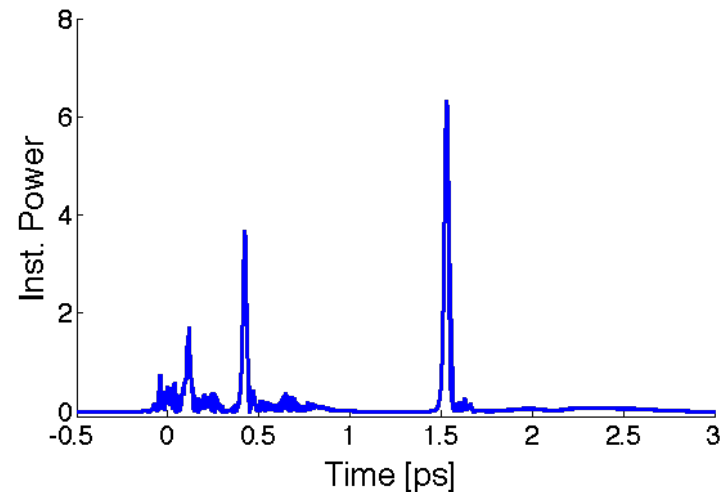
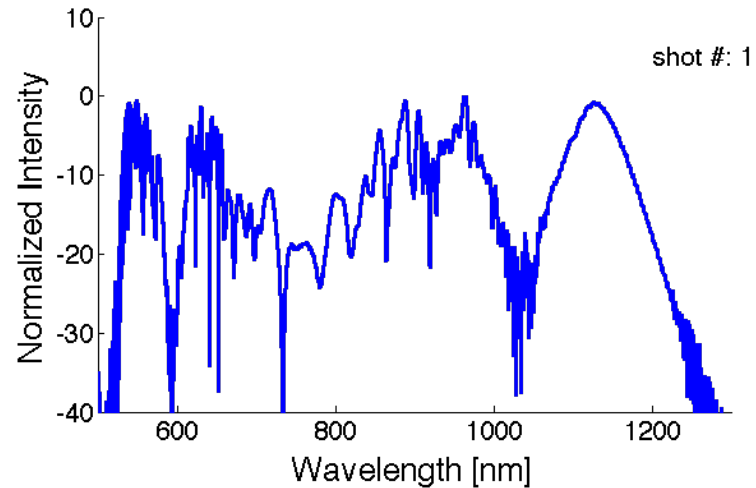
Conventional supercontinuum

Shot-to-shot fluctuations: **50 fs** pump pulse (10 kW)



Simulations including
shot noise

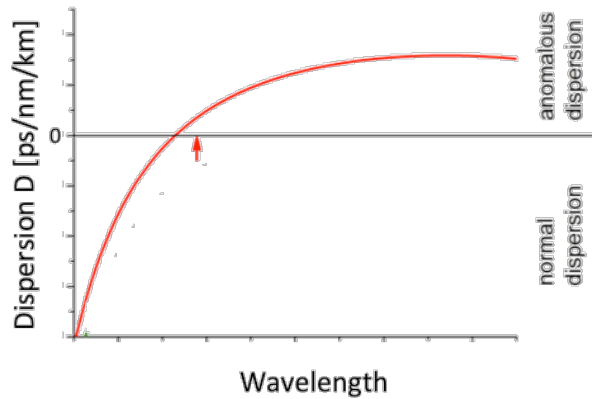
(best case scenario excluding
any technical noise)



