Donna Strickland

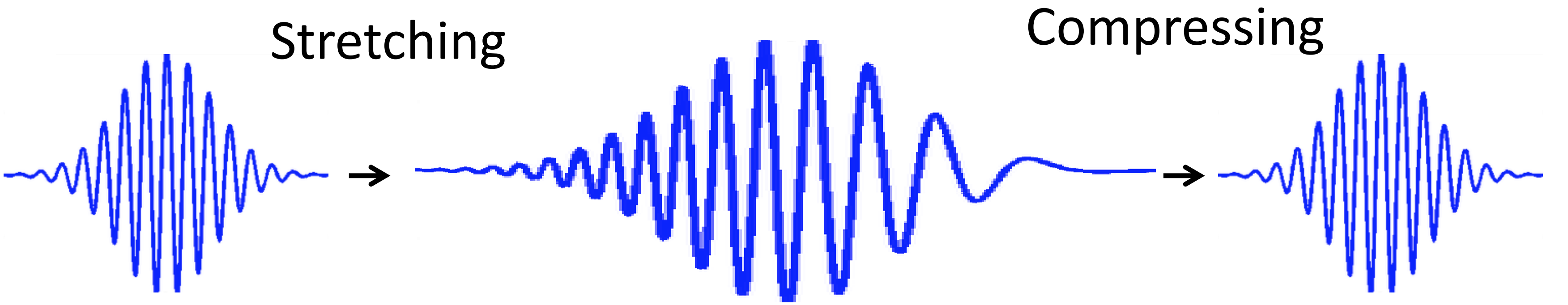
Department of Physics and Astronomy

Siegman International School on Lasers 2017

August 8, 2017

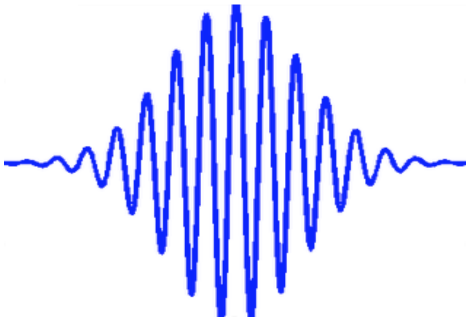
UNIVERSITY OF WATERLOO

Pulse Compression



Reference:
"Lasers" , Anthony E. Siegman,
University Science Books
Chapter 9

Ideal Short Pulse: Fourier Transform Limit



Transform limited pulse – phase changes linearly in time throughout the pulse

Eg: Gaussian amplitude profile pulse: $E(t) = E_t e^{-\mathrm{a}t^2} e^{i\omega_0 t}$

The Fourier Transform of this wave is: $E(\omega) = E_\omega e^{-\frac{(\omega-\omega_0)^2}{4a}}$

We don't measure the amplitudes, we measure the irradiance, I, and spectral power, S.

$I(t) \propto |E(t)|^2 = I_0 e^{-2\mathrm{a}t^2}$

$S(\omega) \propto |E(\omega)|^2 = S_0 e^{-\frac{2(\omega-\omega_0)^2}{4a}}$

Ideal Short Pulse: Fourier Transform Limit

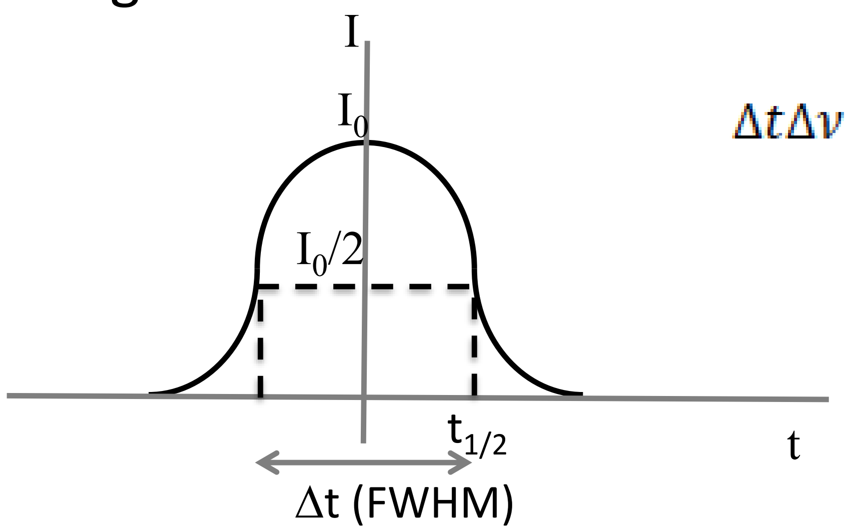
$$I(t) \propto |E(t)|^2 = I_0 e^{-2at^2} \qquad S(\omega) \propto |E(\omega)|^2 = S_0 e^{-\frac{2(\omega-\omega_0)^2}{4a}}$$

$$\Delta t = 2t_{1/2} = 2\sqrt{\frac{ln2}{2a}} = \sqrt{\frac{2\ ln2}{a}}$$

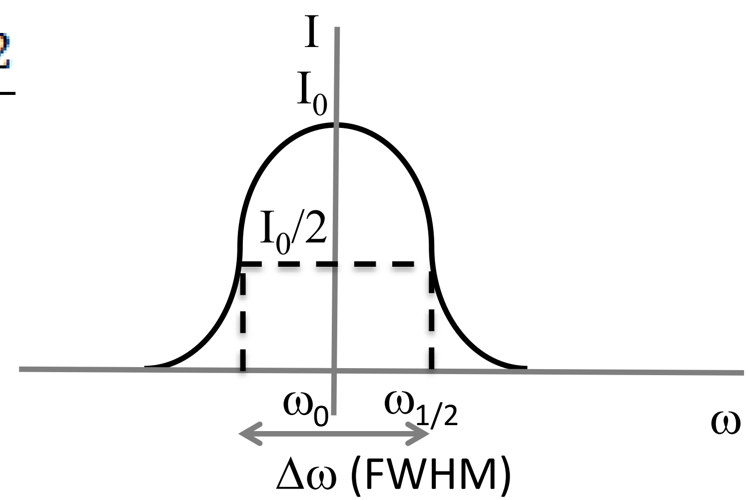
$$\Delta \nu = \frac{\Delta\omega}{2\pi} = \frac{2(\omega_{1/2} - \omega_0)}{2\pi}$$

