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OSA Quantum Optical Science and Technology Technical Group

Diffraction Induced Entanglement Losses

Giacomo Sorelli, Laboratoire Kastler Brossel

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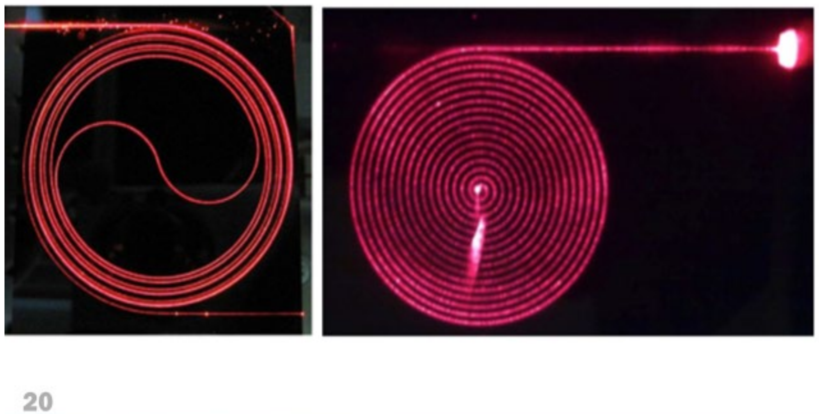
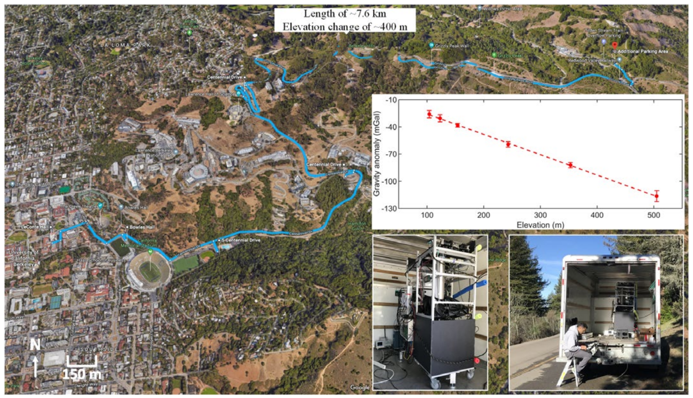
Our Technical Group at a Glance

- Nearly 3000 members worldwide!
- Our Webpage https://www.osa.org/oq we post news regularly
- Engagement activities: Webinars, Networking Events, Campfire Sessions, etc
- Suggestions, ideas for events, email us at OSA TGActivities/gpuentes@df.uba.ar
- **Nominated 20x20 talks for our latest event at Quantum 2.0**

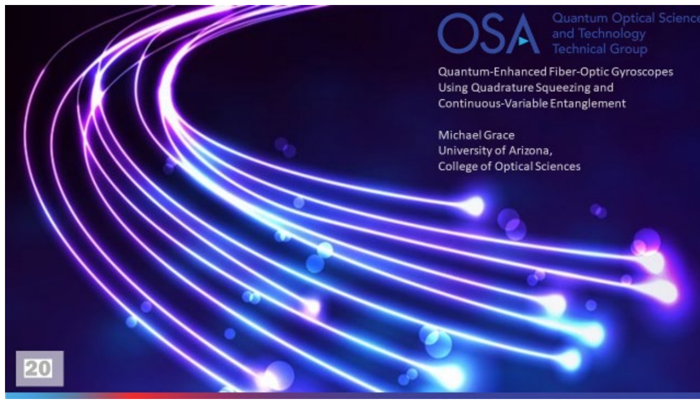
Trevor Steiner (UCSB)



Xuejian Wu (UCBerkeley)



Michael Grace (UArizona)



Welcome to the Quantum Optical Science and Technology Technical Group Webinar!