Noise properties of SC sources

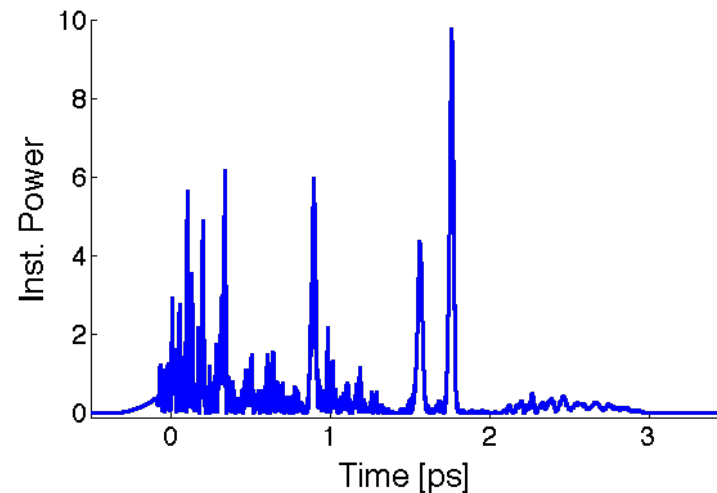
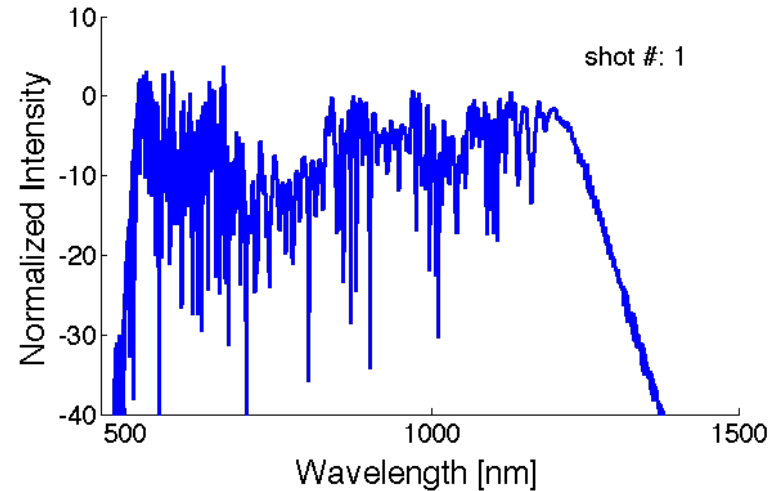
Conventional supercontinuum

Shot-to-shot fluctuations: **150 fs** pump pulse (10 kW)



Simulations including
shot noise

(best case scenario excluding
any technical noise)



Noise in GNLSE simulations

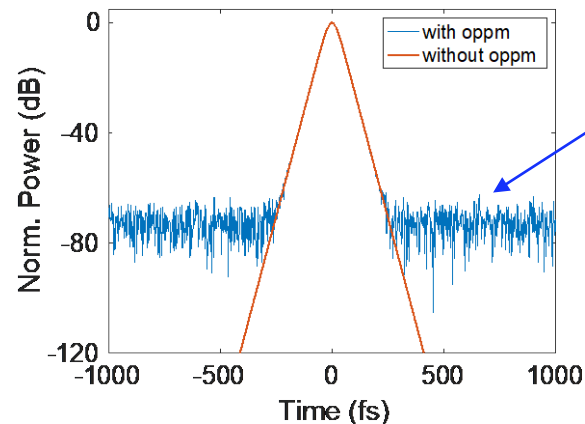
One photon per mode model

Shot noise can be included into the simulations by injecting **one photon with random phase** into each spectral simulation bin : ω_m

$$\tilde{A}_{\text{oppm}}(\omega_m) = \sqrt{\hbar(n_p - 1)dT\omega_m} \exp(-i\Phi(\omega_m)) \quad \Phi(\omega_m) \text{ randomly sampled in interval } [0, 2\pi]$$

This oppm field is then added to the input pulse:

$$A(z = 0, T) = A_{\text{input pulse}}(T) + \mathcal{F}^{-1} \left(\tilde{A}_{\text{oppm}}(\omega) \right)$$