$$= \frac{2}{2\pi}\sqrt{2a\ ln2} = \frac{1}{\pi}\sqrt{2a\ ln2}$$

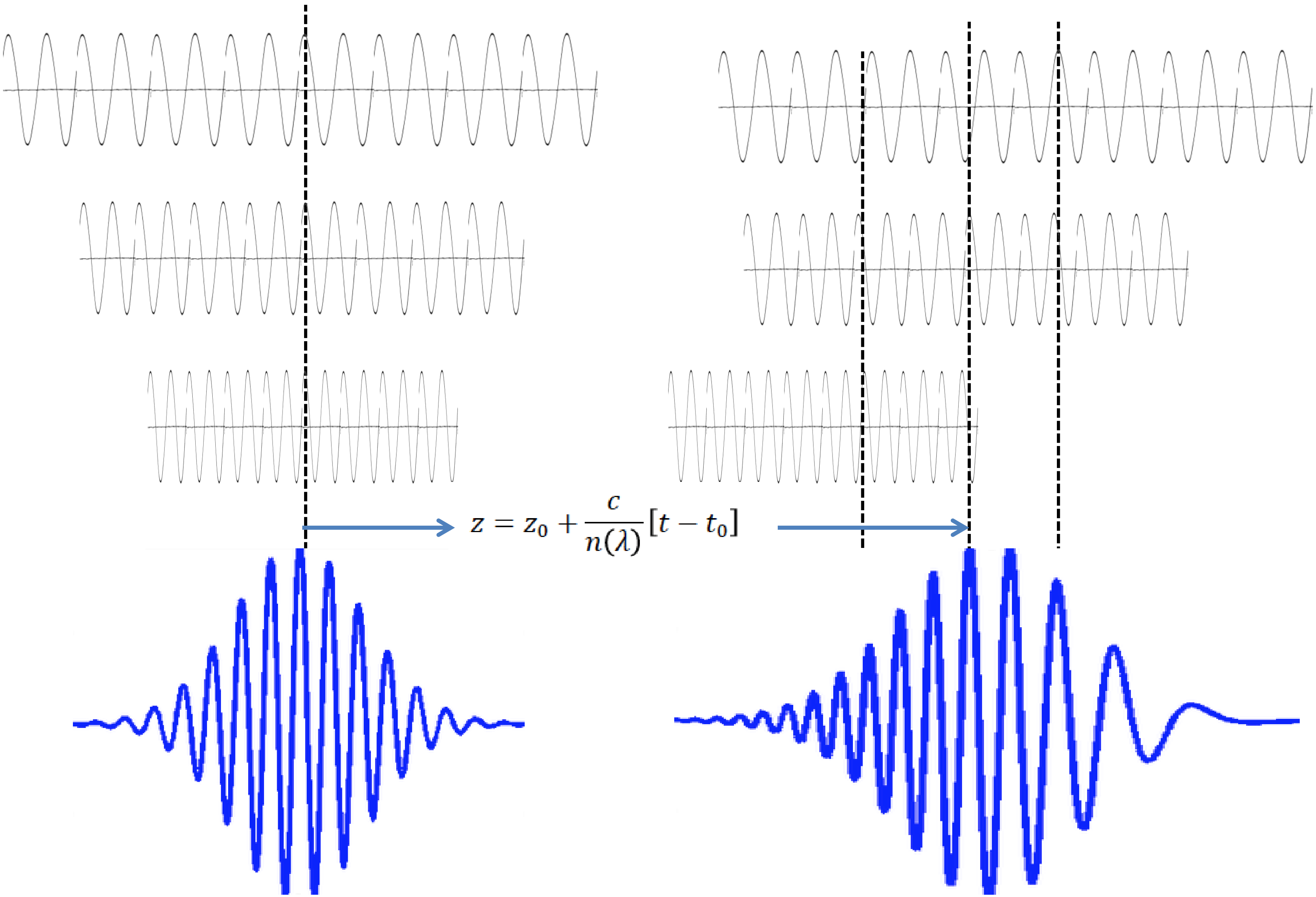
Giving the minimum time-bandwidth product:



$$\Delta t \Delta \nu = \frac{2ln2}{\pi}$$



Material Dispersion, n(λ)



$$z = z_0 + \frac{c}{n(\lambda)}[t - t_0]$$

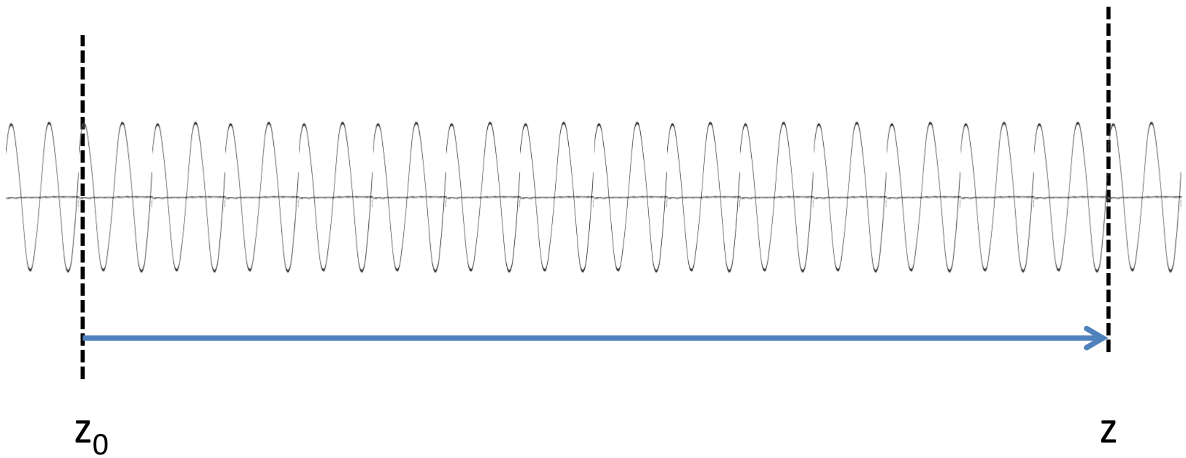
Material Dispersion

Single wavelength: $E(z,t) = E_0 cos(\omega t - kz + \emptyset_0)$

where $k = \frac{\omega n}{c} = \frac{2\pi n}{\lambda_0}$

ω is a constant of the wave, and k is material dependent, λ_0 is the wavelength in vacuum

Holding time constant, $t = t_0$

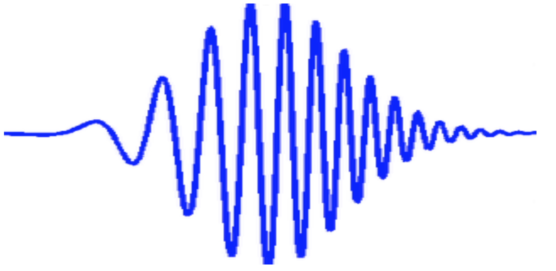


$$\phi_1 = (\omega t_0 - kz_0 + \emptyset_0) \qquad \phi_2 = (\omega t_0 - kz + \emptyset_0)$$

$$\Delta\emptyset = \phi_2 - \phi_1 = k(z - z_0)$$

Material Dispersion

Chirped Optical Pulses:



$$E(z,t) = Re\{E(t)e^{i(\omega_0 t - k_0 z)}\}$$

Where E(t) is a complex function, and here we will assume the amplitude is Gaussian

$$E(t) = E_0 e^{-at^2} e^{i\phi(t)}$$

The phase of the wave is no longer simply linear with time or distance.

Taking z=0, $\phi_{tot} = \omega_0 t + \phi(t)$

The instantaneous frequency is given by:

$$\omega = \frac{d\phi_{tot}}{dt}$$

Material Dispersion

$$\omega = \frac{d\phi_{tot}}{dt}$$

So what is ϕ_{tot} as a function of distance z?

As we have seen, the phase changes with propagation length:

$\phi(z) = \phi_0 + kz$

But k is not linearly dependent on frequency

And for some reason, we no longer use the symbol k for wavenumber, but we talk about a propagation constant, $\beta(\omega)$.

Material Dispersion

The propagation constant then varies with ω and the frequency dependent refractive index

$$\beta(\omega) = \frac{\omega n(\omega)}{c}$$

$\beta(\omega)$ depends on the material, but it doesn't depend on time

But we have written the electric field as a function of time and propagation length, and the phase was time dependent

$$E(z,t) = E_0 e^{-\Gamma_0 t^2} e^{i\phi(t)} e^{i(\omega_0 t - \beta(\omega_0) z)}$$

Time and frequency are Fourier Transform pairs so you cannot simply write an expression for $\phi(t)$ using $\beta(\omega)$

Material Dispersion

You must use the Fourier Transform of the electric field E(z, ω)

If we are concerned with propagation, then we can choose to have the starting point be z=0.

$$E(0,\omega) = E_0 e^{-\Gamma(\omega-\omega_0)^2}$$

Where Γ is complex giving both the bandwidth of the spectral amplitude and the frequency dependent phase

As long as the material is not absorbing (gain), only the phase is changed by propagation:

$$E(z,\omega) = E(0,\omega)e^{-i\beta(\omega)z}$$

$$\phi(\omega,z) = \phi(\omega,z=0) + \beta(\omega)z$$

Material Dispersion

$$\phi_{tot}(\omega, z) = \phi_{tot}(\omega, z_0) + \beta(\omega)z$$

$$\beta(\omega) = \frac{\omega n(\omega)}{c}$$

But $n(\omega)$ is a complicated function so instead we can use a Taylor Expansion, as long as the bandwidth is small compared to the central frequency, ω_0 .

