# Frequency combs to detect phase changes of $10^{-8}$: Intracavity Phase Interferometry Part I

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University of New Mexico

Siegman School
Barcelona
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The magic of intracavity interaction

How do you make the most sensitive sensor out of your purchased laser?

*...???*

You cannot, because you have to built the laser around the sensor,

not the sensor outside of the laser

## The laser itself is the most accurate interfometer

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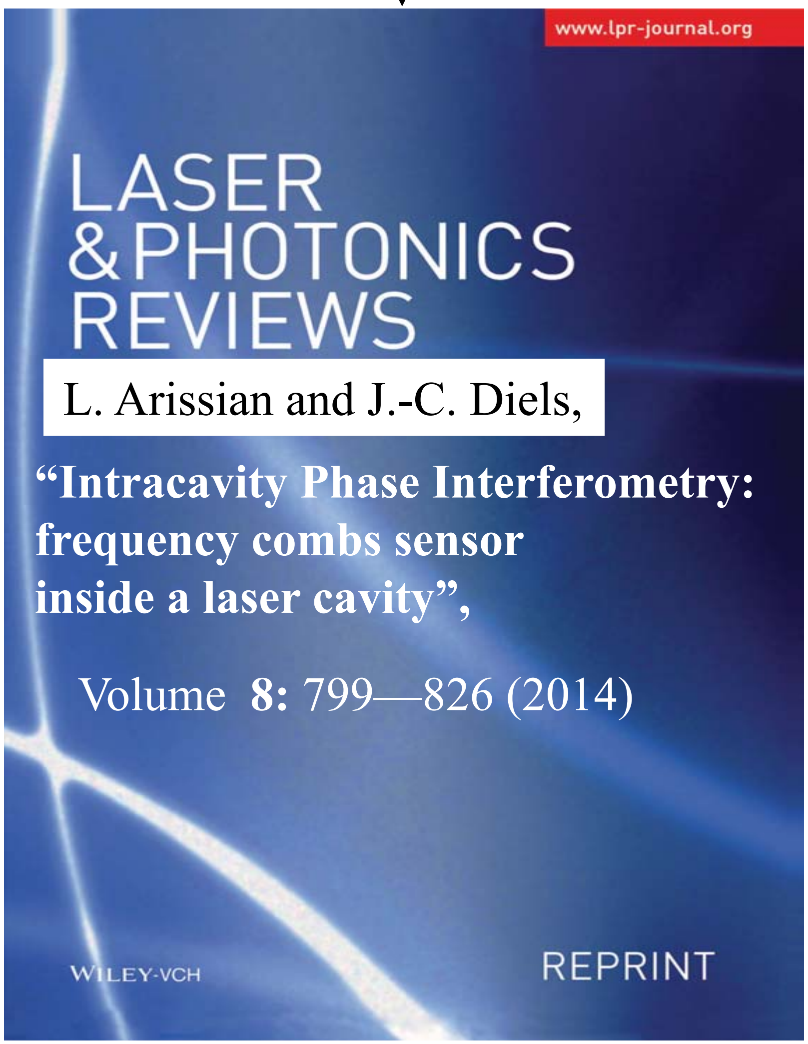
T. W. Haensch, A. L. Schawlow and P. E. Toschek
Ultra-sensitive response of a cw dye laser to selective extinction
IEEE Journal of Quantum Electronics **QE-8:** 802—808 (1972)

The laser as an amplitude sensor




**The laser as a phase sensor**

Basic properties:

The laser is a Fabry-Perot of infinite finesse

Frequency combs combine frequency and time resolution

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# Frequency combs to detect phase changes of $10^{-8}$: Intracavity Phase Interferometry Part I

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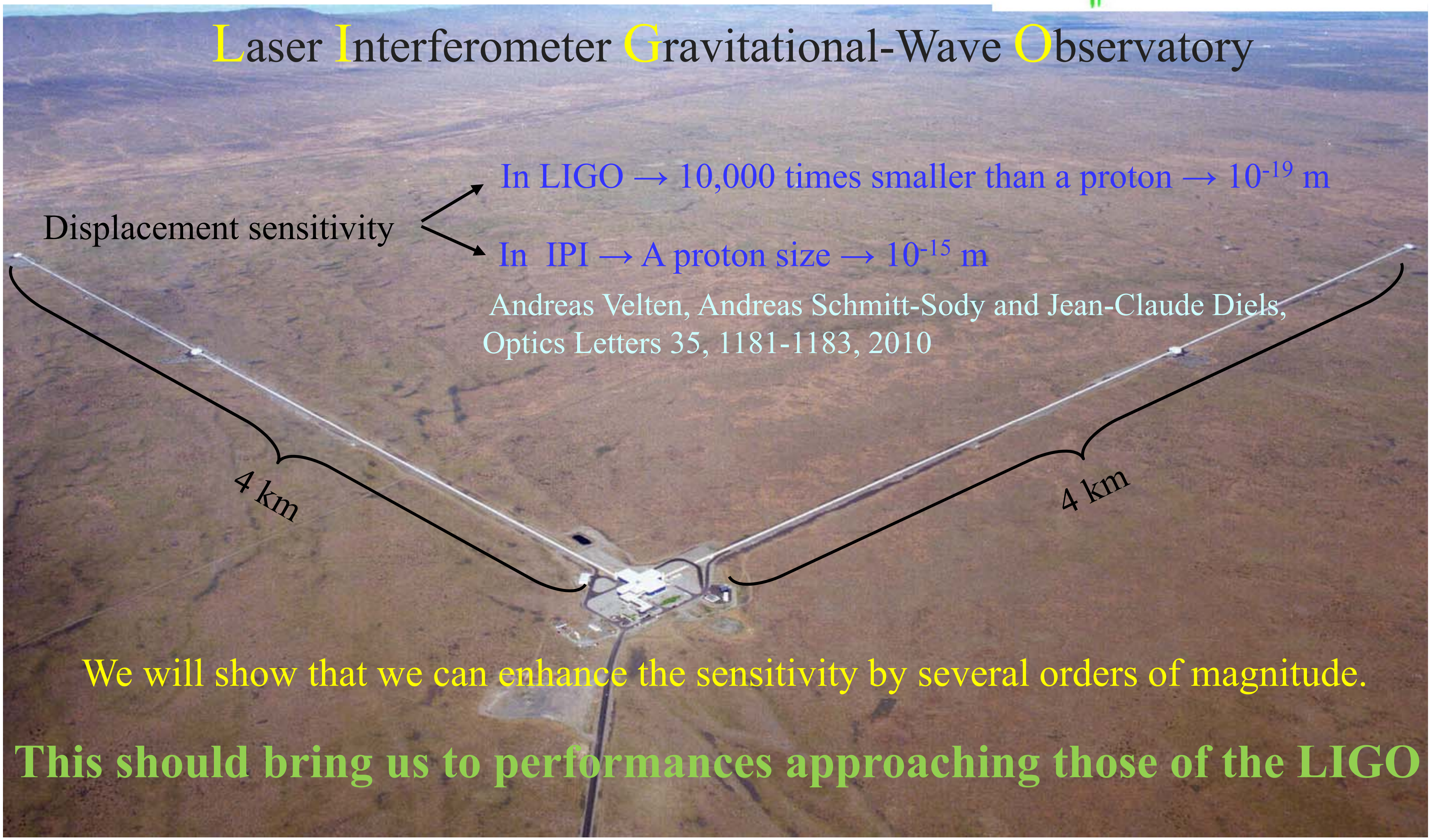
What does it mean to have a phase sensitivity of $10^{-8}$?

$10^{-8}$ of the wavelength is $10^{-15}$ m or a fm or … the diameter of the proton

To put this is perspective…..

# LIGO

Laser Interferometer Gravitational-Wave Observatory




Displacement sensitivity

- In LIGO → 10,000 times smaller than a proton → $10^{-19}$ m
- In IPI → A proton size → $10^{-15}$ m

Andreas Velten, Andreas Schmitt-Sody and Jean-Claude Diels, Optics Letters 35, 1181-1183, 2010

We will show that we can enhance the sensitivity by several orders of magnitude.

**This should bring us to performances approaching those of the LIGO**

## Frequency combs to detect phase changes of $10^{-8}$:
## Intracavity Phase Interferometry
## Part I

What does it mean to have a phase sensitivity of $10^{-8}$?

$10^{-8}$ of the wavelength is $10^{-15}$ m or a fm or … the diameter of the proton

We will show in part II that the sensitivity of IPI can be enhanced 10000 times, bringing it on par with the LIGO

LIGO is a multimillion $ project

IPI is not even a million pennies research…

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# Frequency combs to detect phase changes of $10^{-8}$: Intracavity Phase Interferometry Part I

The magic of intracavity interaction: it is a field full of surprises

An example: a ring laser is a two level system with Rabi frequencies!?!?

Intracavity Phase Interferometry – What it is NOT

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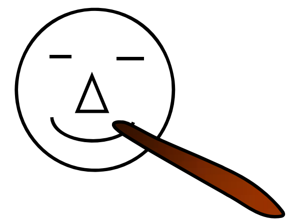
Introduction on frequency combs

Parenthesis on the CEO



Implementation of IPI with free space components

Fiber implementation of IPI

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**Part II**

Manipulating phase response with dispersion

***Is it slow/fast light?***

Can we make a purely optical accelerometer?

# Frequency combs to detect phase changes of $10^{-8}$: Intracavity Phase Interferometry Part I

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## Interactions inside a mode-locked laser: it is a field full of surprises:

The ring laser as a two level system *A. Schmitt-Sody, L. Arissian, A. Velten, J.-C. Diels, and D. Smith, PRA* ***78****:063802 (2008)*

Nested frequency combs *K. Masuda, J. Hendrie, J.-C. Diels and L. Arissian, J. Phys. B* ***49****:085402 (2016)*

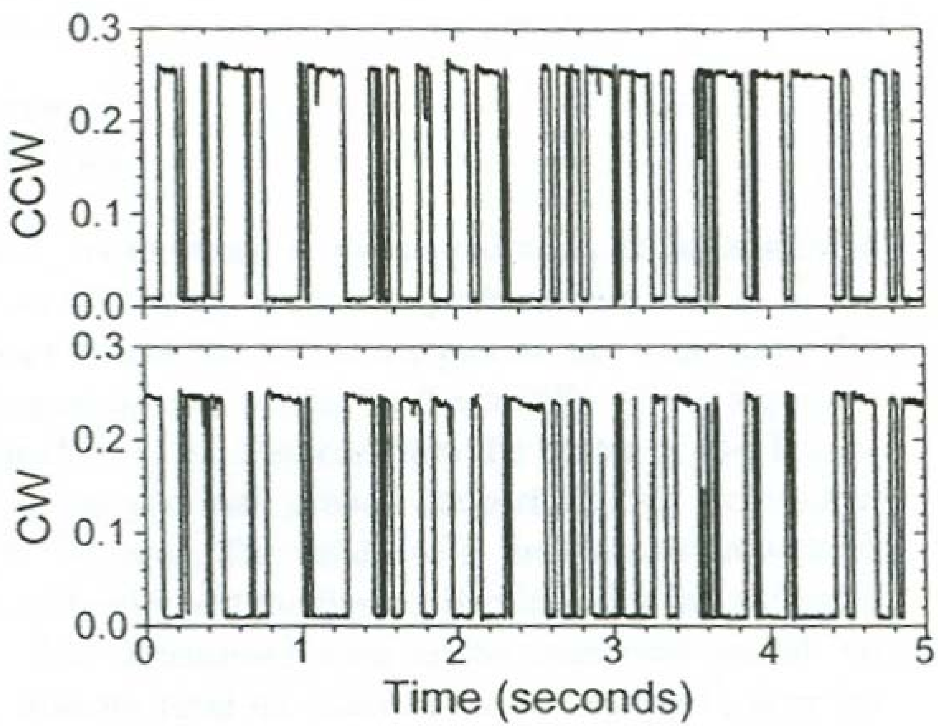
Intracavity Self-Induced transparency *K. Masuda, C. Affolderbach, G. Mileti, J.-C. Diels and L. Arissian, Optics Lett.* ***40****:2146 (2015)*

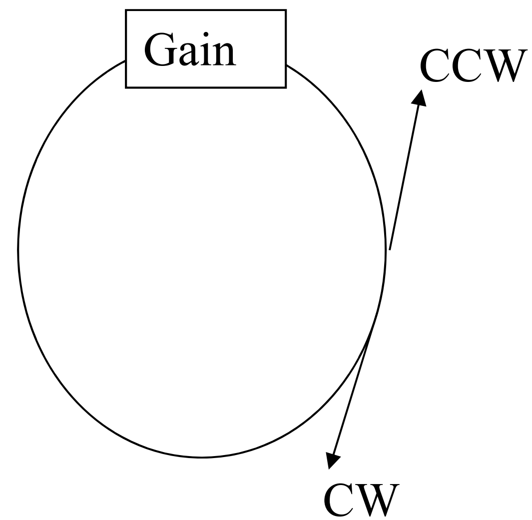
Intracavity dark line resonance leading to magnetometry in *L. Arissian and J.-C. Diels, J. Phys B* ***42****:183001 (2009)*

Intracavity Phase Interferometry, *L. Arissian and J.-C. Diels, Laser and Photonics review 8:799 (2014)*

# A previously unexplained observation, related to the "Rabi Cycling"

- The rate of switching from one direction to the other related to the scattering coefficient in the Ti:sapphire crystal