Diffraction Induced Entanglement Losses

Giacomo Sorelli

Electromagnetism and Radar department (DEMR), ONERA, Palaiseau

Multimode Quantum Optics Group, Laboratoire Kastler Brossel, Paris

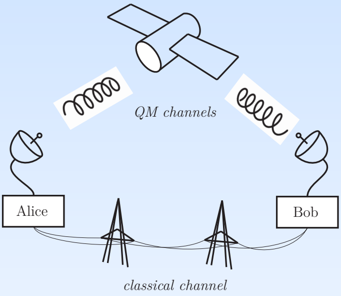
Quantum Information Group, LIP6, Paris

Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

30/09/2020

OSA Technical Group webinar

Motivation: Quantum Communication



Ekert, PRL 67, 661 (1991)

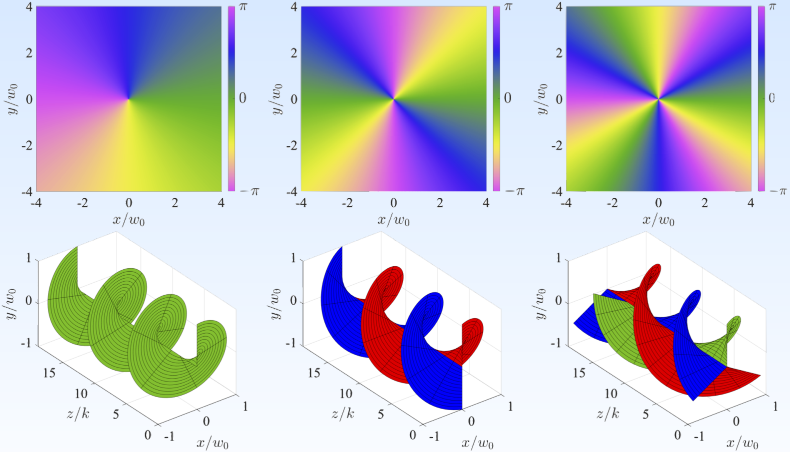
- Information usually encoded in polarization qubits
- The use of qudits allows
  - increased channel capacity
  - enhanced security of quantum key distribution

    Cerf et al., PRL 88, 127902 (2002)
  - enhanced non-locality Dada et al., Nat. Phys. 7, 677 (2011)

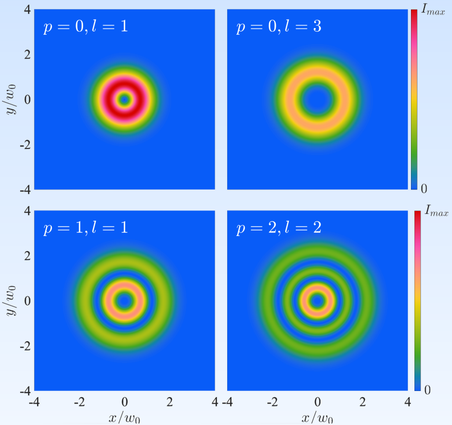
Orbital Angular Momentum

$u(r, \theta, z) = R(r, z)e^{il\theta} \rightarrow$ well defined OAM $\hbar l$

Discrete, infinite dimensional Hilbert space $\longrightarrow$ good for qudits!



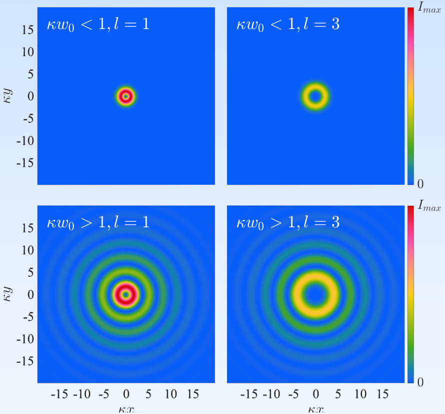
Laguerre-Gauss modes



- $p + 1$ intensity rings
- ring radius: $\sqrt{(2p + |l| + 1)/2}w(z)$

$$u_{p,l}^{LG}(\rho, \phi, z) = \frac{C_{p,l}}{w(z)} \left(\frac{\sqrt{2}\rho}{w(z)}\right)^{|l|} L_p^{|l|}\left(\frac{2\rho^2}{w(z)^2}\right) \exp\left(-\frac{\rho^2}{w(z)^2}\right)$$
$$\times \exp\left(\frac{ik\rho^2 z}{2(z^2 - z_R^2)}\right) \exp(il\phi) \exp\left[-i\Phi(p, l; z)\right]$$