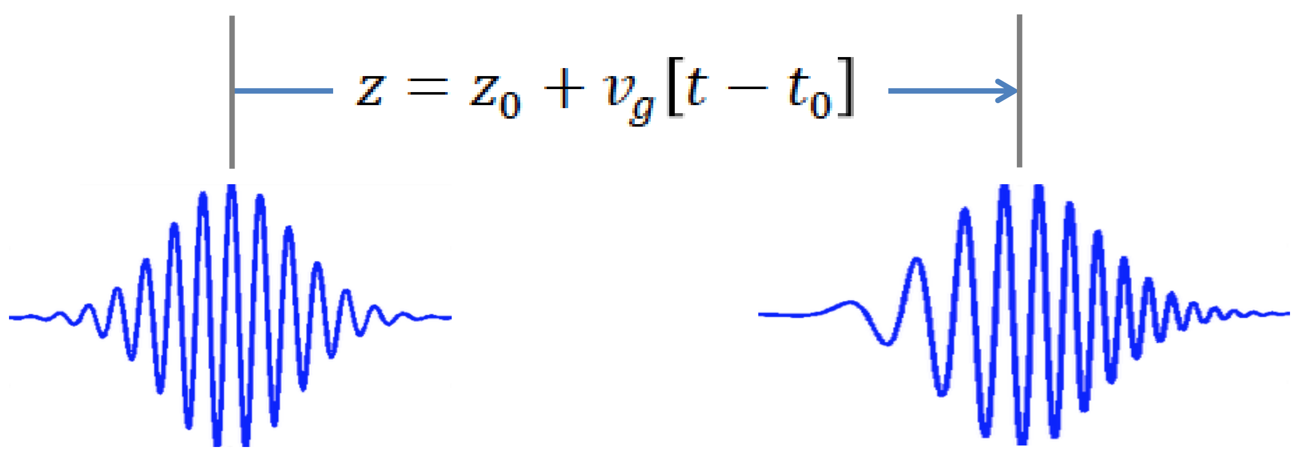
$$\beta(\omega) = \beta(\omega_0) + \frac{d\beta}{d\omega}(\omega - \omega_0) + \frac{1}{2}\frac{d^2\beta}{d\omega^2}(\omega - \omega_0)^2 + \frac{1}{6}\frac{d^3\beta}{d\omega^3}(\omega - \omega_0)^3 + \cdots$$

Dispersion Terms

$$\beta_0 \equiv \beta(\omega)|_{\omega=\omega_0} = \frac{\omega_0}{v_\phi(\omega_0)} \equiv \frac{\omega_0}{phase\ velocity}$$

$$\beta' \equiv \frac{d\beta}{d\omega}|_{\omega=\omega_0} = \frac{1}{v_g(\omega_0)} \equiv \frac{1}{group\ velocity}$$

$$\beta'' \equiv \frac{d^2\beta}{d\omega^2}|_{\omega=\omega_0} = \frac{d}{d\omega}\left(\frac{1}{v_g(\omega_0)}\right) \equiv$$ Group Velocity Dispersion (GVD)



Pulse Stretching

Assume we start with a transform limited Gaussian pulse in time:

$$E(0,t) = E_0 e^{-\Gamma_0 t^2} e^{i\omega_0 t}$$

The Fourier transform of a Gaussian pulse in time is Gaussian in frequency:

$$E(0,\omega) = E_0 e^{-\frac{(\omega-\omega_0)^2}{4\Gamma_0}}$$

After propagating a distance z, in a material the electric field now has the expression:

$$E(z,\omega) = E(0,\omega) e^{-i\beta(\omega) z}$$

We will consider the propagation constant up to second order (GVD) in the Taylor expansion.

Pulse Stretching

$$E(z,\omega) = E_0 e^{-\frac{(\omega-\omega_0)^2}{4\Gamma_0}} exp\left[-i\left(\beta_0 + \beta'(\omega-\omega_0) + \frac{1}{2}\beta''(\omega-\omega_0)^2\right)z\right]$$

Rearranging the expression gives:

$$E(z,\omega) = E_0 exp\left[\left(-i\beta_0 z - i\beta' z(\omega-\omega_0) - \left(\frac{1}{4\Gamma_0} + \frac{i\beta'' z}{2}\right)(\omega-\omega_0)^2\right)\right]$$

We need to Fourier Transform this expression to get the time dependent field.

Pulse Stretching

Fourier Transform, with central frequency term pulled out in front, to have frequency in terms of the difference: $(\omega-\omega_0)$

$$E(z,t) = e^{i\omega_0 t}\int_{-\infty}^{\infty} E(z,\omega)\, e^{i(\omega-\omega_0)t} d\omega$$

Substituting in expression for $E(z,\omega)$ gives:

$$E(z,t) = e^{i(\omega_0 t-\beta_0 z)}\int_{-\infty}^{\infty} E_0 exp\left[\left(i(t-\beta' z)(\omega-\omega_0) - \left(\frac{1}{4\Gamma_0}+\frac{i\beta'' z}{2}\right)(\omega-\omega_0)^2\right)\right] d\omega$$

We can now write a shifted time coordinate: $t' = t - \beta' z$

And write a z-dependent pulse width parameter $\Gamma(z)$

$$\frac{1}{\Gamma(z)} = \frac{1}{\Gamma_0} + i2\beta'' z$$

Pulse Stretching

Substituting in expression for Γ(z) and t' gives:

$$E(z,t) = e^{i(\omega_0 t - \beta_0 z)} \int_{-\infty}^{\infty} E_0 e^{-\frac{(\omega-\omega_0)^2}{4\Gamma(z)}} e^{i(\omega-\omega_0)t'} d(\omega - \omega_0)$$

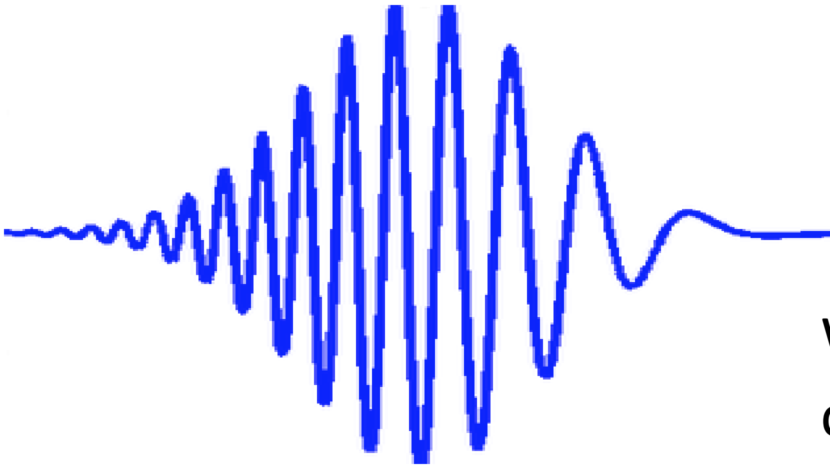
The integral is the inverse Fourier transform of a Gaussian, giving:

$$E(z,t) = exp\left[i\omega_0\left(t - \frac{z}{v_\phi(\omega_0)}\right)\right] exp\left[-\Gamma(z)\left(t - \frac{z}{v_g(\omega_0)}\right)^2\right]$$

Where we have substituted in the expressions for the phase velocity, $v_\phi(\omega_0)=\omega_0/\beta_0$ and group velocity, $v_g=1/\beta'$.