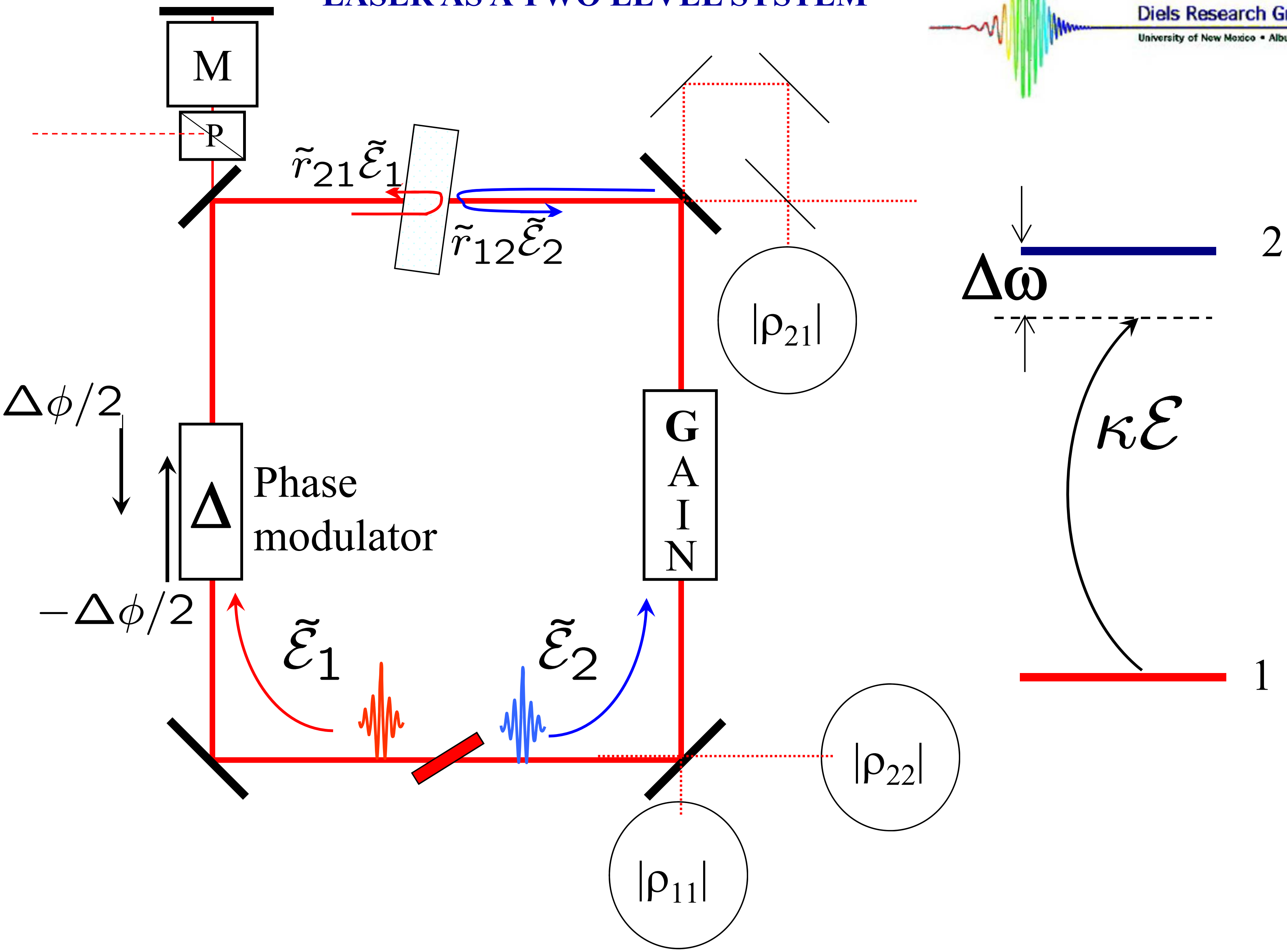



M. J. Bohn, J.-C. Diels: Bidirectional Kerr-lens mode-locked femtosecond ring laser; Optics Communications 141 (1997)

# LASER AS A TWO LEVEL SYSTEM





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## Analogy

| Ring Laser | | Two -level atom |
|---|---|---|
| Clockwise pulse,<br>Counterclockwise pulse | **Level** | $\vert 1\rangle \quad \vert 2\rangle$ |
| $\psi(t) = \tilde{\mathcal{E}}_1(t)\vert 1\rangle + \tilde{\mathcal{E}}_2(t)\vert 2\rangle$ | **Wave function** | $\psi(t) = c_1(t)\vert 1\rangle + c_2(t)\vert 2\rangle$ |
| | **Density matrix elements** | |
| $\propto I_1,\ I_2$ | $\rho_{11},\ \rho_{22}$ | $c_1^* c_1,\ c_2^* c_2$ |
| $\mathcal{E}_2^* \mathcal{E}_1 \propto$ beat note | $\rho_{12}$ | $c_2^* c_1$ |
| $\tilde{r}/\tau_{RT}$ | **Rabi Frequency** | $\kappa\vert\tilde{\mathcal{E}}\vert = (p/\hbar)\vert\tilde{\mathcal{E}}\vert$ |
| $\Delta\phi/(2\tau_{\mathrm{RT}})$ | **Detuning** | $\Delta\omega$ |

# LASER AS A TWO LEVEL SYSTEM

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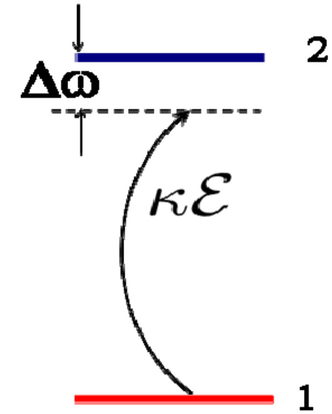
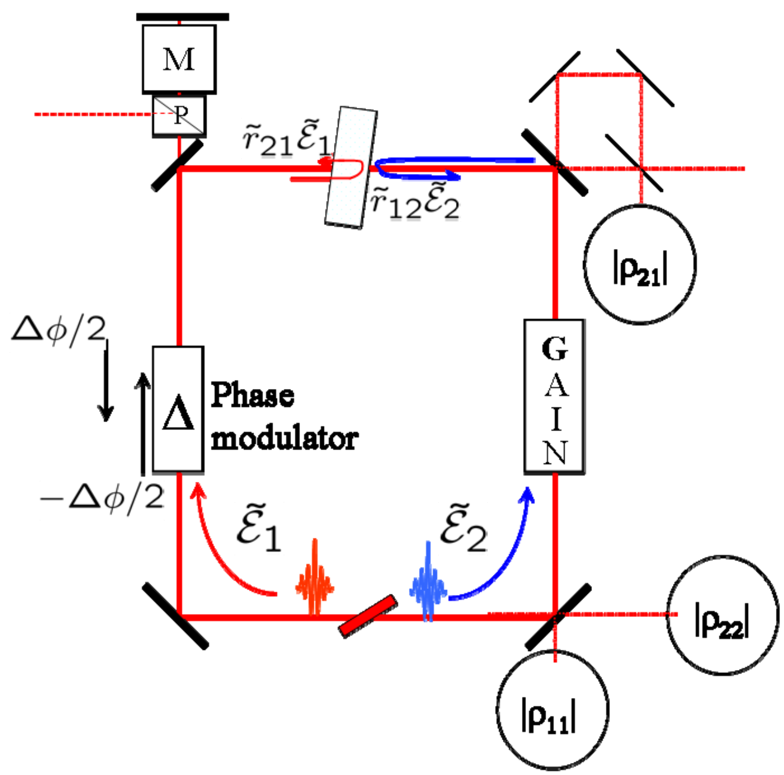
$$\frac{d}{dt}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} i\frac{\Delta\omega}{2} & i\frac{1}{2}\kappa\tilde{\mathcal{E}} \\ -i\frac{1}{2}\kappa\tilde{\mathcal{E}}^* & -i\frac{\Delta\omega}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$




Similar Equation for the variation of the complex field amplitudes of each pulse circulating in a ring cavity

$$\frac{d}{dt/\tau_{RT}}\begin{pmatrix} \tilde{\mathcal{E}}_1 \\ \tilde{\mathcal{E}}_2 \end{pmatrix} = \begin{pmatrix} \tilde{r}_{11} & \tilde{r}_{12} \\ \tilde{r}_{21} & \tilde{r}_{22} \end{pmatrix}\begin{pmatrix} \tilde{\mathcal{E}}_1 \\ \tilde{\mathcal{E}}_2 \end{pmatrix}$$

$$\tilde{r}_{21} = -\tilde{r}_{12}^* = -\tilde{r}^*$$



# Experiment

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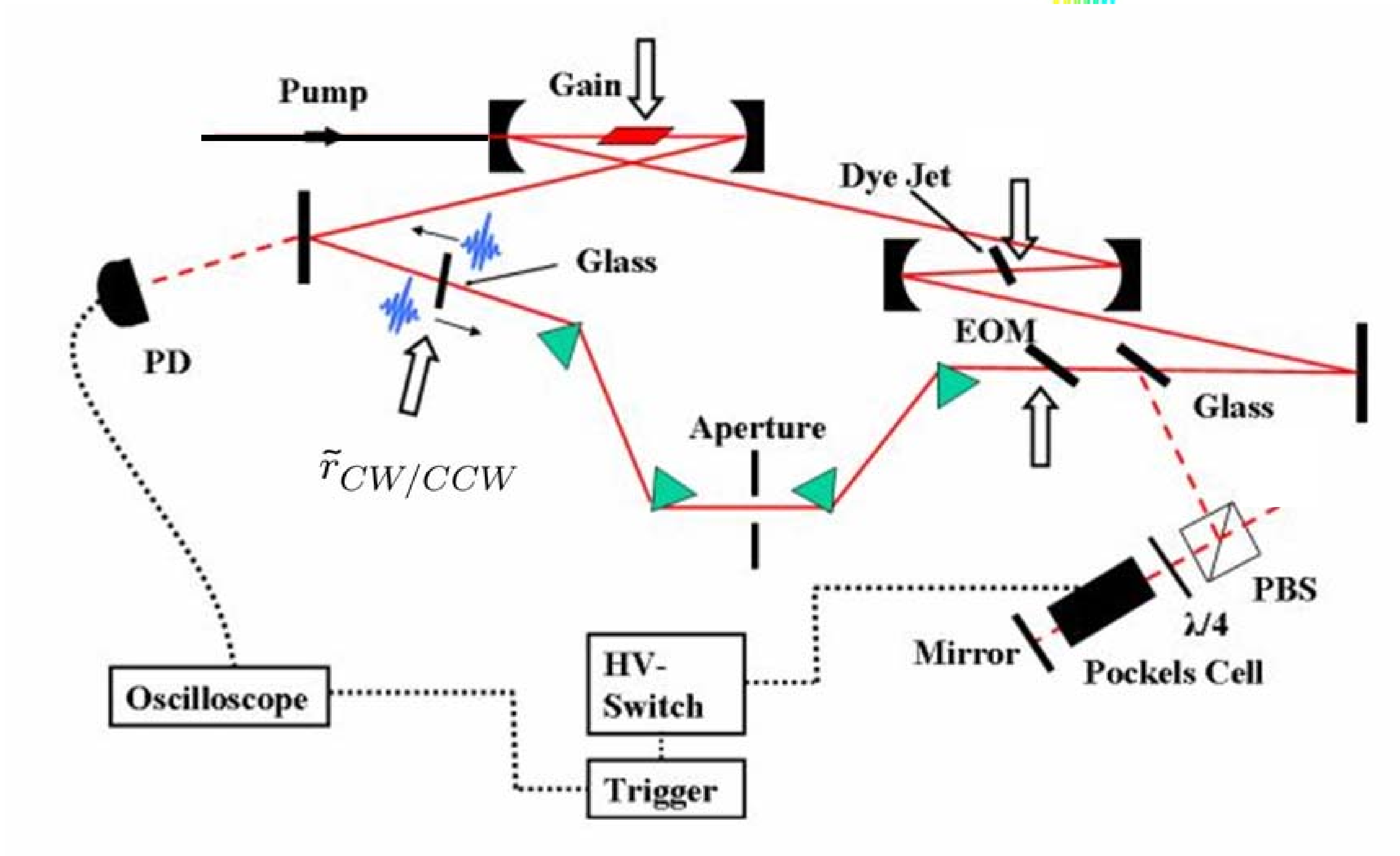
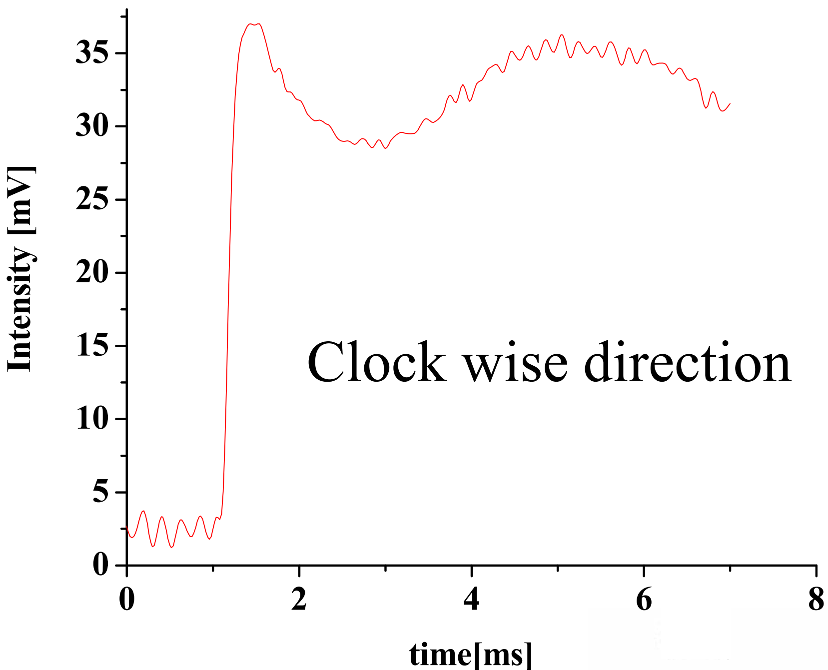
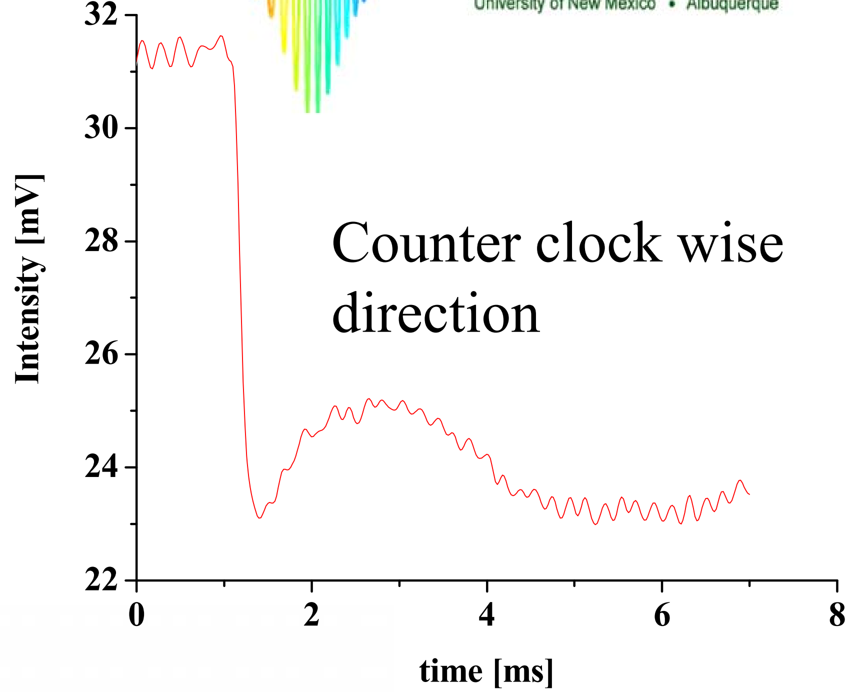





Clock wise direction




Counter clock wise direction

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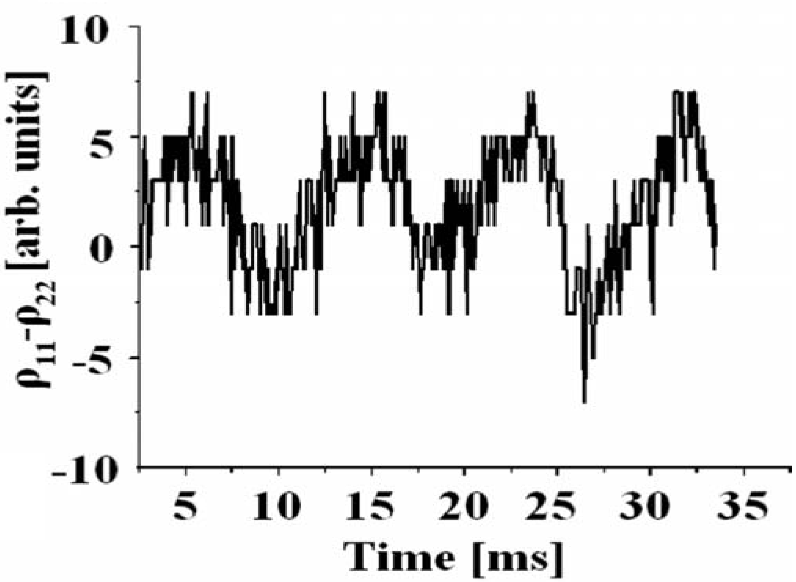
Population difference