Bessel-Gauss modes



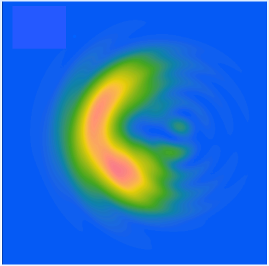
- $\kappa w_0 \lesssim 1 \rightarrow$ single ring
- $\kappa w_0 \gtrsim 1 \rightarrow$ multi ring
- $z \lesssim w_0 k/\kappa$ non-diffracting

$$u^{BG}_{\kappa,l}(\rho,\phi,z) = C_l \frac{w_0}{w(z)} \exp\left(il\phi\right) \exp\left[i\left(\frac{\kappa^2}{2k}\right)z - \Phi(z)\right]$$
$$\times J_l\left(\frac{\kappa\rho z_R}{z_R + iz}\right) \exp\left[\left(-\frac{1}{w^2(z)} - \frac{ik}{2R(z)}\right)\left(\rho^2 + \frac{\kappa^2 z^2}{k^2}\right)\right]$$

Spatial modes are sensitive to disturbances

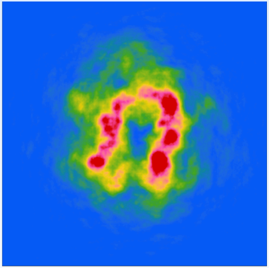
Deterministic distortions:

e.g. Diffraction on obstrucions



Random distortions:

e.g. turbulence distortions



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Deterministic distortions:

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Random distortions:

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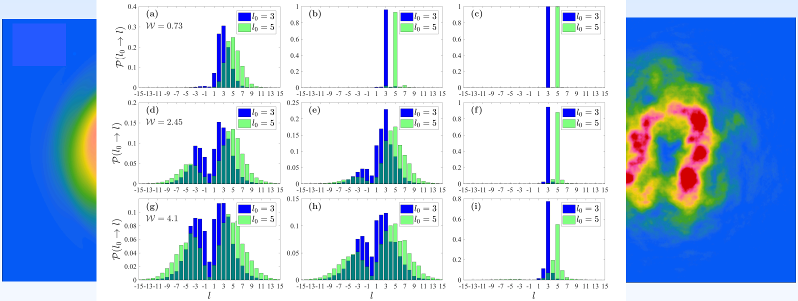

Distortions induce crosstalk!

Spatial modes are sensitive to disturbances

Deterministic distortions:

e.g. Diffraction on obstrucions

Random distortions:

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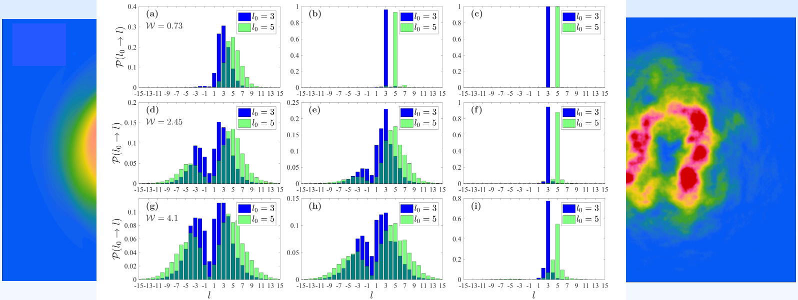

Distortions induce crosstalk!

G. Sorelli *et al.* New J. Phys. (2019)

Spatial modes are sensitive to disturbances

Deterministic distortions:

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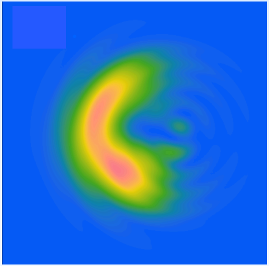
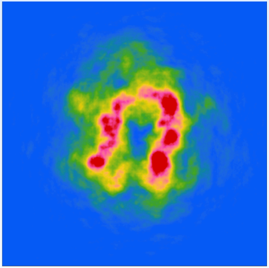


Today's talk!