E(z,t) is still a Gaussian, where the phase travels with the phase velocity, the amplitude profile travels with the group velocity and the width of the pulse is determined by Γ(z)

Chirped Pulse: Non-Fourier Transform Limited



$$E(t) = E_0 e^{-at^2} e^{i\phi(t)}$$

Where the phase can now be a complicated function of time

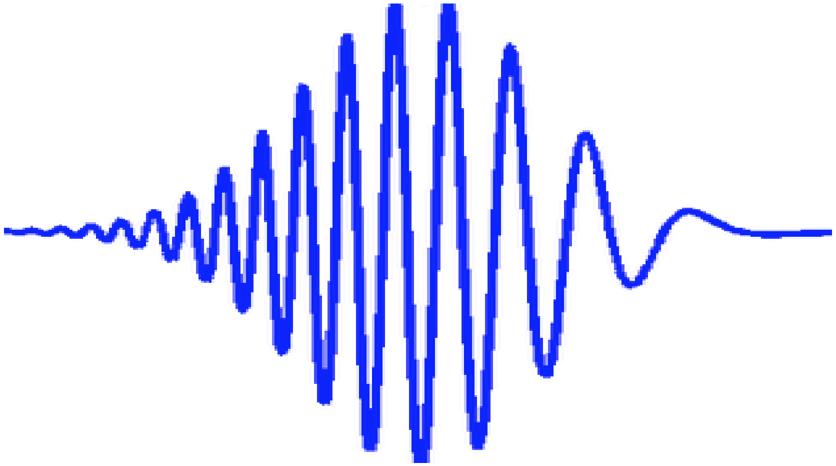
As long as the bandwidth is not too large we can use a Taylor expansion to write, $\phi(t)$ – for now we just need terms up to t^2.

$$\phi(t) = \omega_0 t + bt^2$$

This phase gives a linear chirp:

$$\omega = \omega_0 + bt$$

Chirped Pulse: Non-Fourier Transform Limited



$$E(t) = E_0 e^{-at^2} e^{i(\omega_0 t + bt^2)}$$

$$E(t) = E_0 e^{-(a-ib)t^2} e^{i(\omega_0 t)}$$

Compare this to the Gaussian pulse stretched by dispersion:

$$E(t) = exp(i\omega_0 t) exp[-\Gamma t^2]$$

Chirped Pulse: Non-Fourier Transform Limited

For the pulse chirped from dispersion, we want to write Γ in terms of a and b:

$$\Gamma(z) = a - ib$$

$$\frac{1}{\Gamma(z)} = \frac{1}{\Gamma_0} + i2\beta''z$$

$$\Gamma(z) = \frac{1}{\frac{1}{\Gamma_0} + i2\beta''z}\left(\frac{\frac{1}{\Gamma_0} - i2\beta''z}{\frac{1}{\Gamma_0} - i2\beta''z}\right) = \frac{\frac{1}{\Gamma_0} - i2\beta''z}{\left(\frac{1}{\Gamma_0}\right)^2 + (2\beta''z)^2}$$

$$a = Re\{\Gamma(z)\} = \frac{\frac{1}{\Gamma_0}}{\left(\frac{1}{\Gamma_0}\right)^2 + (2\beta''z)^2} \qquad b = Im\{\Gamma(z)\} = \frac{2\beta''z}{\left(\frac{1}{\Gamma_0}\right)^2 + (2\beta''z)^2}$$

Chirped Pulse: Non-Fourier Transform Limited

$$\text{a} = \frac{\frac{1}{\Gamma_0}}{\left(\frac{1}{\Gamma_0}\right)^2 + (2\beta'' z)^2}$$

$$\frac{1}{\text{a}} = \left(\frac{1}{\Gamma_0}\right) + \Gamma_0 (2\beta'' z)^2$$

$$\Delta t_{stretched} = \sqrt{\frac{2ln2}{\text{a}}} = \sqrt{\frac{2ln2}{\Gamma_0}\left[1 + {\Gamma_0}^2 (2\beta'' z)^2\right]}$$

The shorter the original pulse, the larger the value Γ_0, which shows that for the same dispersion, β'', the pulse can get longer for a shorter original pulse.

Chirped Pulse: Non-Fourier Transform Limited

$$\Delta t_{stretched} = \sqrt{\frac{2ln2}{a}} = \sqrt{\frac{2ln2}{\Gamma_0}\left[1 + {\Gamma_0}^2 (2\beta'' z)^2\right]}$$

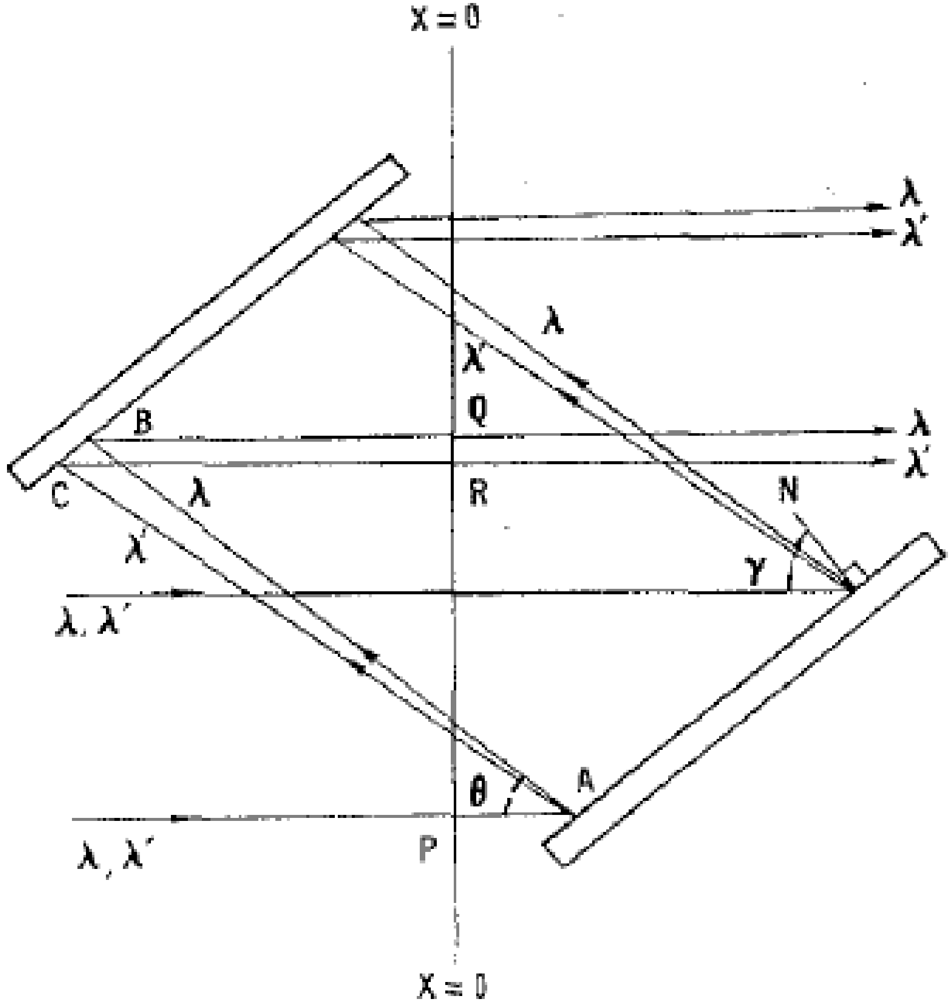
Note: the spectral bandwidth has not been altered by dispersion,

$$\Delta \nu = \frac{1}{\pi}\sqrt{2\Gamma_0\, ln2}$$

$$\Delta t \Delta \nu = \left(\sqrt{\frac{2ln2}{\Gamma_0}\left[1 + {\Gamma_0}^2 (2\beta'' z)^2\right]}\right)\left(\frac{1}{\pi}\sqrt{2\Gamma_0\, ln2}\right)$$