# Experimental Results

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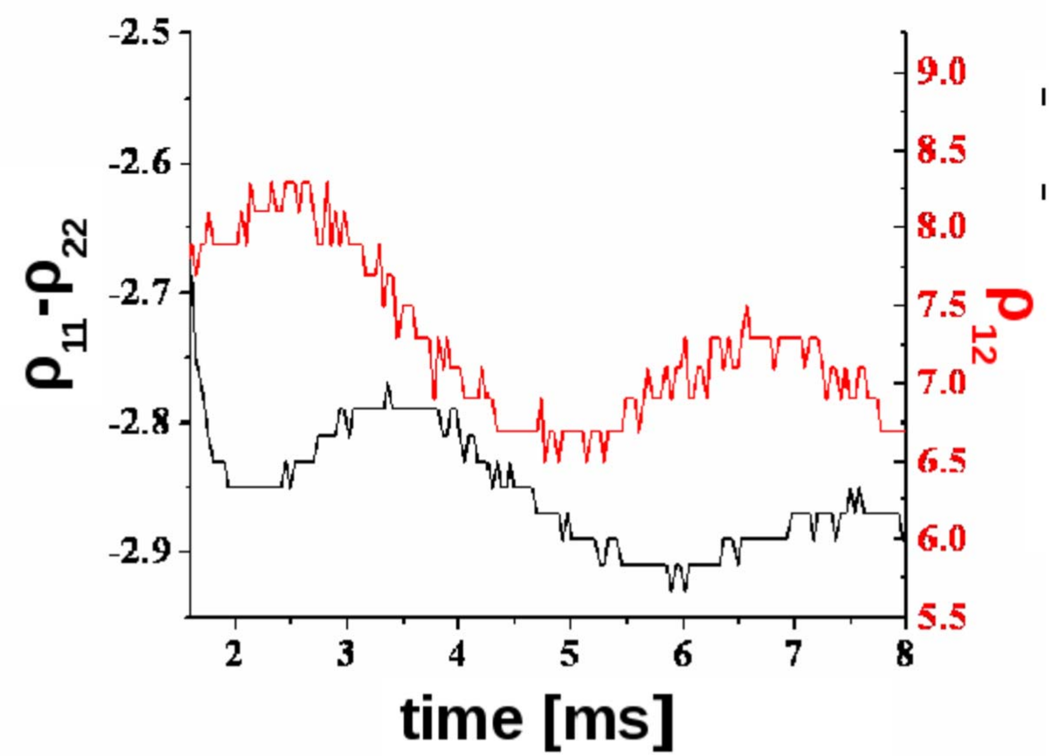
Diagonal element are 90° out of phase with off-diagonal elements



## Intracavity Phase Interferometry (IPI)

# What it is NOT

NOT TO BE CONFUSED WITH:

Ultrasensitive frequency-modulation spectroscopy enhanced by a high-finesse optical cavity
by Long-Sheng Ma, Jun Ye, Pierre Dube and John L. Hall
J. Opt. Soc. Am. B **16**: 2255 (1999)

In the "Intracavity Nonlinear Spectroscopy " of John Hall, an extreme finesse p***assive*** FP cavity transforms a few ppm absorption into some % change in transmission.

In **Intracavity Phase Interferometry** there is no electronic stabilization

***An active laser is a cavity of infinite finesse***

Intracavity Phase Interferometry, *L. Arissian and J.-C. Diels, Laser and Photonics review 8:799 (2014)*

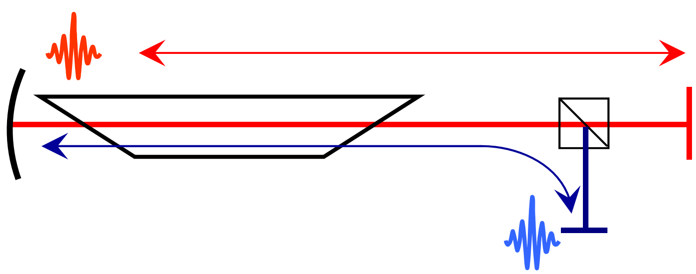
# Intracavity Phase Interferometry (IPI)

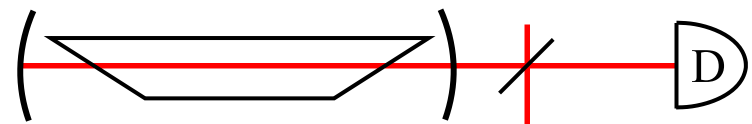


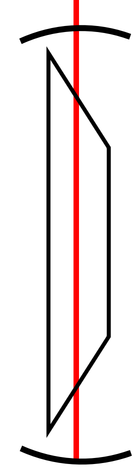
## What it is NOT

NOT TO BE CONFUSED WITH:

In IPI: noise correlated between the two pulse trains





Called
“active cavity interferometer”
by A. Abramovici and Z. Vager
PRA **33**:3181 (1985)

Phase noise uncorrelated between two arms

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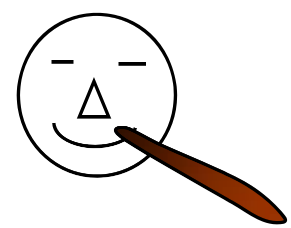
Parenthesis on the CEO



Implementation of IPI with free space components

Fiber implementation of IPI

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**Part II**

Manipulating phase response with dispersion

***Is it slow/fast light?***

Can we make a purely optical accelerometer?

## Metrology with lasers

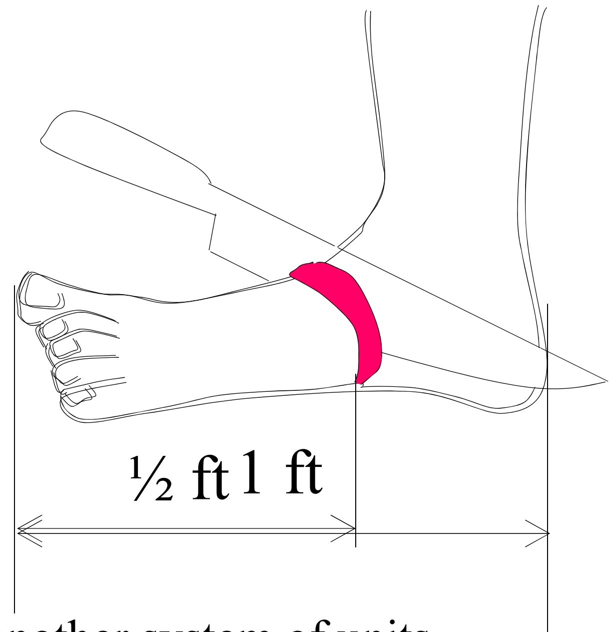
Oldest unit of measurement:

More precise measurements:

divide by 2, 4, 8, 16 …

There is still a tribe in the world

that uses this system of units!




Ronald reagan declared "antipatriotic" the use of another system of units.

(A new US presidential candidate would do the same if he new what a unit is)

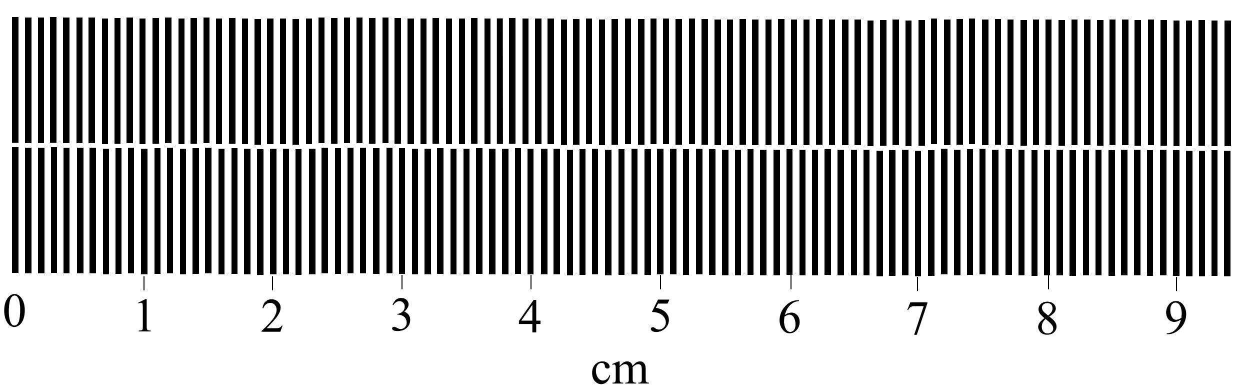
One can do better…

There comes a point at which you cannot divide anymore. Is that the resolution limit?

Given a reference ruler below. Another one has spacing differing only by 1%

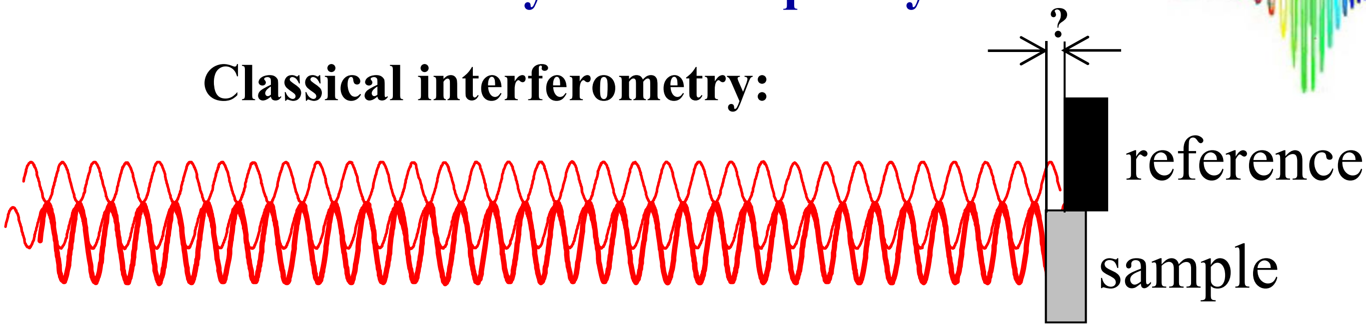
How to tell easily the difference?

By superimposing the two rulers.



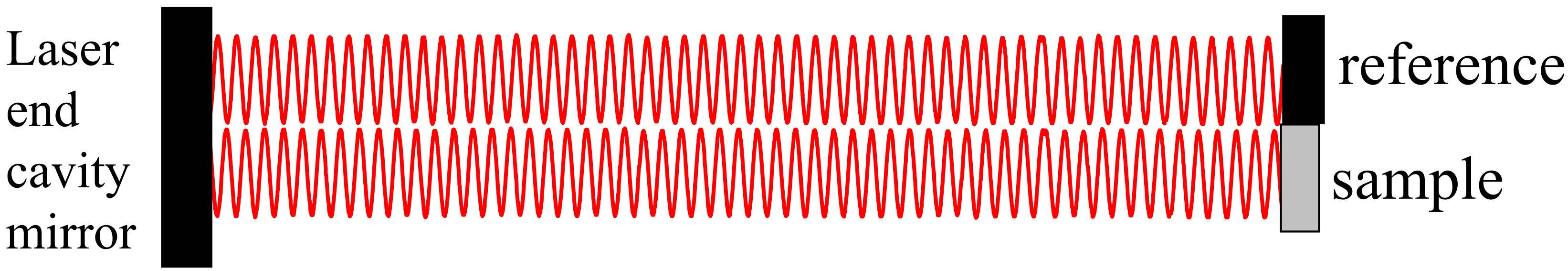
## IPI = Moire interferometry in the frequency domain

**Classical interferometry:**




**Intracavity interferometry:**

The two beams have slightly different wavelengths. How can you tell the difference?




**The ''Moire" pattern allows you to tell a difference of wavelength as small as $(\lambda/L)\lambda$**

**Instead of observing in space a superposition of two standing wave patterns, one measures the time equivalent: the "beating" of the two frequencies $\nu_1 = c/\lambda_1$ and $\nu_2 = c/\lambda_2$; a difference that can be smaller than 1 Hz.**

**This is "Moire" in the frequency domain.**

## Intracavity Phase Interferometry: Moire in the frequency domain

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***Intracavity***: only radiation for which the phase shift/round-trip = $2N\pi$ survives

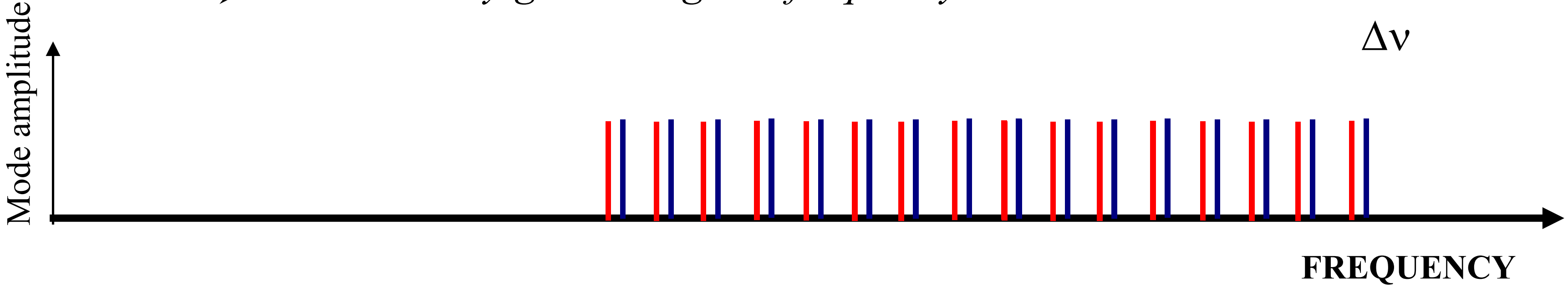
***Differential interferometer***: can two beams of nearly same frequency coexist?