Random distortions:

e.g. turbulence distortions

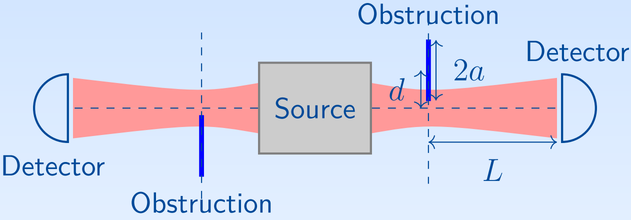


Maybe next time...

Outline of the talk

1. Entanglement of diffracted states: general formula
2. Entanglement losses and radial structure
3. Entanglement losses induced by angular uncertainty
4. Outline and conclusion

Setup and initial state



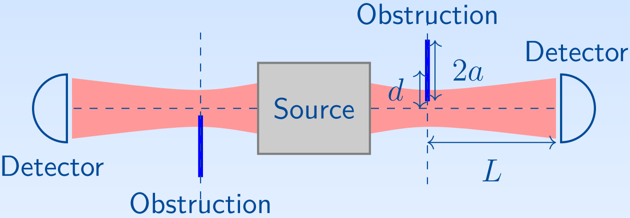
Maximally entangled two-photon states:

$$|\psi_0\rangle = \frac{|l_0, -l_0\rangle + |-l_0, l_0\rangle}{\sqrt{2}}$$

Single photon states:

$$|\pm l_0\rangle = \int d\mathbf{r} u_{\pm l_0}(\mathbf{r}) |\mathbf{r}\rangle$$

Setup and initial state



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$u_{\pm l_0}$ can be arbitrary orthogonal spatial modes...

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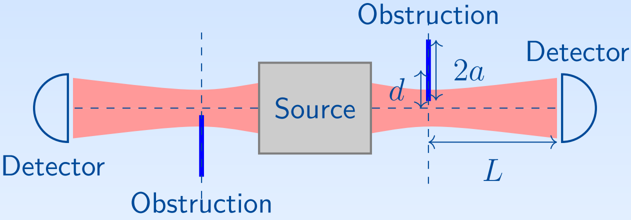

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Single photon states:

$$|\pm l_0\rangle = \int d\mathbf{r} u_{\pm l_0}(\mathbf{r}) |\mathbf{r}\rangle$$

$u_{\pm l_0}$ can be arbitrary orthogonal spatial modes...
...in this talk: $u_{\pm l_0}$ modes with opposite OAM!

Diffraction of single photons

$$|\pm l_0\rangle = \int d\mathbf{r} u_{\pm l_0}(\mathbf{r}) |\mathbf{r}\rangle \xrightarrow{diffraction} |\psi_{\pm l_0}\rangle = \int d\mathbf{r} \psi_{\pm l_0}(\mathbf{r}) |\mathbf{r}\rangle$$

$$\psi_{\pm l_0}(x, y) = \mathcal{F}^{-1}\left\{T(k_x, k_y)\mathcal{F}\left[t(x, y)u_{\pm l_0}(x, y)\right]\right\}$$

$t(x, y)$: obstacle transmission function

Fresnel (angular-spectrum) propagator

$$T(k_x, k_y) = \exp[ikz - (k_x^2 + k_y^2)z/2k\ ]$$

Crosstalk:

$$|\psi_{\pm l_0}\rangle = \sum_l C_{l,\pm l_0} |l\rangle$$

$$C_{l,\pm l_0} = \int d\mathbf{r} u_l^*(\mathbf{r})\psi_{\pm l_0}(\mathbf{r})$$

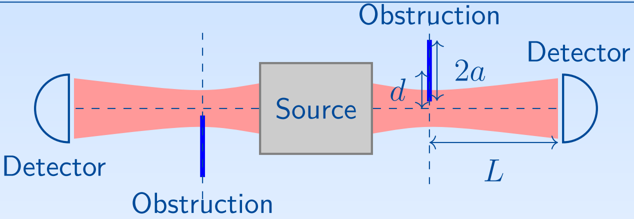
General approach:

Quantify the coefficients $C_{l,\pm l_0}$ (work in the OAM basis)

Our approach:

Work with the non-orthogonal modes $\psi_{\pm l_0}(\mathbf{r})$

Entanglement of diffracted biphotons



Diffracted state:

$$|\Psi\rangle = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \Psi(\mathbf{r}_1, \mathbf{r}_2) |\mathbf{r}_1, \mathbf{r}_2\rangle$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2(1+b^2)}} \left[\psi_{l_0}^{d}(\mathbf{r}_1) \psi_{-l_0}^{-d}(\mathbf{r}_2) + \psi_{-l_0}^{d}(\mathbf{r}_1) \psi_{l_0}^{-d}(\mathbf{r}_2) \right],$$

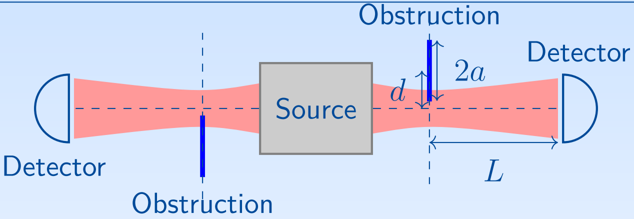
Mutual overlap:

$$b = \int \psi_{-l_0}^{d\,*}(\mathbf{r}) \psi_{l_0}^{d}(\mathbf{r}) d\mathbf{r}$$

Concurrence:

$$C(|\Psi\rangle) = \sqrt{2\left(1 - \mathrm{Tr}[\varrho_1^2]\right)}$$

Entanglement of diffracted biphotons



Diffracted state:

$$|\Psi\rangle = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \Psi(\mathbf{r}_1, \mathbf{r}_2) \, |\mathbf{r}_1, \mathbf{r}_2\rangle$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2(1+b^2)}} \left[\psi_{l_0}^{d}(\mathbf{r}_1) \psi_{-l_0}^{-d}(\mathbf{r}_2) + \psi_{-l_0}^{d}(\mathbf{r}_1) \psi_{l_0}^{-d}(\mathbf{r}_2) \right],$$

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$$b = \int \psi_{-l_0}^{d\,*}(\mathbf{r}) \psi_{l_0}^{d}(\mathbf{r}) d\mathbf{r}$$

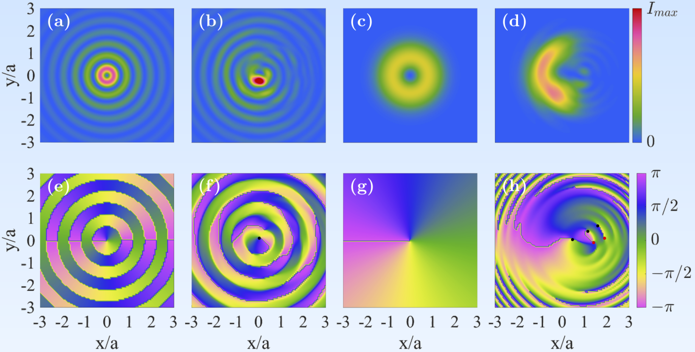
Concurrence:

$$C(|\Psi\rangle) = \sqrt{2\left(1 - \mathrm{Tr}[\varrho_1^2]\right)} = \frac{1-b^2}{1+b^2}$$

Entanglement only depends on b!

Role of radial structure on entanglement loss

Single-ring (LG) modes vs multi-ring (BG) modes



Displaced (smooth) circular obstacles:

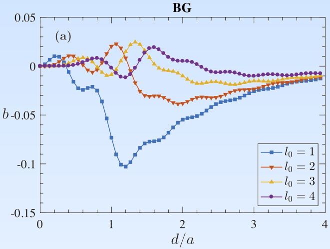
$$t(x-d,y) = 1 - \exp\left\{-\left[\frac{(x-d)^2+y^2}{a^2}\right]^m\right\}$$

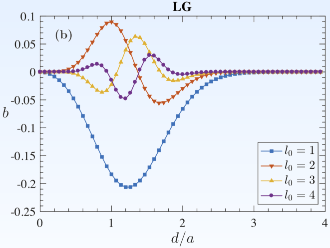
$d =$ displacement, $a =$ obstacle radius.