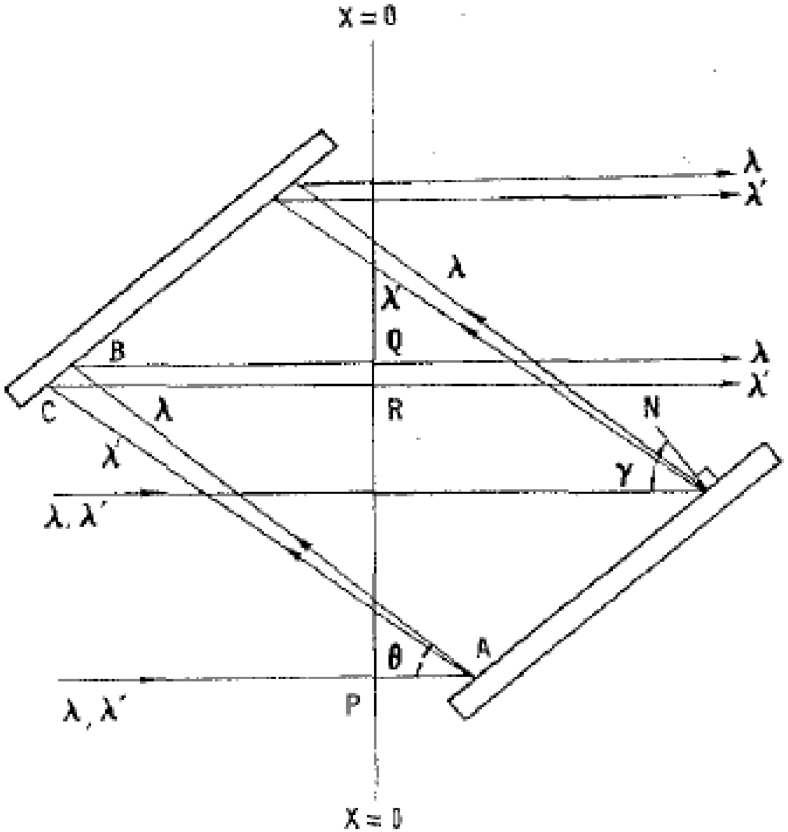
$$\Delta t \Delta \nu = \frac{2\, ln2}{\pi}\sqrt{1 + {\Gamma_0}^2 (2\beta'' z)^2} = \frac{2\, ln2}{\pi}\sqrt{(1 + (b/a)^2)}$$

Angular dispersion - parallel gratings



"Optical Pulse Compression With Diffraction Gratings" Edmond B. Treacy, IEEE J. Quantum Electron, 1969, QE-5, p. 454,

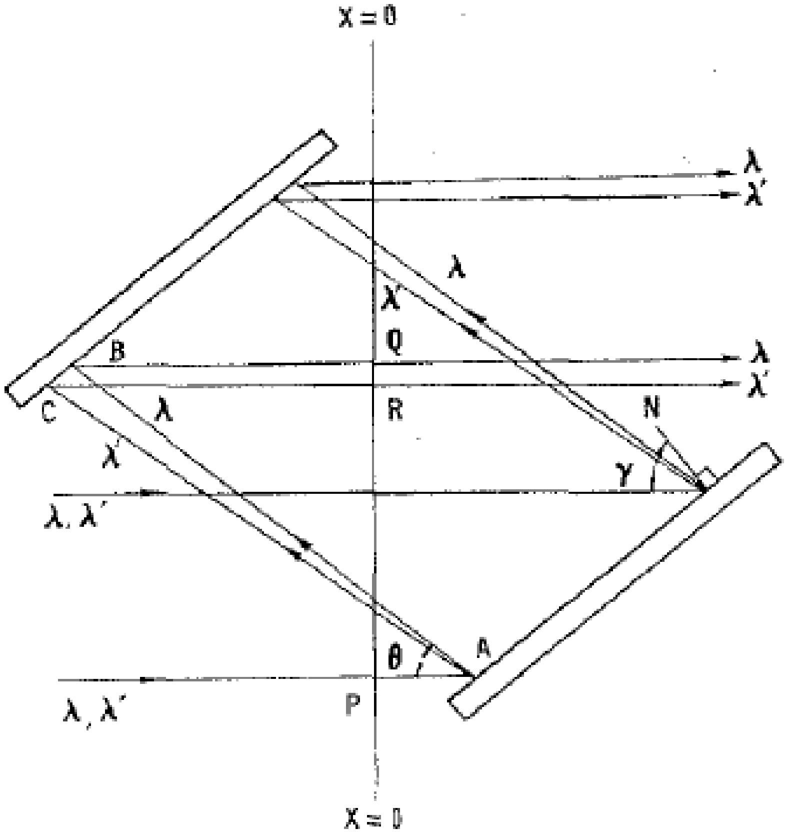
Angular dispersion - parallel gratings



Because the gratings are parallel, all wavelengths that travel in the same beam incident on the first grating will leave the second grating parallel to the incoming beam

Because each wavelength diffracts from the gratings at different angles, they travel different distances to get from the x=0 plane before striking the first grating back to x=0 after diffracting off the second grating

Angular dispersion - parallel gratings



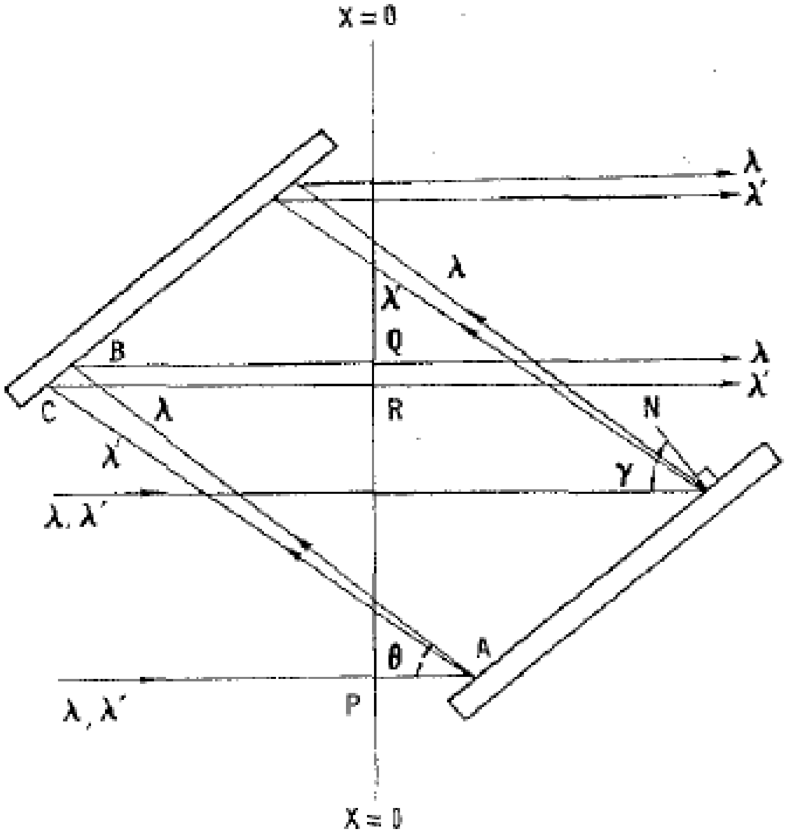
The phase accumulated from x=0 back to x=0 for each wavelength is given by:

ϕ = kp, where p is the path length traveled

Now we are assuming that the gratings are in vacuum so there is no material dispersion so we can write:

$$k = \frac{\omega}{c} \qquad \text{and} \qquad \emptyset = \frac{\omega}{c} p$$

Angular dispersion - parallel gratings



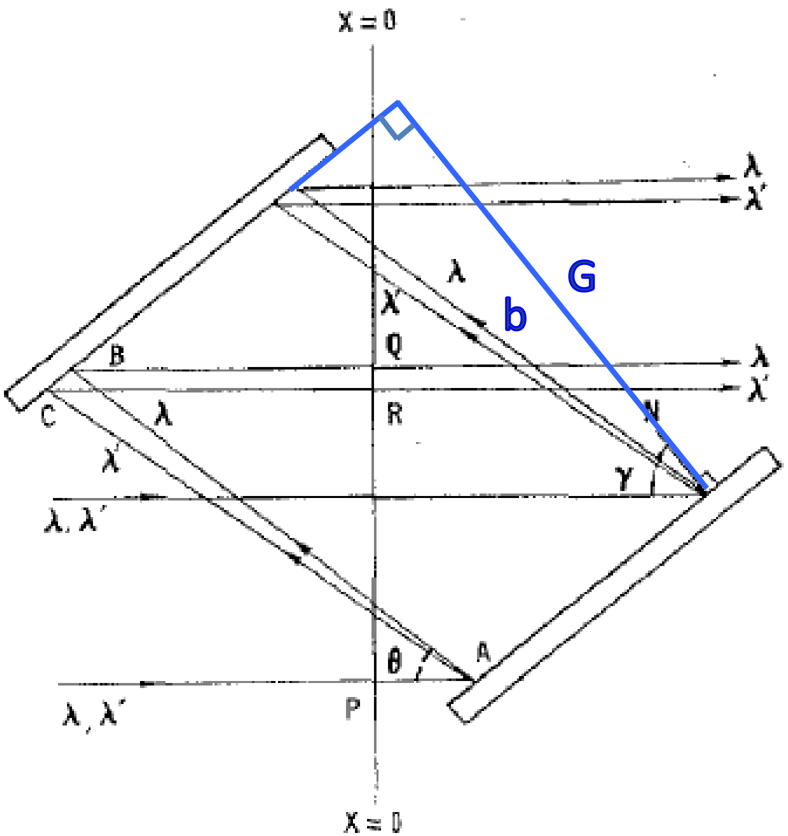
Now we need an expression for p as a function of ω

First we use geometry to get:

$p = b(1 + \cos\theta)$

Where b is the distance travelled between the two gratings and is wavelength dependent

Angular dispersion - parallel gratings



p = b(1 +cos θ)

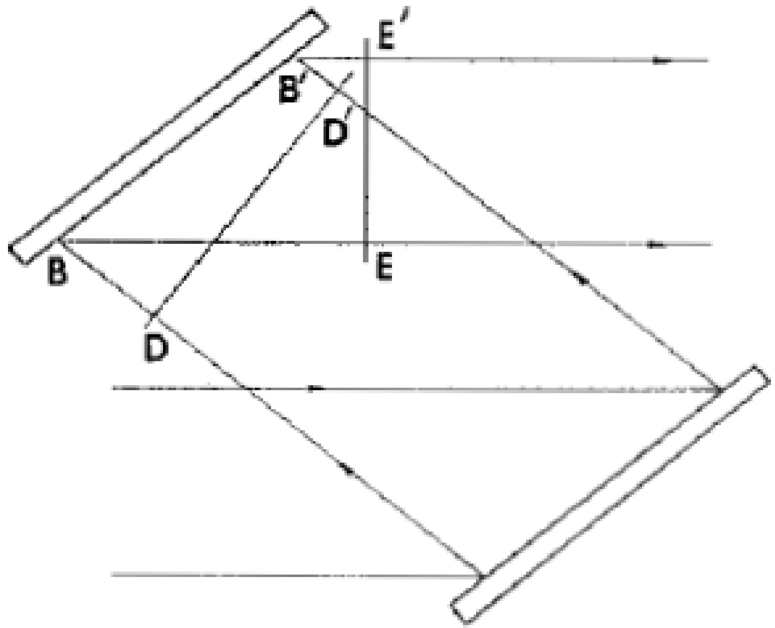
$$b = \frac{G}{\cos(\gamma - \theta)}$$

Where G is the perpendicular distance between the two gratings and (γ–θ) is the angle between b and the grating normal

And then we use the grating equation to get an expression for (γ–θ) as a function of λ and d, the grating constant.

$$sin(\gamma - \theta) = \frac{\lambda}{d} - sin\gamma$$

Angular dispersion - parallel gratings



The path length DBE and D'B'E' from the same phase front to another phase front have the same phase difference, but the path lengths are clearly different. The grating phase of -2π for every d along the grating distance, BB' must be added to the equation for ϕ

$$\emptyset = -\frac{2\pi}{d} G tan(\gamma - \theta)$$

Angular dispersion - parallel gratings

The total phase is then given by:

$$\emptyset = \frac{\omega}{c}p - \frac{2\pi}{d}Gtan(\gamma - \theta)$$

$$\emptyset = \frac{\omega}{c}b(1 + cos\theta) - \frac{2\pi}{d}Gtan(\gamma - \theta)$$

$$\emptyset = \frac{\omega}{c}\frac{G}{\cos(\gamma - \theta)}(1 + cos\theta) - \frac{2\pi}{d}Gtan(\gamma - \theta)$$

Angular dispersion - parallel gratings

$$\emptyset = \frac{\omega}{c}\frac{G}{\cos\left(\gamma-\theta\right)}\left(1+cos\theta\right)-\frac{2\pi}{d}Gtan(\gamma-\theta)$$

Once again the phase is a complicated function, so we will use a Taylor expansion again. This time we write ϕ as a Taylor expansion:

$$\phi(\omega) = \phi(\omega_0) + \frac{d\phi}{d\omega}(\omega-\omega_0) + \frac{1}{2}\frac{d^2\phi}{d\omega^2}(\omega-\omega_0)^2 + \frac{1}{6}\frac{d^3\phi}{d\omega^3}(\omega-\omega_0)^3 +$$

Dispersion

$$\phi(\omega) = \phi(\omega_0) + \frac{d\phi}{d\omega}(\omega - \omega_0) + \frac{1}{2}\frac{d^2\phi}{d\omega^2}(\omega - \omega_0)^2 + \frac{1}{6}\frac{d^3\phi}{d\omega^3}(\omega - \omega_0)^3 +$$

From the Taylor expansion of the propagation constant, we had:

$$\beta' \equiv \frac{d\beta}{d\omega}\Big|_{\omega=\omega_0} = \frac{1}{v_g(\omega_0)} \equiv \frac{1}{group\ velocity}$$

The first derivative term in the expansion for ϕ, would be:

$$\frac{d\phi}{d\omega} = \frac{d\beta'}{d\omega} z \equiv \frac{z}{v_g(\omega_0)} \equiv group\ delay \equiv \tau$$