Not in a cw laser: injection locking of one beam into the other (dead band)

but

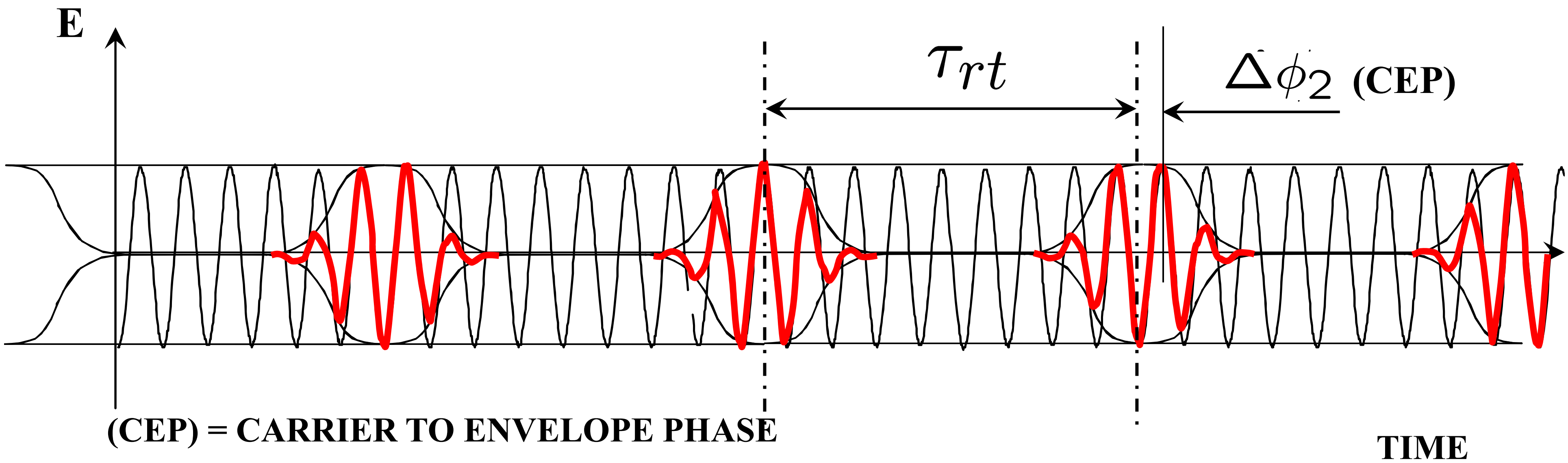
Differential interferometry with a *mode-locked* laser is possible

⟹ *Laser cavity generating two frequency combs*

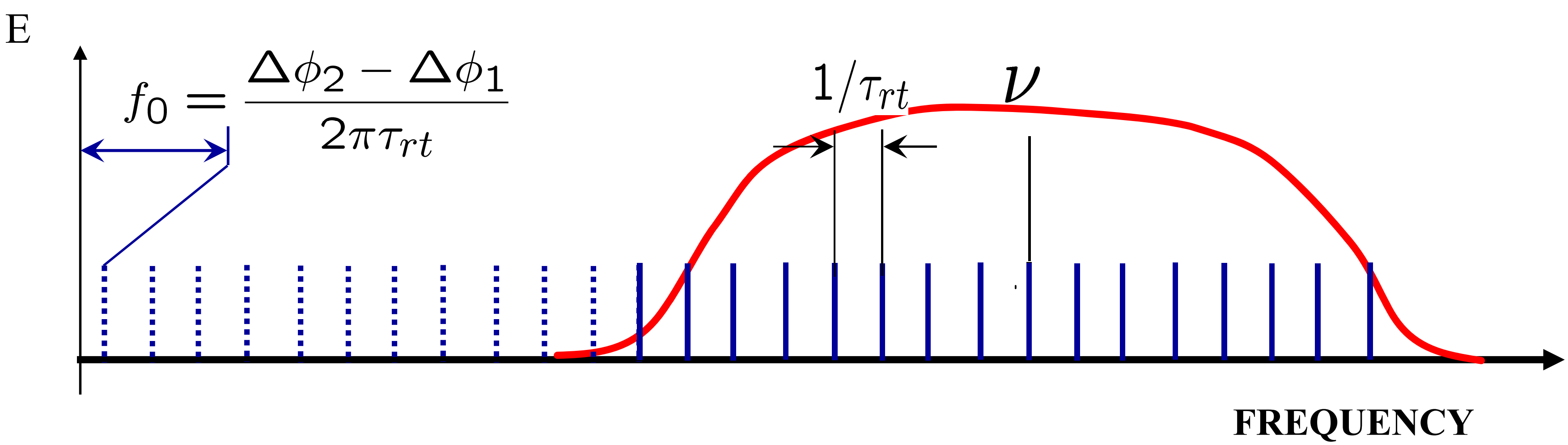



⟹ *Parenthesis on frequency combs*

## Parenthesis: the frequency comb




CEP/$\tau_{rt}$ = CEO = CARRIER TO ENVELOPE OFFSET $f_0$



## Summarizing the frequency comb parameters

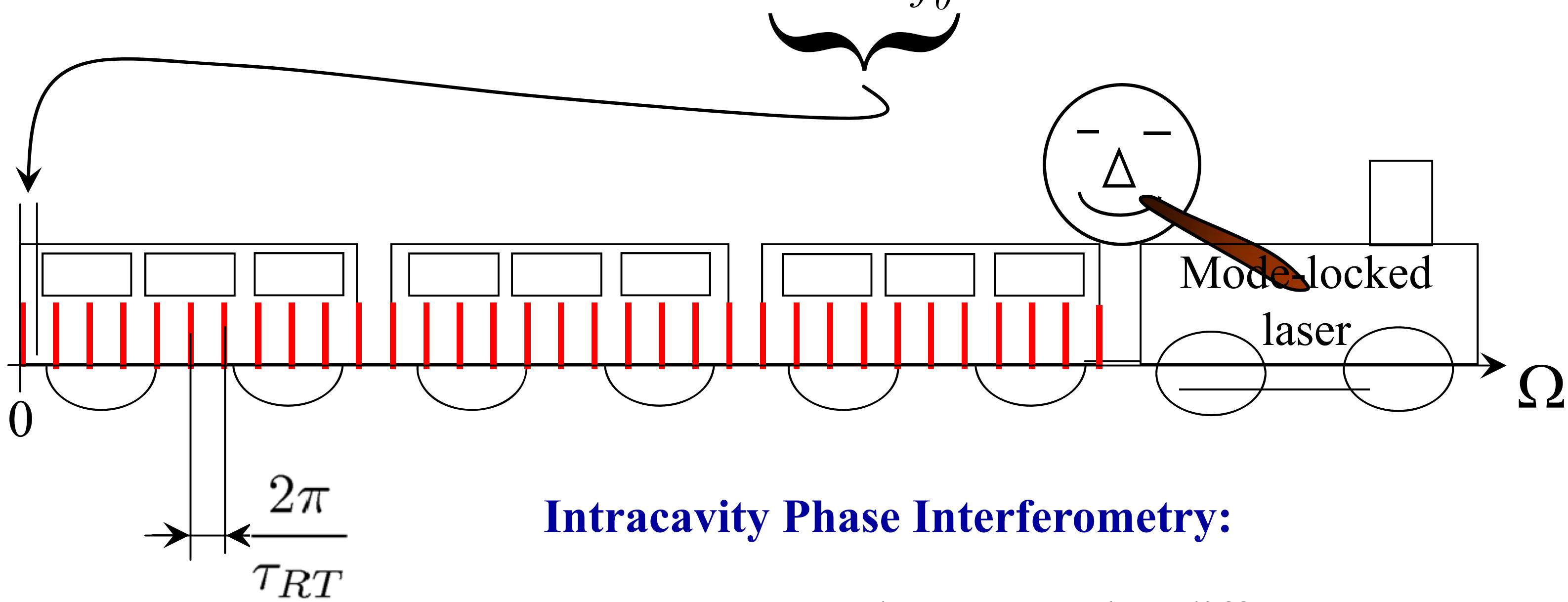
The Carrier to Envelope Phase (CEP) applies to a **single pulse**

The Carrier to Envelope Phase Offset (CEO) applies to a **pulse train**

Tooth spacing: $\Delta = \frac{2\pi}{\tau_{RT}}$

What controls the pulse train is the CEO

The mode comb does not start at zero but at $\Omega = 2\pi f_0$




**Intracavity Phase Interferometry:**

2 combs, same **Δ**, but different **CEO**

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*Parenthesis:*

[ *frequency combs* (*CEP – CEO*) ]

↓

**Start of (**

# CEP: a tricky definition

The Carrier to Envelope Phase

The Careful to Experimental Physicist

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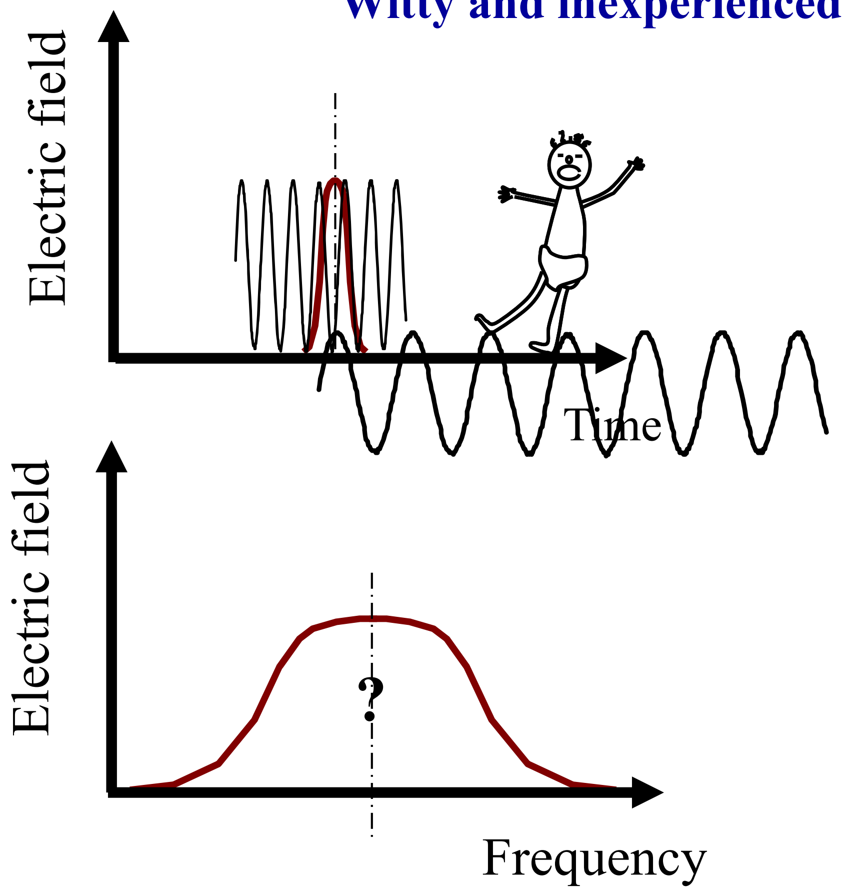
$$E(t) = \mathcal{E}(t)e^{i[\omega_0 t + \varphi(t)]} + c.c.$$

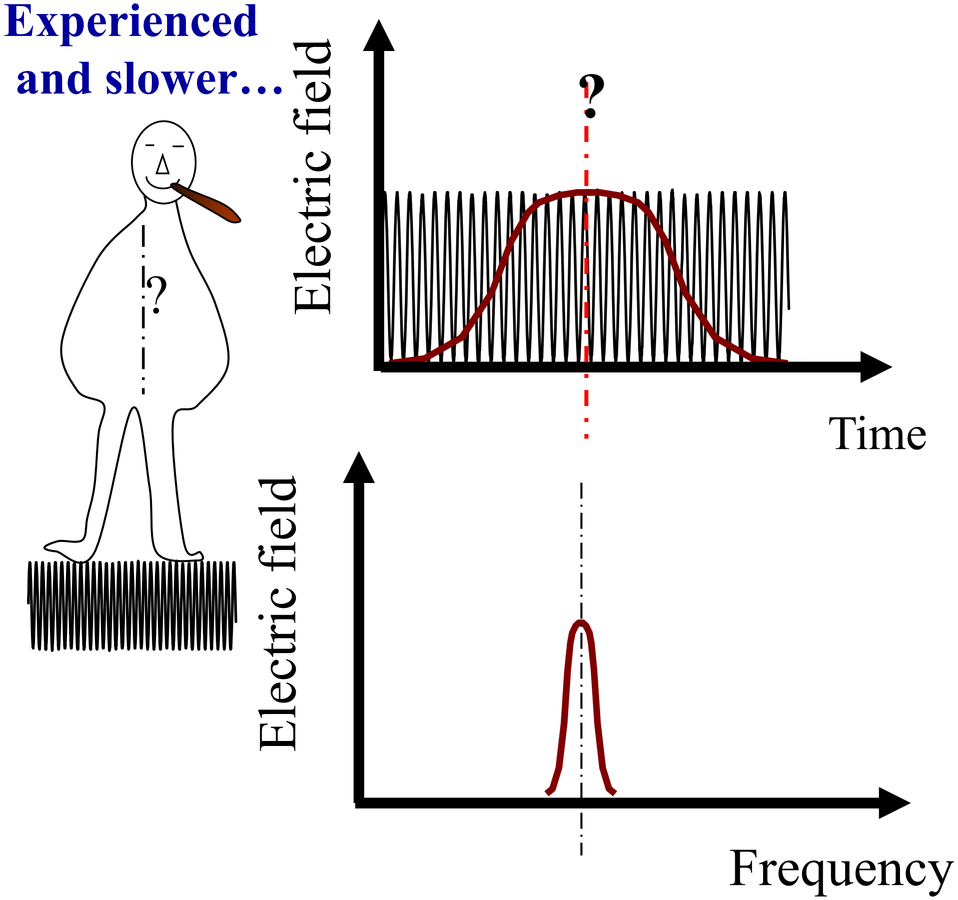
Short pulse: what is the carrier?

Long pulse: what is the envelope center?

**Witty and inexperienced**




**Experienced and slower…**




**Traditional CEP definition does not work for chirped pulses**

# Electric field is just an oscillation in time
# CEP independent of C and E

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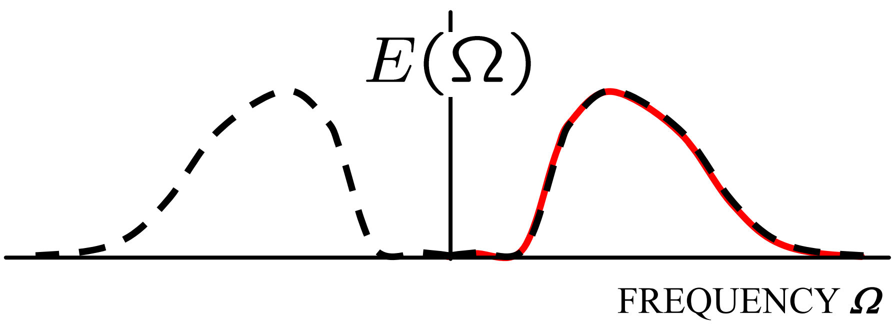
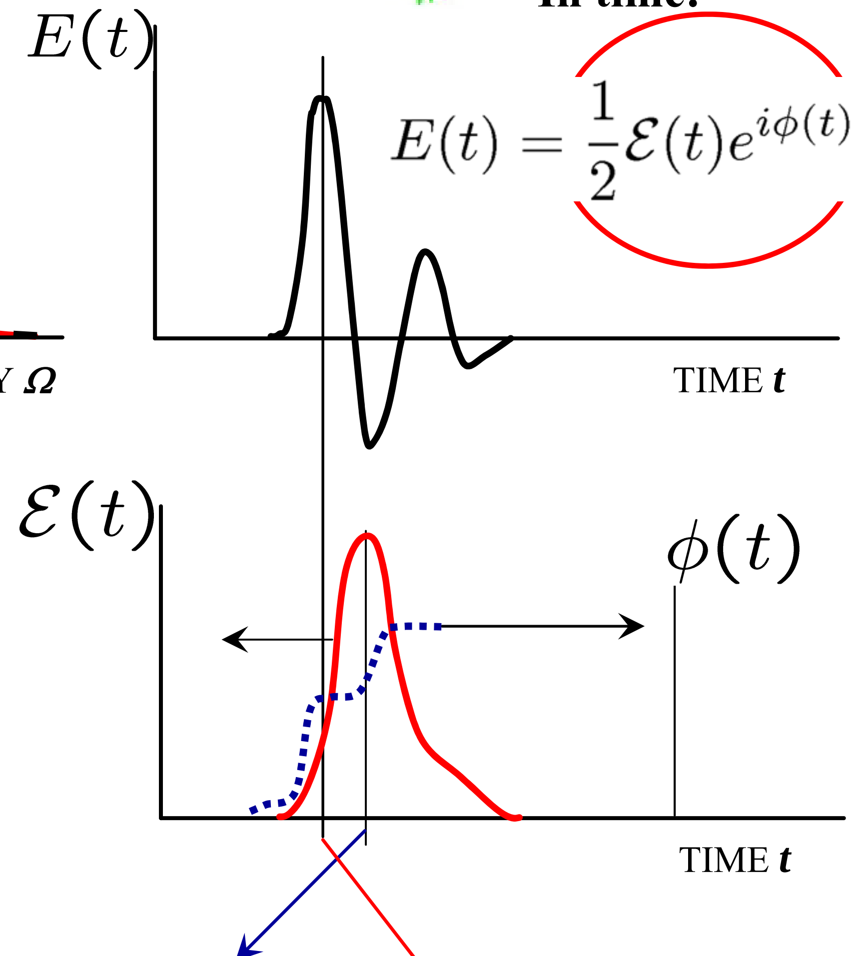
**In frequency:**