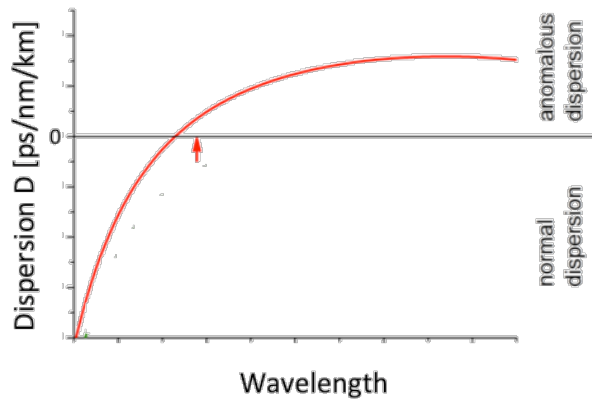


Noise floor important for correct simulation of noise-seeded nonlinearities

- Modulational instability
- Raman effect

Noise properties of SC sources

Conventional supercontinuum

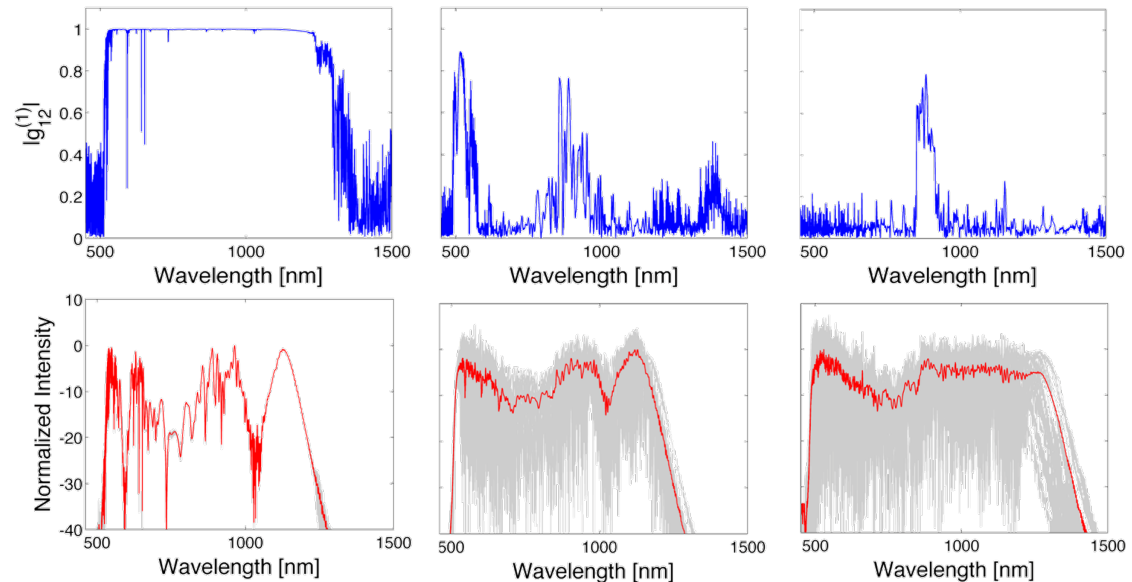


Pump pulse duration (10 kW peak power)

50 fs

100 fs

150 fs



Quantify shot-to-shot fluctuations by
first-order coherence function
at zero path difference

$$|g_{12}^{(1)}(\omega)| = \left| \frac{\langle \tilde{A}_i^*(\omega) \tilde{A}_j(\omega) \rangle_{i \neq j}}{\sqrt{\langle |\tilde{A}_i(\omega)|^2 \rangle \langle |\tilde{A}_j(\omega)|^2 \rangle}} \right|$$

$g = 1$: perfect amplitude / phase stability

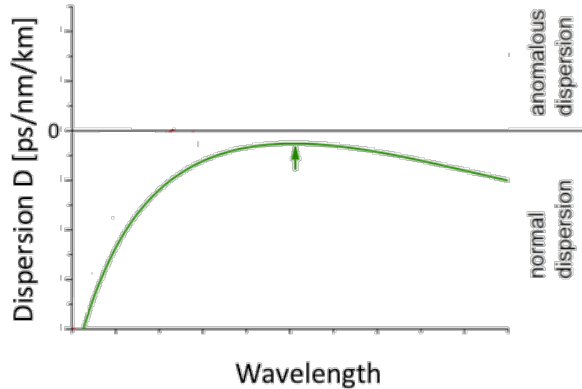
$g = 0$: random fluctuations

20 independent simulations \rightarrow 190 unique pairs

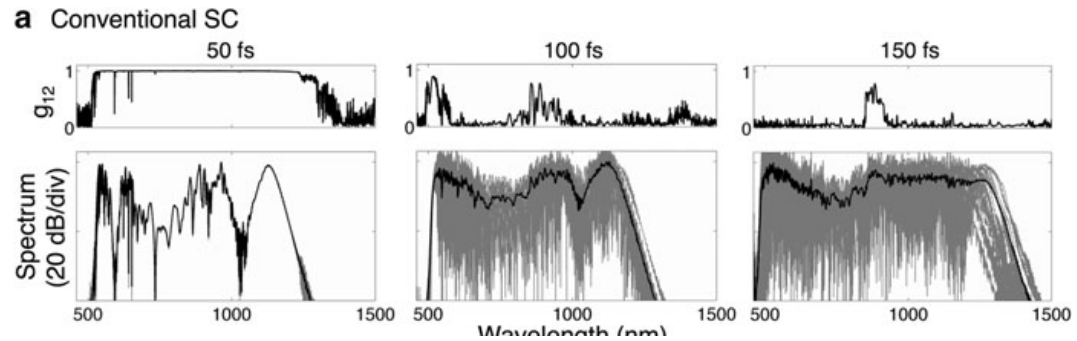
Increasing noise amplification
Increasing “apparent” flatness