The second derivative term in the expansion for ϕ, would be:

$$\frac{d^2\phi}{d\omega^2} = \frac{d}{d\omega}\left(\frac{1}{v_g(\omega_0)}\right) z \equiv \frac{d\tau}{d\omega} \equiv group\ delay\ dispersion \equiv GDD$$

Angular dispersion - parallel gratings

$$\emptyset = \frac{\omega}{c}\frac{G}{\cos(\gamma-\theta)}(1+cos\theta) - \frac{2\pi}{d}Gtan(\gamma-\theta)$$

$$\phi(\omega) = \phi(\omega_0) + \frac{d\phi}{d\omega}(\omega-\omega_0) + \frac{1}{2}\frac{d^2\phi}{d\omega^2}(\omega-\omega_0)^2 + \frac{1}{6}\frac{d^3\phi}{d\omega^3}(\omega-\omega_0)^3 +$$

If you carry out the first derivative you get:

$$\frac{\partial\phi}{\partial\omega} = \frac{p}{c} + \left(\frac{\omega}{c}\right)\frac{\partial p}{\partial\omega} - \frac{2\pi}{d}G\frac{\partial}{\partial\omega}[tan(\gamma-\theta)]$$

You can show that the second and third terms cancel to get:

$$\frac{\partial\phi}{\partial\omega} = \frac{p}{c}$$

Angular dispersion - parallel gratings

$$\tau = \frac{\partial \phi}{\partial \omega} = \frac{p}{c}$$

If you carry out the second derivative and substitute in the grating equation where needed you get:

$$GDD = \frac{\partial \tau}{\partial \omega} = \frac{-4\pi^2 cb}{\omega^3 d^2 \left\{1 - \left[\left(\frac{2\pi c}{\omega d}\right) - sin\gamma\right]^2\right\}}$$

Take note that the GDD of a parallel grating dispersion line is always negative – it can balance positive material dispersion

Angular dispersion - parallel gratings

$$GDD = \frac{\partial \tau}{\partial \omega} = \frac{-4\pi^2 cb}{\omega^3 d^2 \left\{1 - \left[\left(\frac{2\pi c}{\omega d}\right) - sin\gamma\right]^2\right\}}$$

This gives us the dispersion of the pulse peaks as a function of the bandwidth

$$\delta_\omega \tau = \frac{-4\pi^2 cb\, \delta\omega}{\omega^3 d^2 \left\{1 - \left[\left(\frac{2\pi c}{\omega d}\right) - sin\gamma\right]^2\right\}}$$

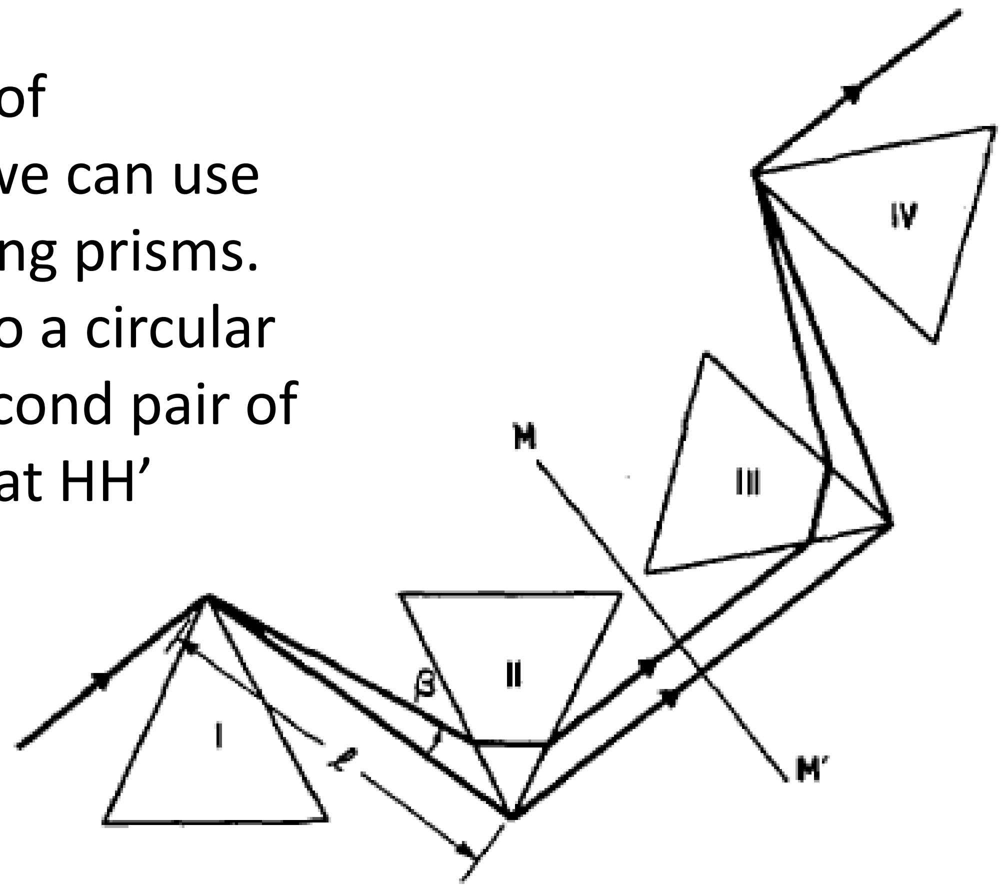
Since we measure the bandwidth in wavelength, λ, Treacy wrote the dispersion as:

$$\delta_\lambda \tau = \frac{b\,(\lambda/d)\delta\lambda}{cd\,\{1 - [(\lambda/d) - sin\gamma]^2\}}$$

These expressions tell you how big the separation b, needs to be to achieve the wanted stretched pulse duration

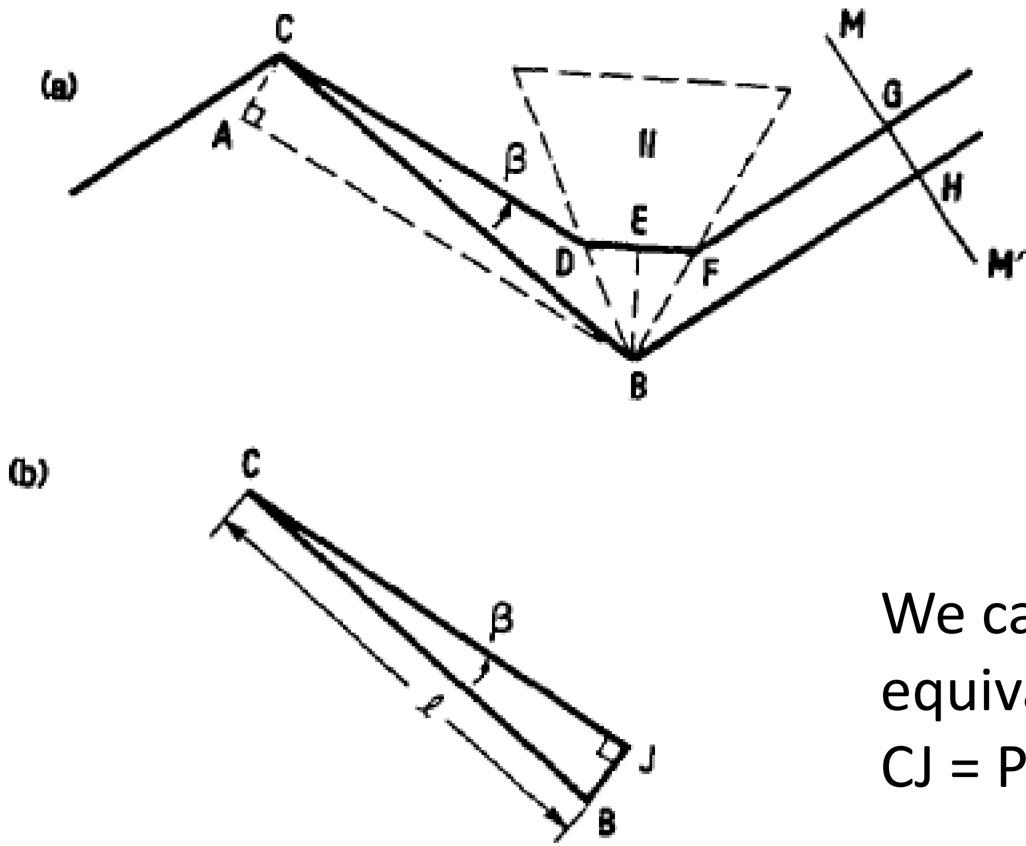
Dispersion with Prisms – Material and Angular