**In time:**

$$E(t) = \frac{1}{2}\mathcal{E}(t)e^{i\phi(t)} + c.c.$$




**Definition of CEP: difference between phase $\phi_1(t)$**

**at peak of amplitude, and phase $\phi_2(t)$ at peak of real field**

# How to correctly propagate an ultrashort pulse without phase and group velocity



$$\tilde{E}(t) = \frac{1}{2}\mathcal{E}(t)e^{i\phi(t)} + c.c.$$

**If we do not define a carrier $\omega_0$ and envelope, the notion of phase delay $k_0/\omega_0$ and group delay $\left.\frac{dk}{d\Omega}\right|_{\omega_0}$ disappear,**

We have to return to Maxwell's propagation equation: $\left(\frac{\partial^2}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}(z,t) = 0$

In frequency

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\Omega^2 n^2(\Omega)}{c^2}\right]\tilde{E}(z,\Omega) = 0$$

$$\left[\frac{\partial}{\partial z} + i\frac{\Omega n(\Omega)}{c}\right]\left[\frac{\partial}{\partial z} - i\frac{\Omega n(\Omega)}{c}\right]\tilde{E}(z,\Omega) = 0.$$

$$n(\Omega) = n_0 + a\Omega$$

$$\tilde{E}(z,\Omega) = \tilde{E}(0,\Omega)e^{i\left(-\frac{n_0}{c}\Omega z - a\Omega^2\frac{z}{c}\right)}$$

# Single pulse    Propagation maxwell Eq.

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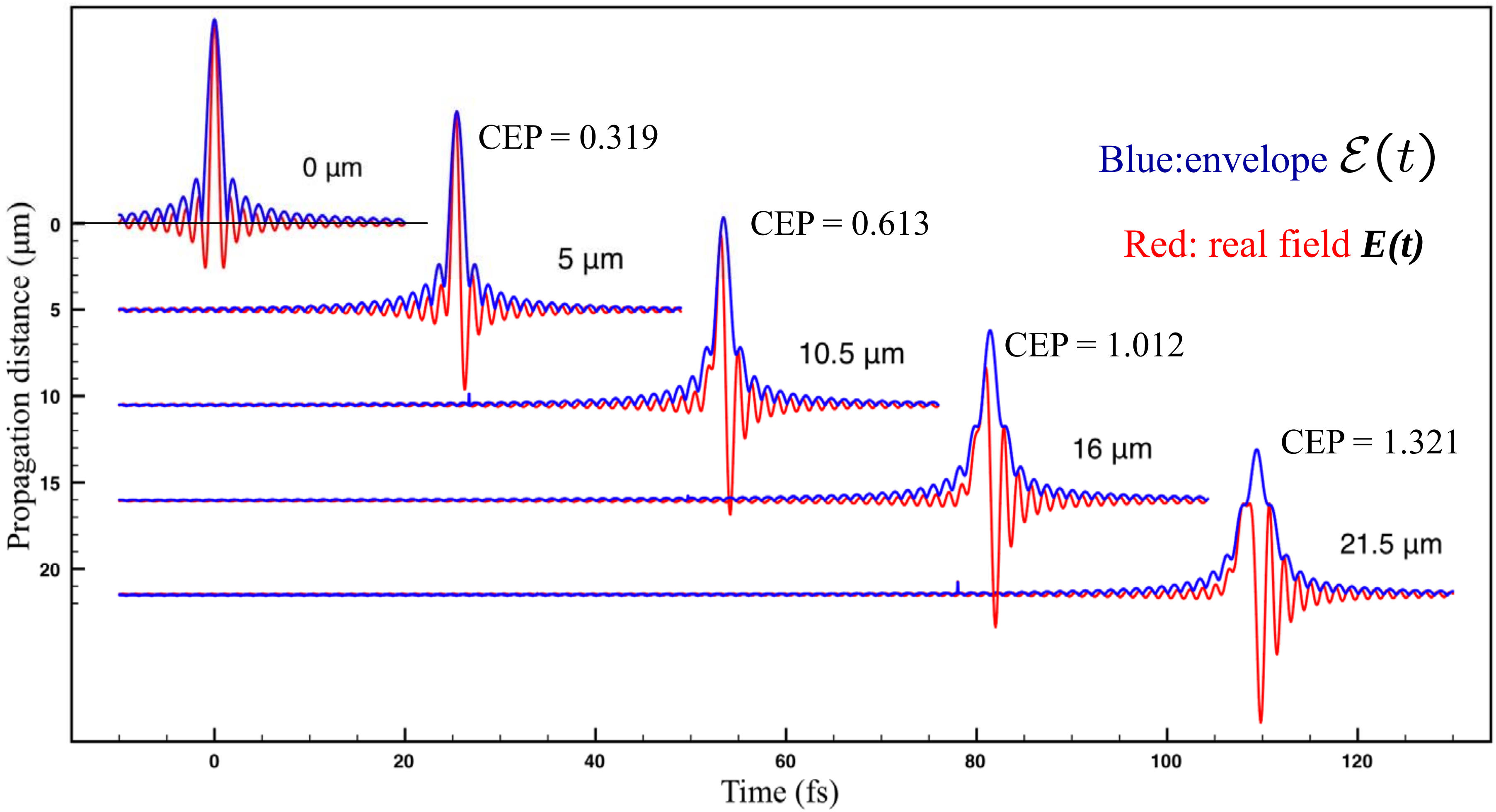
$$E(t) = \frac{1}{2}\mathcal{E}(t)e^{i\phi(t)} + c.c.$$

**Definition of CEP: difference between phase $\phi_1(t)$ at peak of amplitude and phase $\phi_2(t)$ at peak of real field**



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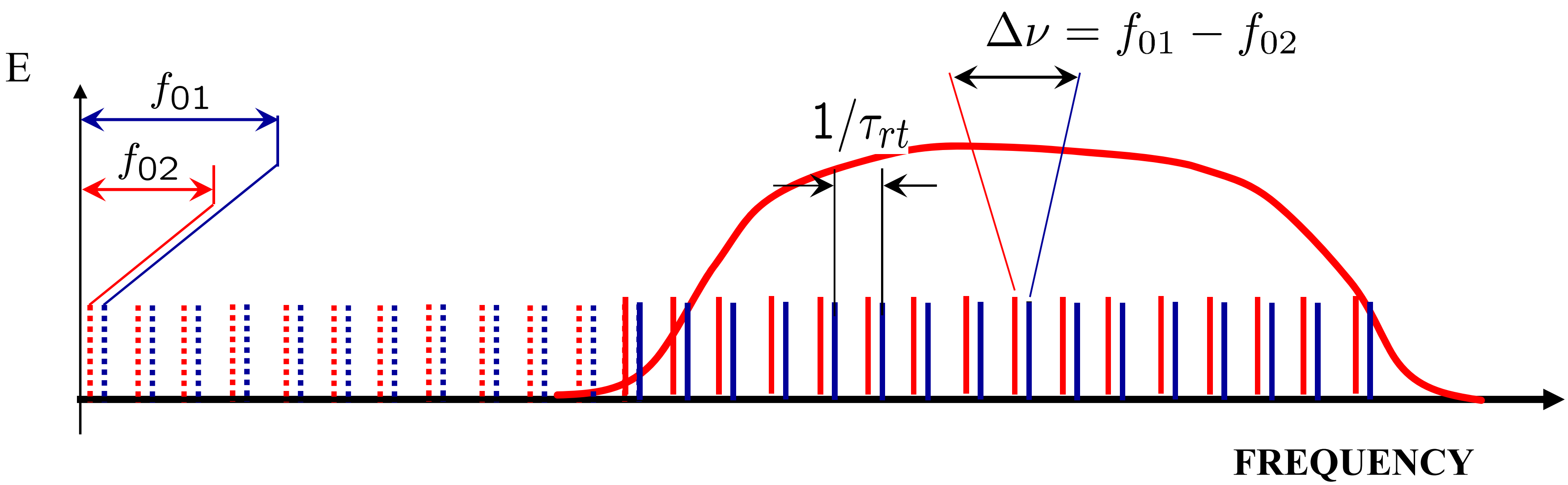
## Two frequency combs in one laser cavity

Intracavity phase interferometry: 2 circulating pulses in the mode-locked laser

⟹ *Two frequency combs of the same repetition rate*

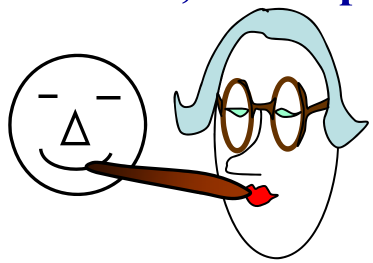


# Intracavity Phase Interferometry: 2 combs, same spacing, 2 CEO's

As the laser cavity fluctuates, do we need to stabilize $f_{01}$ and $f_{02}$ ?

No! The difference between CEO's remains constant!

CEO 1



CEO 2

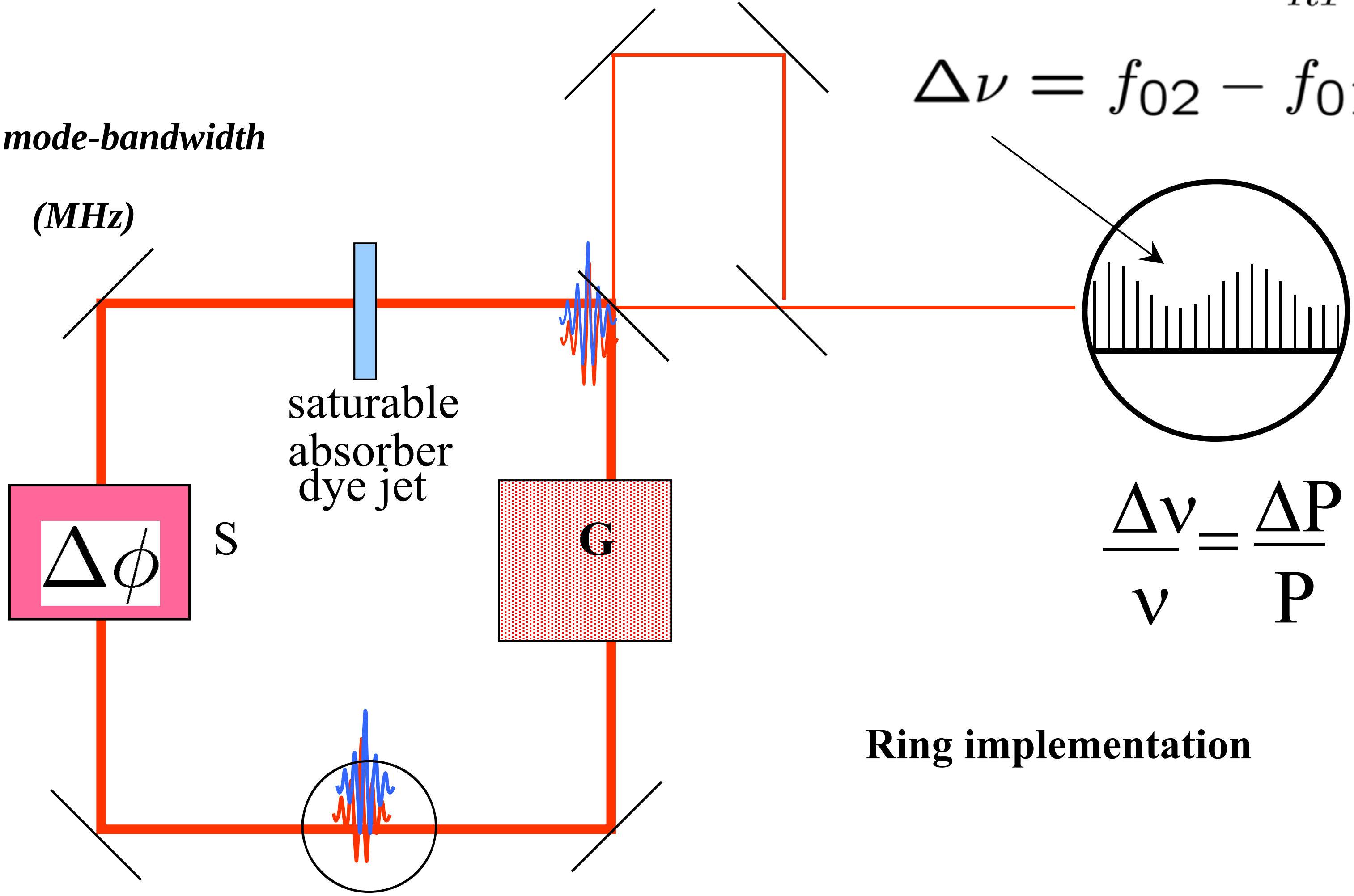
$$\nu_{m1} = f_{01} + m\frac{1}{\tau_{RT}}$$

$$\nu_{m2} = f_{02} + m\frac{1}{\tau_{RT}}$$

$$\Delta\nu = f_{02} - f_{01}$$

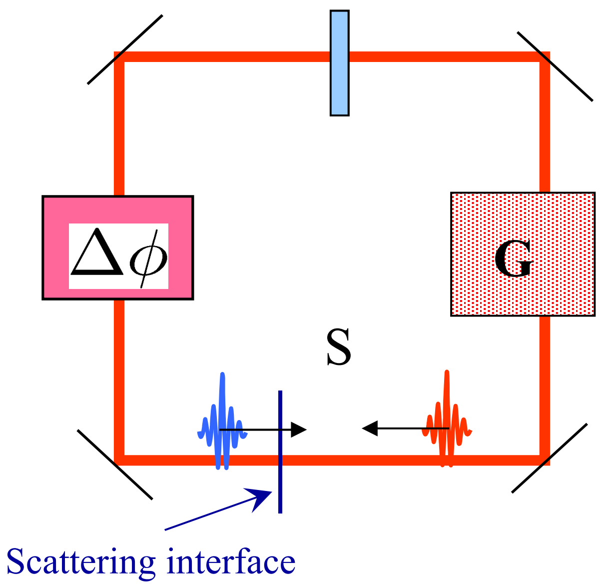
***Bandwidth $\Delta\nu$ << mode-bandwidth***