Noise properties of SC sources

ANDi supercontinuum



Pump pulse duration (50 kW peak power)



Decoherence only becomes significant at $T_p \approx 1$ ps
(vs. ~ 100 fs in conventional SC generation)

Quantify shot-to-shot fluctuations by
first-order coherence function
at zero path difference

$$|g_{12}^{(1)}(\omega)| = \left| \frac{\langle \tilde{A}_i^*(\omega) \tilde{A}_j(\omega) \rangle_{i \neq j}}{\sqrt{\langle |\tilde{A}_i(\omega)|^2 \rangle \langle |\tilde{A}_j(\omega)|^2 \rangle}} \right|$$

$g = 1$: perfect amplitude / phase stability

$g = 0$: random fluctuations

20 independent simulations \rightarrow 190 unique pairs

Noise properties
are sensitive to fiber design!

Conclusions

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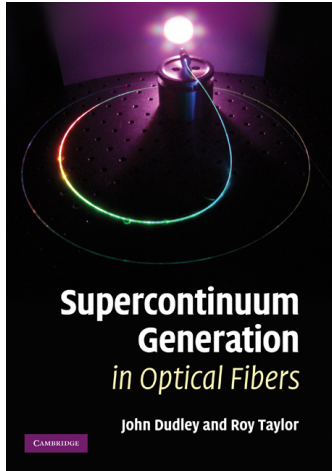
- **Nonlinear fiber optics** provides powerful tools to shape laser pulses
 - in the spectral domain
 - In the temporal domain
 - in their noise and coherence properties
- **Numerical simulations** based on the GNLSE help to
 - understand nonlinear effects and their interaction
 - design new light sources with properties tailored to specific applications
- Using tips of this webinar and mentioned resources **coding your own simulation** is not difficult!

Have fun exploring nonlinear optics!

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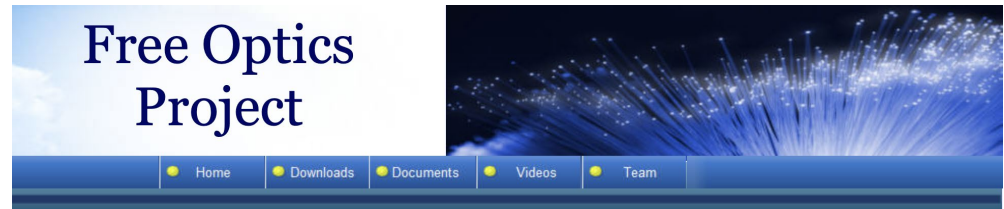


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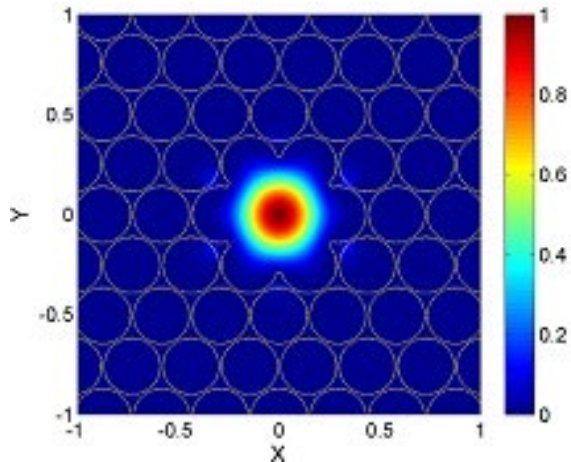
- **Book “Supercontinuum Generation in Optical Fibers”**
 - contains some simple sample code suitable for solving with Matlab’s internal ODE solver
 - This code does not contain full complexity of GNLSE

- www.freeopticsproject.org



- Complete Matlab scripts to download
- GNLSE with RK4IP solver and adaptive step size control using photon number conservation
- Good starting point to customize your own code

Where to get fiber parameters?



- **Commercial mode solving software**

- e.g. COMSOL Multiphysics
- Solves stationary Maxwell equations with the boundary conditions of the fiber geometry
- extracts dispersion and mode field parameters

- **Empirical models**

- Exist for an increasing number of specialty optical fibers
- Provide empirical fitting values to generate dispersion profiles directly from fiber design parameters
- Excellent for quickly scanning over a large range of fiber designs
- for hexagonal PCF structures:
 - Koshiha and Saitoh, Opt. Lett. 29, 1739 (2004).
 - Saitoh and Koshiha, Opt. Express 13, 267 (2005)