For smaller values of frequency chirps, we can use parallel but opposing prisms. Again to get back to a circular beam, we use a second pair of prisms or a mirror at HH’



“Negative dispersion using pairs of prisms” R.L Fork, O. E. Martinez, and J.P. Gordon, Opt. Lett., 1984, 9, 15-17

Dispersion with Prisms – Material and Angular



Optical path lengths, P, of CDE and CB must be equal

$$P = \int n(z)dz$$

We can use the equivalent path length CJ = P

$$P = l \cos \beta$$

Dispersion with Prisms – Material and Angular

$\phi = kP$

$$\tau = \frac{d\phi}{d\omega} = \frac{d}{d\omega}(kP) = \frac{d}{d\omega}\left(\frac{2\pi}{\lambda}P\right)$$

$$= \frac{d\lambda}{d\omega}\frac{d}{d\lambda}\left(\frac{2\pi}{\lambda}P\right) = -\frac{\lambda^2}{2\pi c}\left[2\pi\left(\frac{-1}{\lambda^2}P\right) + \frac{2\pi}{\lambda}\frac{dP}{d\lambda}\right]$$

$$= \frac{1}{c}\left[P - \lambda\frac{dP}{d\lambda}\right]$$

Dispersion with Prisms – Material and Angular

$$\mathrm{GDD} = \frac{d\tau}{d\omega} = \frac{d\lambda}{d\omega}\frac{d\tau}{d\lambda} = -\frac{\lambda^2}{2\pi c}\frac{d\tau}{d\lambda}$$

$$\mathrm{GDD} = -\frac{\lambda^2}{2\pi c}\left\{\frac{1}{c}\frac{d}{d\lambda}\left[P - \lambda\frac{dP}{d\lambda}\right]\right\}$$

$$\mathrm{GDD} = -\frac{\lambda^2}{2\pi c}\left\{\frac{1}{c}\left[\frac{dP}{d\lambda} - \frac{dP}{d\lambda} - \lambda\frac{d^2P}{d\lambda^2}\right]\right\}$$

$$\mathrm{GDD} = +\frac{\lambda^2}{2\pi c}\left\{\frac{\lambda}{c}\frac{d^2P}{d\lambda^2}\right\}$$

$$\frac{d\tau}{d\lambda} = -\left\{\frac{\lambda}{c}\frac{d^2P}{d\lambda^2}\right\}$$

Dispersion with Prisms – Material and Angular

$$\delta\tau = \frac{d\tau}{d\omega}\delta\omega = \frac{d\tau}{d\lambda}\delta\lambda$$

$$\delta\tau = GDD\delta\omega = DL\delta\lambda$$

$$D = -\frac{1}{L}\frac{d\tau}{d\lambda} = \frac{\lambda}{cL}\frac{d^2P}{d\lambda^2}$$

Where P is the optical path length and L is the length

Dispersion with Prisms – Material and Angular

$$P = 2l cos\beta$$

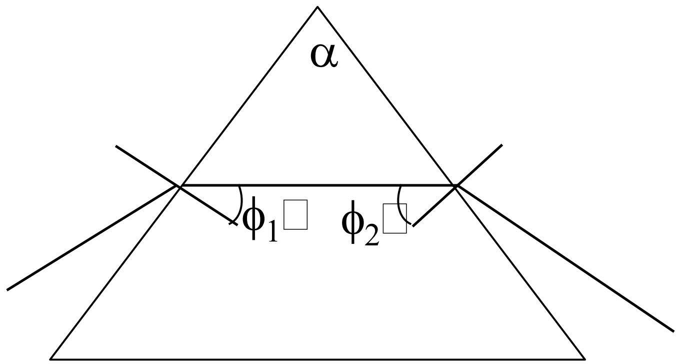
The 2 is for the double set of prisms

$$\frac{dP}{d\beta} = -2l sin\beta$$

$$\frac{d^2P}{d\beta^2} = -2l cos\beta$$

$$\frac{d^2P}{d\lambda^2} = \left[\frac{d^2n}{d\lambda^2}\frac{d\beta}{dn} + \left(\frac{dn}{d\lambda}\right)^2 \frac{d^2\beta}{dn^2}\right]\frac{dP}{d\beta} + \left(\frac{dn}{d\lambda}\right)^2 \left(\frac{d\beta}{dn}\right)^2 \frac{d^2P}{d\beta^2}$$

Dispersion with Prisms – Material and Angular



From Snell's Law, where the prime angles are inside prism

$$\sin\phi_1 = n\sin\phi'_1$$

$$\sin\phi_2 = n\sin\phi'_2$$

For the apex angle, α and using Brewster's angle :

$$\alpha = \phi'_1 + \phi'_2 \qquad\qquad \phi'_1 = \phi'_2$$

$$\frac{d\phi_2}{dn} = \frac{1}{\cos\phi_2}[\sin\phi'_2 + \cos\phi'_2 \tan\phi'_1]$$

$$\frac{d^2\phi_2}{dn^2} = \tan\phi_2\left(\frac{d\phi_2}{dn}\right)^2 - \frac{\tan^2\phi'_1}{n}\left(\frac{d\phi_2}{dn}\right)$$

Dispersion with Prisms – Material and Angular

$$\frac{d\beta}{dn} = -\frac{d\phi_2}{dn} \qquad \frac{d^2\beta}{dn^2} = -\frac{d^2\phi_2}{dn^2}$$

$$\frac{d\beta}{dn} = -2 \qquad \frac{d^2\beta}{dn^2} = -4n + \frac{2}{n^3}$$

*Note there is a typo in the paper

$$\frac{d^2P}{d\lambda^2} = 4l\left\{\left[\frac{d^2n}{d\lambda^2} + \left(2n - \frac{1}{n^3}\right)\left(\frac{dn}{d\lambda}\right)^2\right] sin\beta - 2\left(\frac{dn}{d\lambda}\right)^2 cos\beta\right\}$$

$$sin\beta \approx 0, \quad cos\beta \approx 1$$

The second term typically dominates giving negative dispersion – like a grating pair, but much smaller magnitude – used in short pulse oscillators.

Chirp Compensation

For wavelengths < 1.5μm, material dispersion is typically positive

In the original optical pulse compression systems, optical fibers were used to stretch the pulses with this positive GDD, and then grating compressors with negative GDD were used to compress the bandwidth

In the first ~ 100 fs oscillators, the positive dispersion from the crystals and mirrors was balanced by the negative GDD of prism compressors placed inside the oscillators

Thank You

CENTRO DE INVESTIGACIONES EN OPTICA, A.C.

OSA The Optical Society