***(sub Hz)*** ***(MHz)***

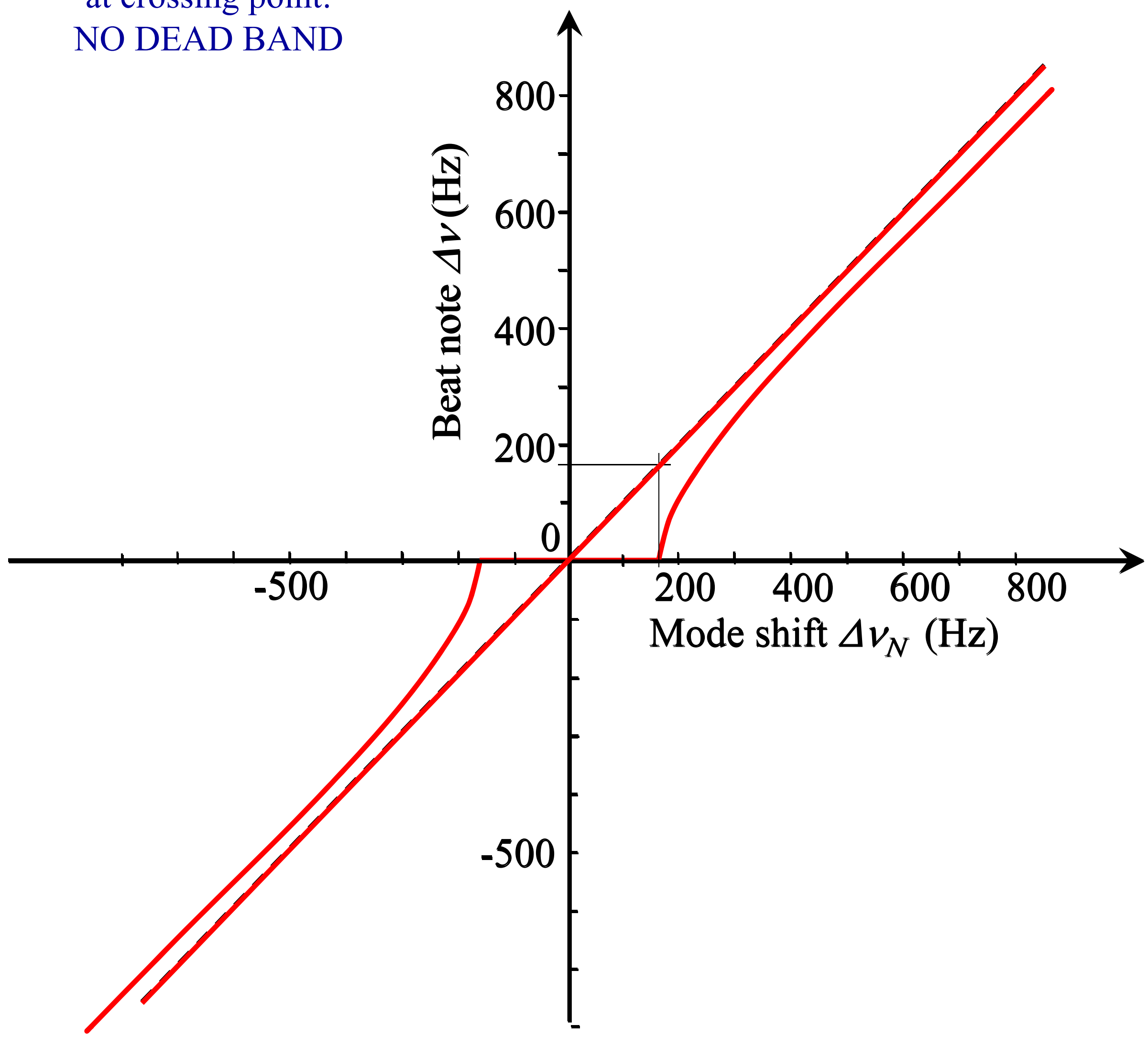



$$\frac{\Delta\nu}{\nu} = \frac{\Delta P}{P}$$

**Ring implementation**




Scattering interface not
at crossing point:
NO DEAD BAND




Bring scattering interface
at crossing point:
DEAD BAND

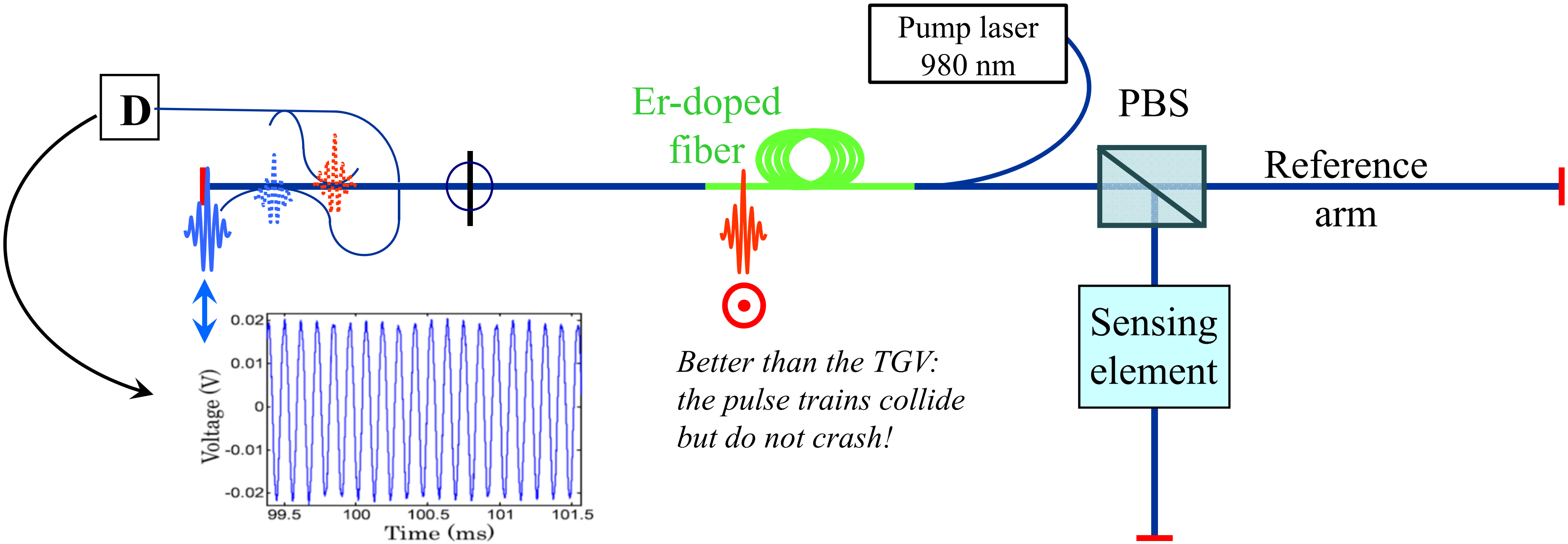
$\Delta\nu_{lock} = 150$ Hz

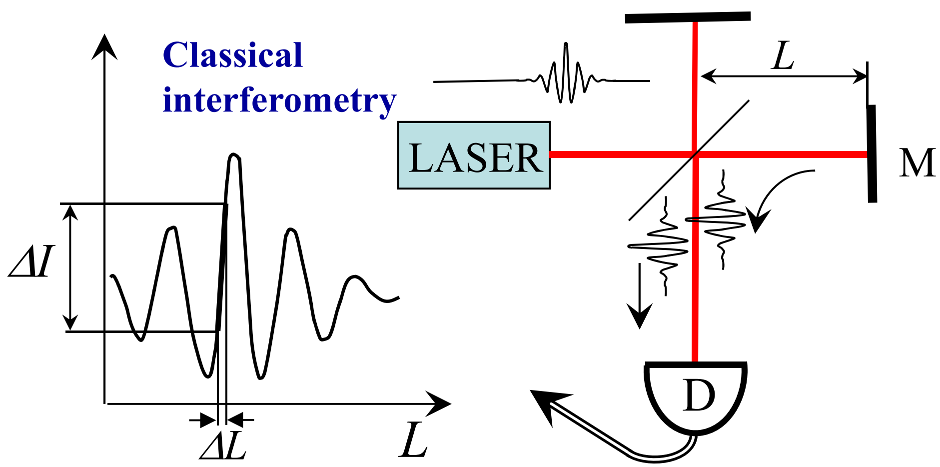
$$r = 2\pi \Delta\nu_{lock} \tau_{rt}$$

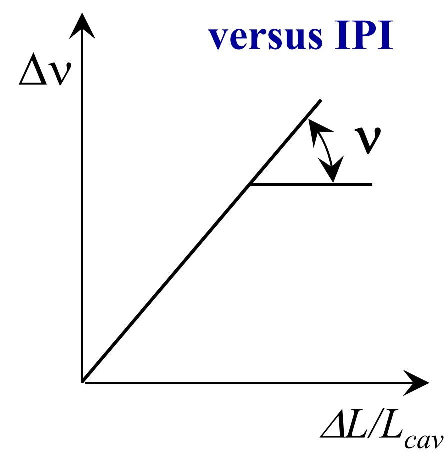
$$r^2 \approx 10^{-10}$$

## Intracavity Phase Interferometry: 2 combs, same spacing, 2 CEO's
## Linear fiber laser implementation

Polarization maintaining fiber laser: unique possibility to have two orthogonally polarized pulses circulating inside the cavity



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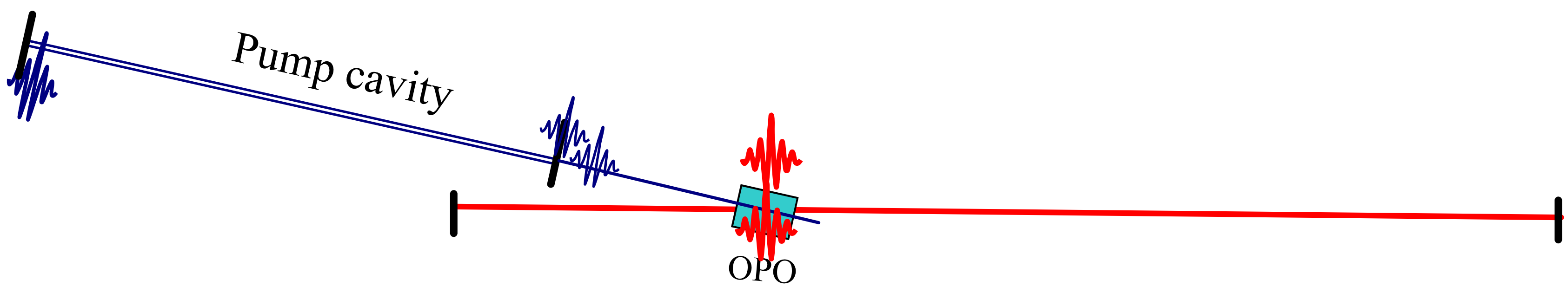
## State of the art with free space optics: Optical Parametric Oscillators (OPO)




External pumping, with pump cavity ½ length or signal cavity.

Advantage: Stability – no feedback from OPO to pump

Disadvantage: high power needed (> 1 nJ/pulse)

## IPI applied to the measurement of $n_2$

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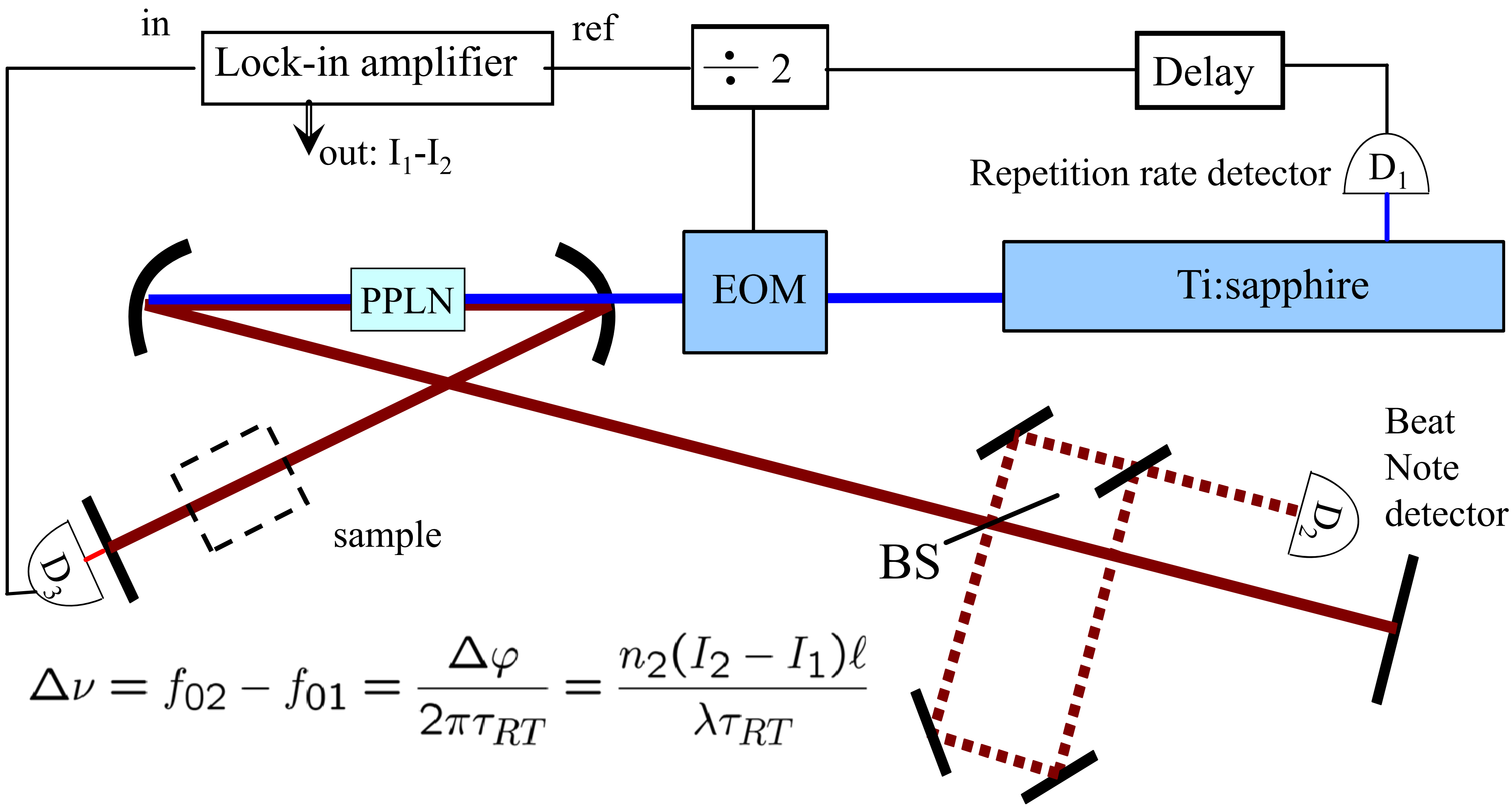
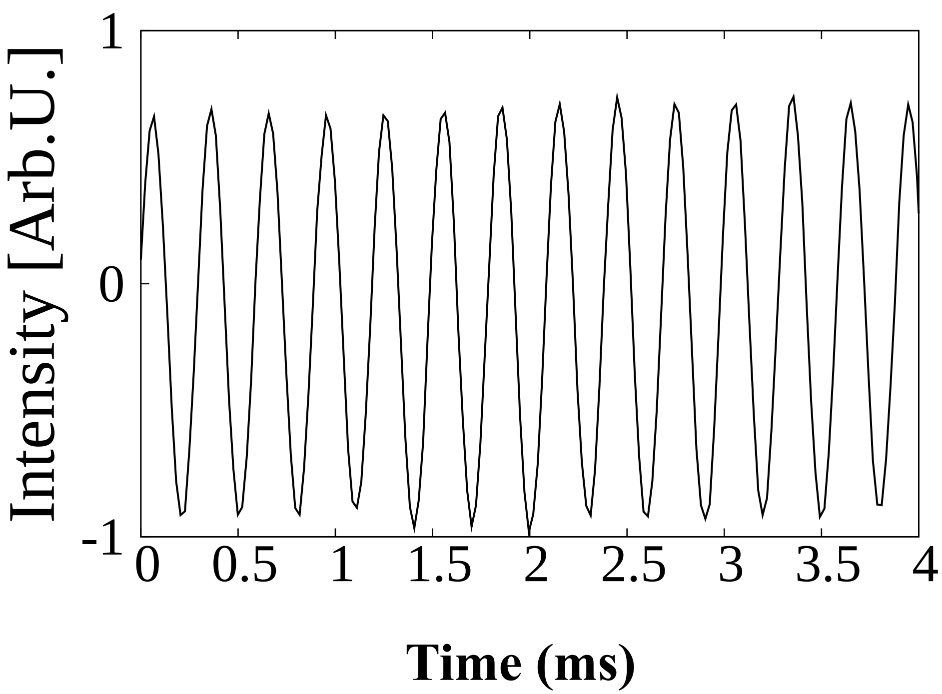
### External pumping, with pump cavity ½ length or signal cavity.

EOM: Pockel's cell to induce an intensity difference $I_1$-$I_2$ between the two OPO pulses



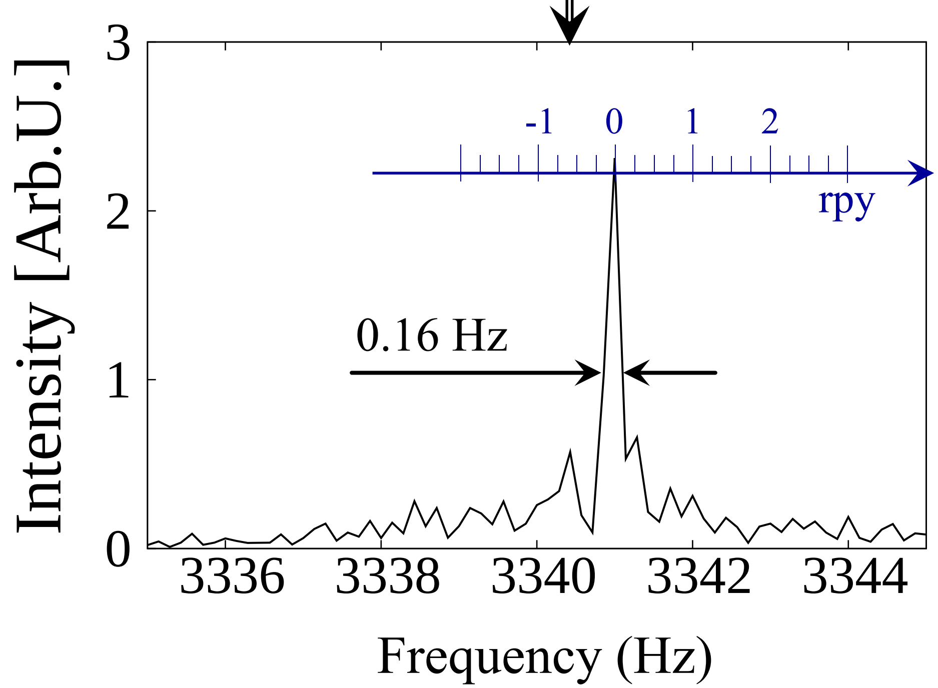

$$\Delta\nu = f_{02} - f_{01} = \frac{\Delta\varphi}{2\pi\tau_{RT}} = \frac{n_2(I_2 - I_1)\ell}{\lambda\tau_{RT}}$$

# Measurement of $n_2$: beat note and resolution




Section of beat note

Fourier transform of the beat note




Optimum resolution from 0.16 Hz bandwidth:

$\Delta(n_2) = 2\ 10^{-19}$ cm$^2$/W

Optical path resolution $\Delta\ell = 0.4$ fm

rpy = turn/year

# Z-scan versus Intracavity Phase Interferometry (IPI)

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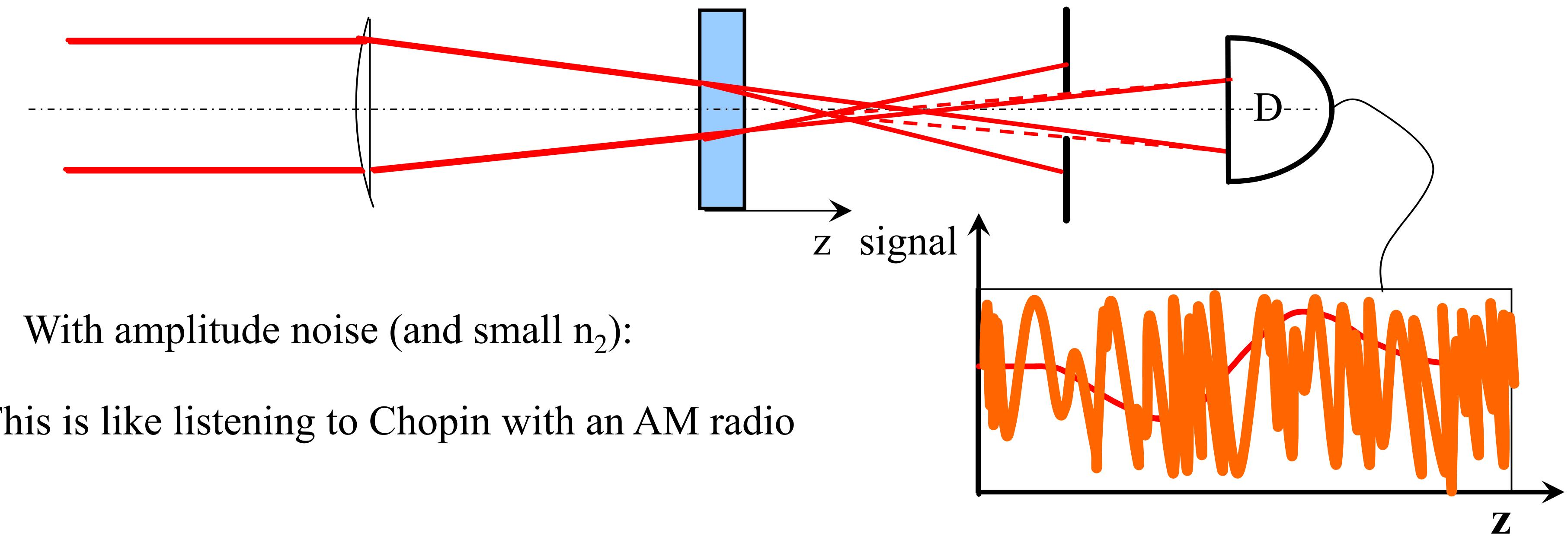
## Measurement of $n_2$ is a measurement of phase

Most phase measurements convert the phase in intensity, hence sensitive to amplitude noise

Example: zscan




With amplitude noise (and small $n_2$):

This is like listening to Chopin with an AM radio

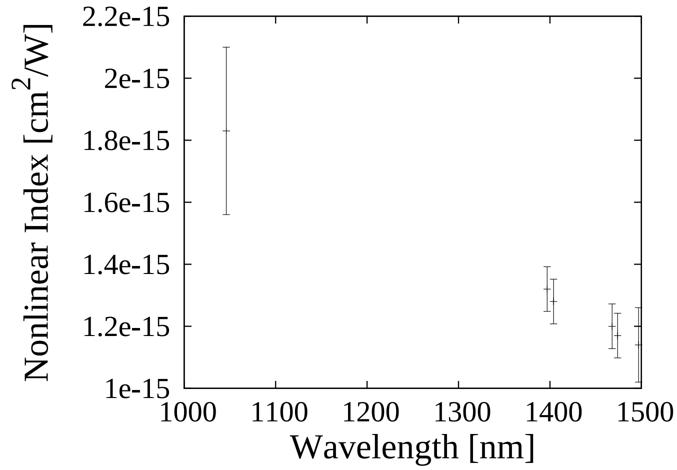
SOLUTION: go to an FM station!

This is what **IPI** is

# Measurement of $n_2$ --- IPI vs z-scan

| I P I | Z-scan |
|---|---|
| No scan required | Requires a … z-scan |
| Single intensity difference provides $n_2$ | |
| Intensity measurements on continuous beam | Requires single shot determination of the intensity |
| Frequency measurement (larger dynamic range) | Intensity measurement |
| Not affected by amplitude noise | Amplitude noise sensitive |

OPO tunable ⟶ Dispersion of $n_2$



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# Frequency combs to detect phase changes of $10^{-8}$: Intracavity Phase Interferometry Part I

The magic of intracavity interaction: it is a field full of surprises

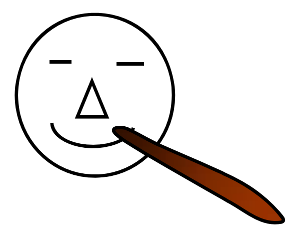
An example: a ring laser is a two level system with Rabi frequencies!?!?

Intracavity Phase Interferometry – What it is NOT

Intracavity Phase Interferometry – What it is: metrology with laser

Introduction on frequency combs

Parenthesis on the CEO



Implementation of IPI with free space components

Fiber implementation of IPI

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**Part II**

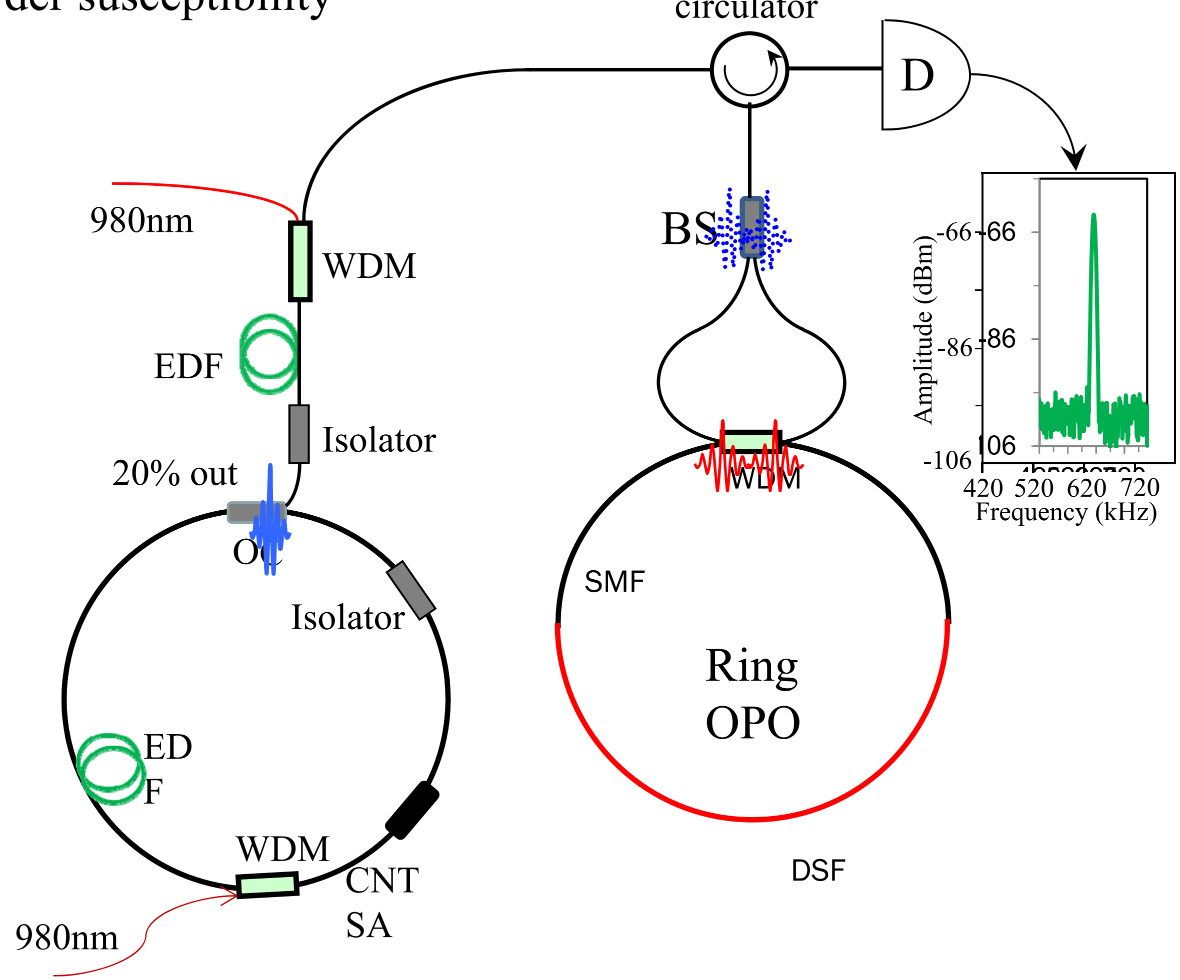
Manipulating phase response with dispersion

***Is it slow/fast light?***

Can we make a purely optical accelerometer?

## Fiber implementation: optical Parametric Oscillator laser gyro

Nonlinearity: third order susceptibility




All-fiber bidirectional optical parametric oscillator for precision sensing
R. Gowda, N. Nguyen, J.-C. Diels, R. A. Norwood, N. Peyghambarian, and K. Kieu
Optics Letters 40: 2033—2036 (2015)

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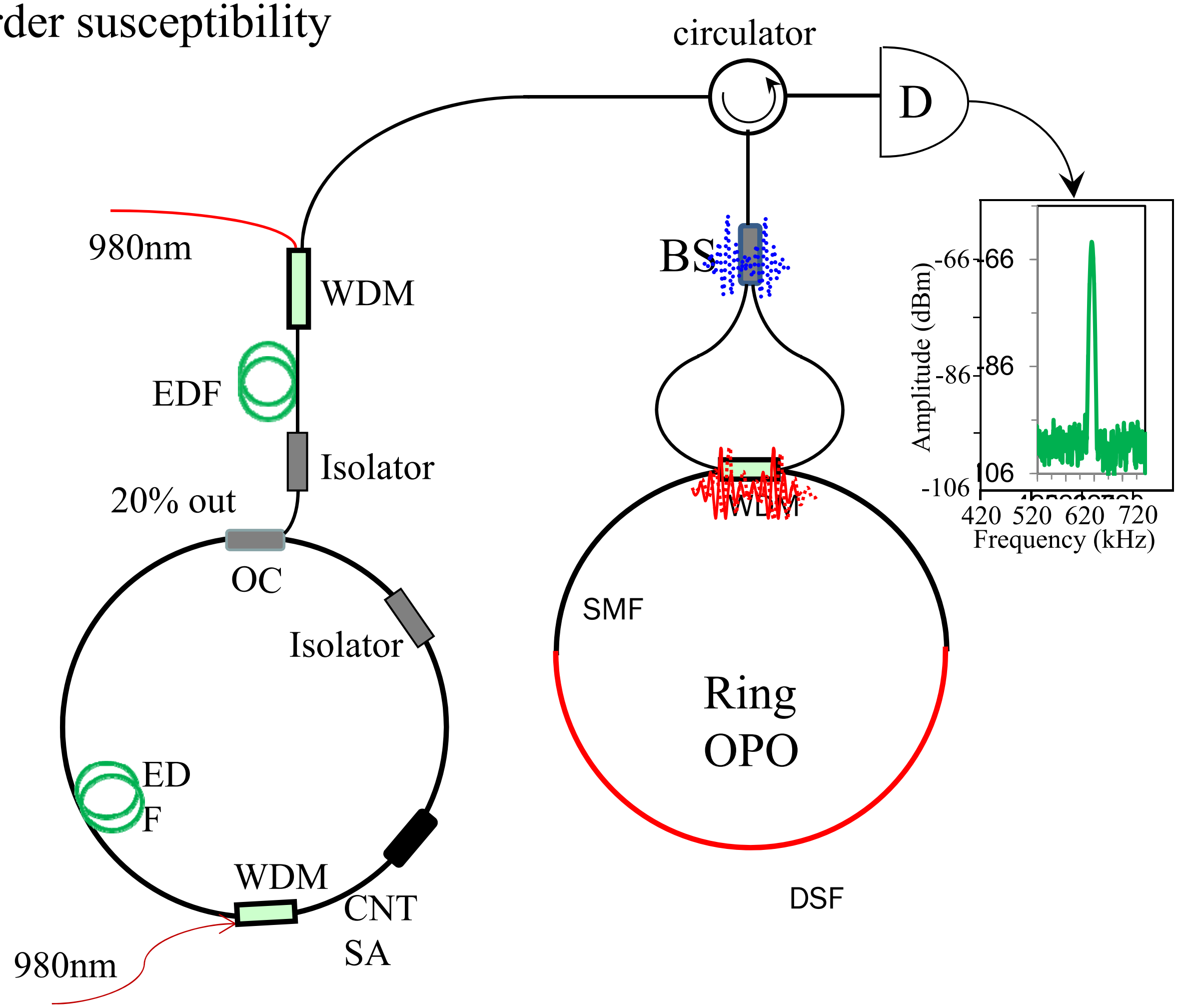


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# Fiber implementation: optical Parametric Oscillator laser gyro

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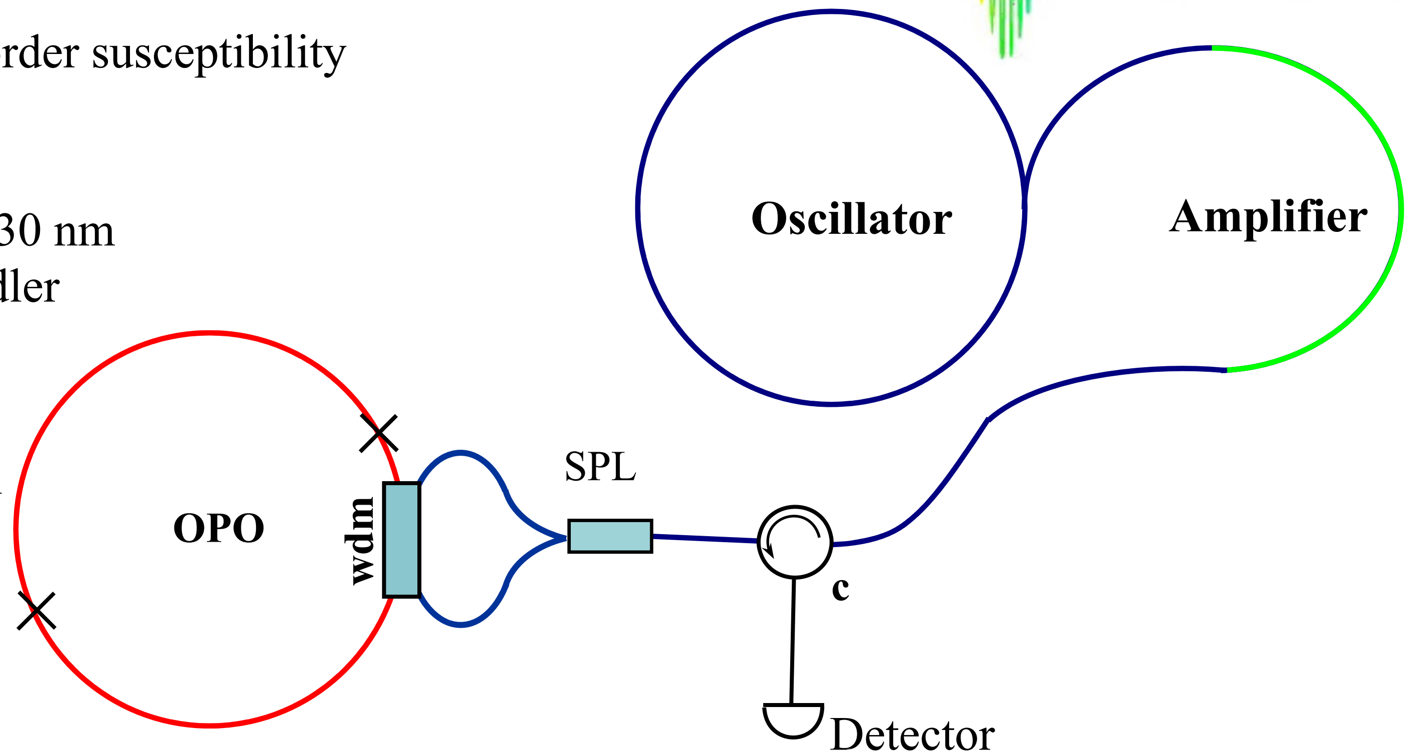
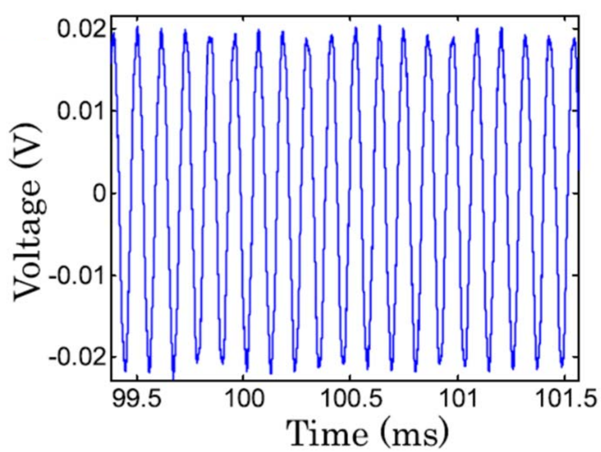
Nonlinearity: third order susceptibility

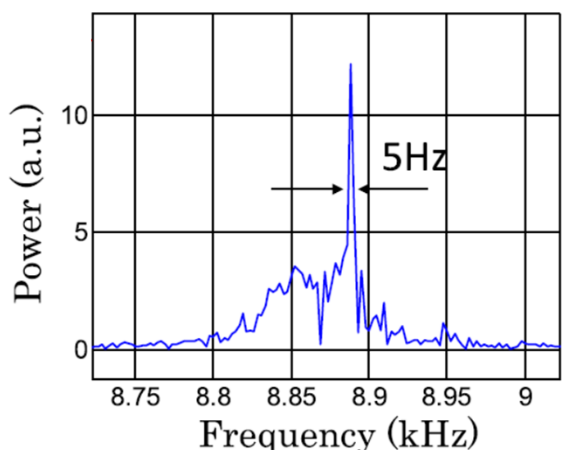
1550 nm pump    signal    1630 nm idler

$$\omega_p+\omega_p=\omega_s+\omega_i$$

The nonlinear gain in the FOPO loop is four wave mixing




R. Gowda, N. Nguyen, J.-C. Diels, R. Norwood, N. Peyghambarian, and K. Kieu, *Optics Letters*, submitted, 2015

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# Frequency combs to detect phase changes of $10^{-8}$: Intracavity Phase Interferometry Part I

The magic of intracavity interaction: it is a field full of surprises

**Hamburgers for thoughts:** fm resolution with mirrors with a surface roughness like the Everest???

An example: a ring laser is a two level system with Rabi frequencies!?!?

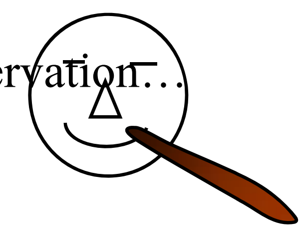
Intracavity Phase Interferometry – What it is NOT

Intracavity Phase Interferometr $\tilde{r}_{21} = -\tilde{r}^*_{12} = -\tilde{r}^*$ ser ???

Introduction on frequency combs

Parenthesis on the CEO

It is all about energy conservation…



Implementation of IPI with free space components

Fiber implementation of IPI

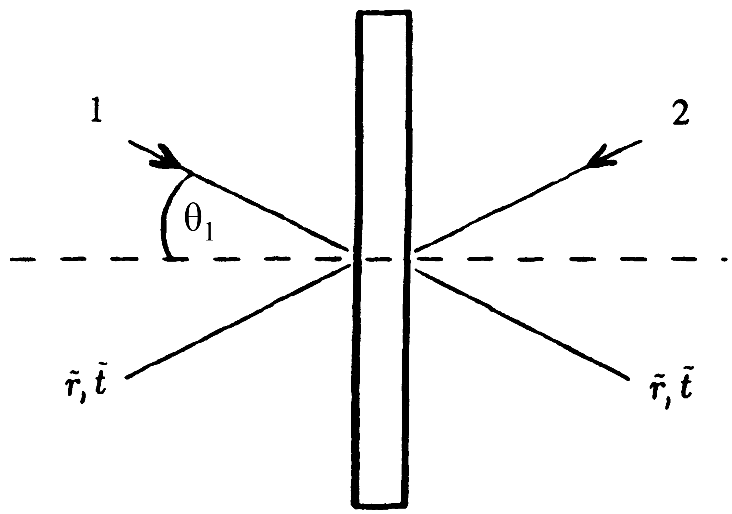
**Part II**

Manipulating phase response with dispersion

***Is it slow/fast light?***

Can we make a purely optical accelerometer?

Can we compete with LIGO?




$$|\tilde{r}|^2 + |\tilde{t}|^2 = 1$$

$$|\tilde{r} + \tilde{t}|^2 + |\tilde{r} + \tilde{t}|^2 = 2$$

$$2[\tilde{r}\tilde{t}^* + \tilde{r}^*\tilde{t}] = 0$$

For a non-symmetric interface, one finds:

$$\tilde{t}_1\tilde{t}_2 - \tilde{r}_1\tilde{r}_2 = 1$$

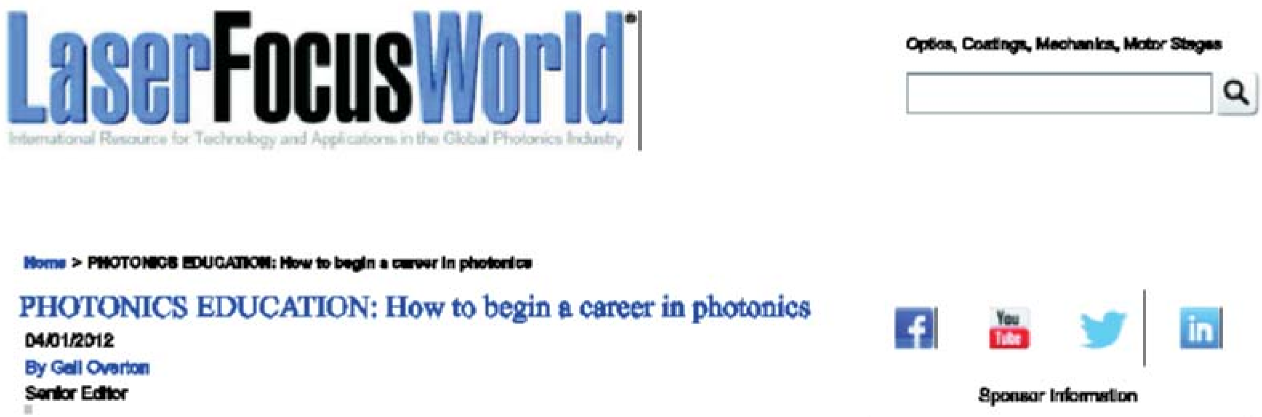
Energy conservation in the Gires-Tournois interferometer imposes that:

$$\tilde{r}_1 = -\tilde{r}_2^*$$

Optical Sciences Program at UNM – the pride of the nation in the 1980’s – 1990.

One out of 3 Universities to offer a PhD in Optics

NOW:




This article highlights the optics/photonics educational programs at several institutions worldwide

...top optics/photonics institutions like **CREOL at UCF** or **École Polytechnique**,
+ ``myriad lesser-known colleges and universities”

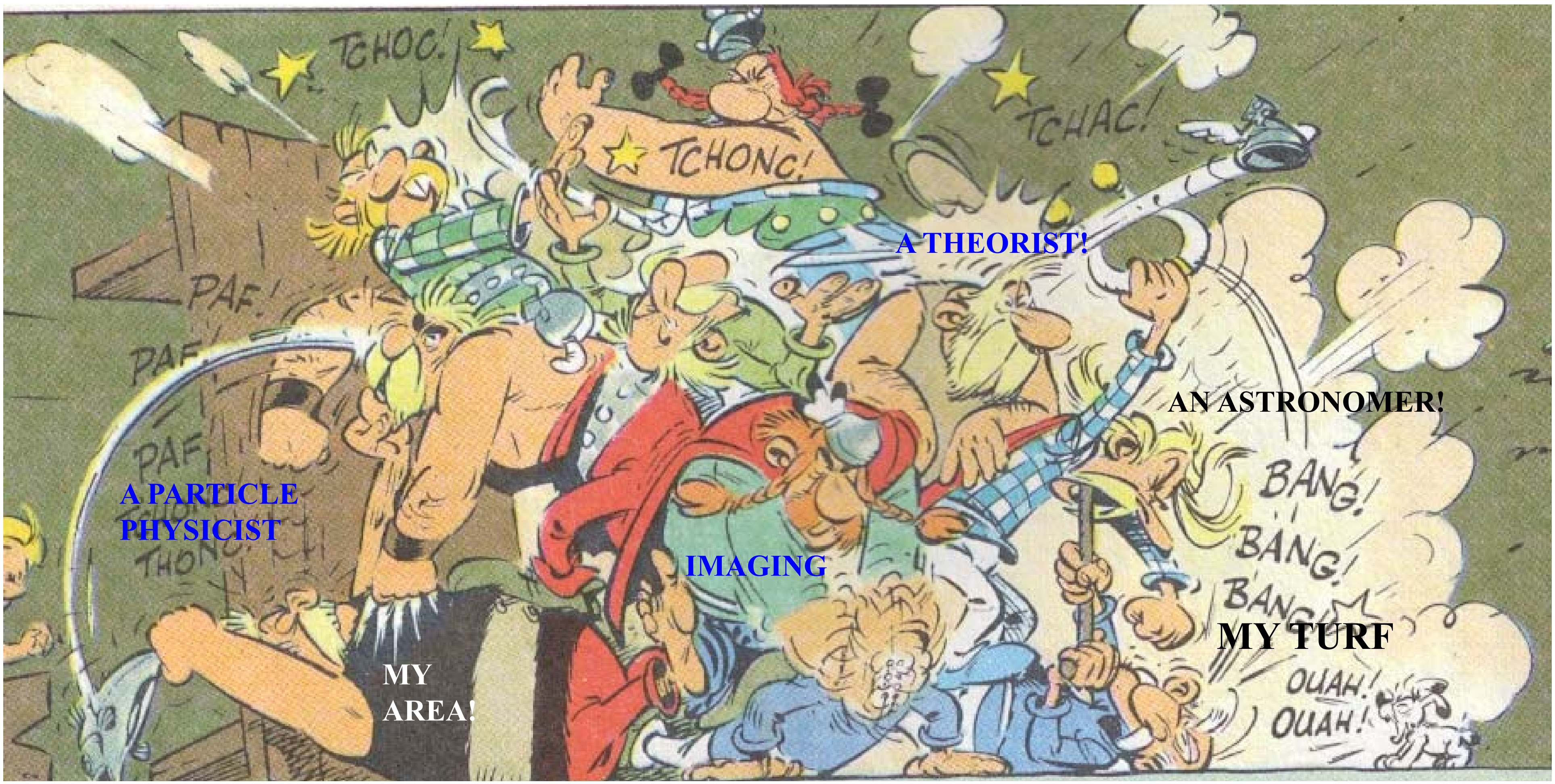
The US cream of the crop **…OSC; Tucson, AZ,** U. of **Rochester** director Xi-Cheng Zhang

Heriot-Watt University Tianjin University, etc et … but NOT UNM

Semiconductor Manufacturing

Lost to the program – Atomic and Molecular Optics (Howard Bryant, Charles Beckel, Wilhelm Becker) M.O Scully and too many to list whose operation was eliminated

The position of a retiree is left to intra-departmental demagogy, rather than rational considerations about preserving the intellectual and material legacy of Federal grants.



In view of the great vision of UNM,
their mascot should not be:

The lobo

There is a need for a
policy to transfer the infrastructure
destined to be scraped, to another institution
having a successful and promising program

but the MOLE

…with blinders