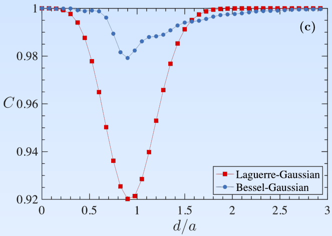
Multi-ring case: transmitted intensity $\sim$ on l_0

Single-ring case: l_0-dependent waist $\rightarrow$ obstacle on intensity max.

Role of radial structure on entanglement loss







$$C(|\Psi\rangle) = \frac{1-b^2}{1+b^2}$$

$b \to 0$ for $d/a \to 0$ (no OAM change)

l_0-dependent b modulations:
phase plays a role!

Entanglement of multi-ring modes is more robust!!

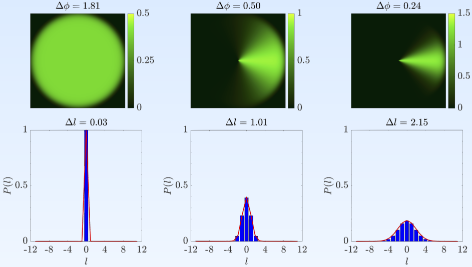
Entanglement loss and angular uncertainty

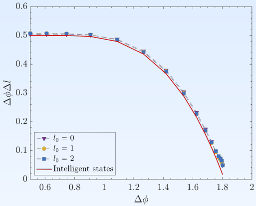
Angular position and momentum uncertainty relation:

Barnett & Pegg, Phys. Rev. A (1990)

$$\Delta\phi\Delta L_z \geq \frac{\hbar}{2}|1-2\pi P(\pi)|$$

$\phi = [-\pi, \pi]$, $L_z/\hbar = l \in \mathbb{Z}$, $P(\phi) = |\psi(\phi)|^2$ angular probability density





Intelligent states:

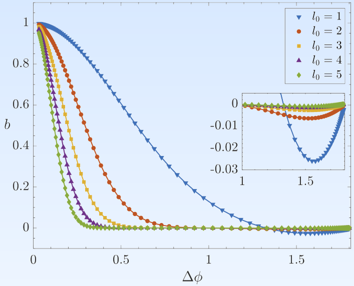
$$g(\phi) = \frac{(\lambda/\pi)^{1/4}}{\sqrt{\mathrm{erf}\left(\pi\sqrt{\lambda}\right)}} e^{i\bar{l}\phi} e^{-\lambda\phi^2/2}$$

$$g(l) = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{-il\phi} g(\phi) d\phi$$

Entanglement loss and angular uncertainty

Maximally entangled states diffracted on angular apertures:

Mutual overlap:



$$b = \frac{e^{-l_0^2/\lambda} \Re\left\{ \operatorname{erf}\left(\frac{\pi\lambda + i l_0}{\sqrt{\lambda}} \right) \right\}}{\operatorname{erf}\left(\pi\sqrt{\lambda}\right)},$$

$b \sim 1$ at $\Delta\phi \sim 0$

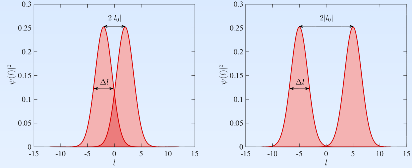
(narrow aperture, broad OAM)

$b \sim 0$ at $\Delta\phi \sim \pi/\sqrt{3}$

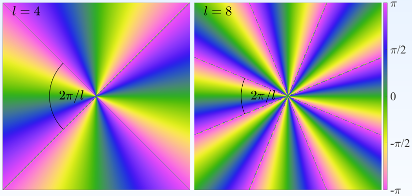
(open aperture, defined OAM)

b decays faster for larger l_0:

1. Δl independent on l_0



2. Finer phase structure



$\Delta\phi$ is effectively smaller!!

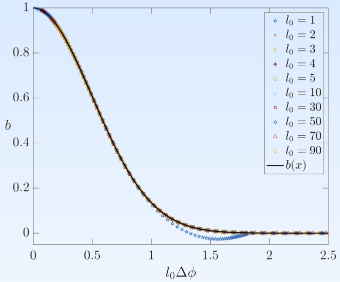
Entanglement loss and angular uncertainty

What if we rescale $\Delta\phi$ with l_0?

Entanglement loss and angular uncertainty

What if we rescale $\Delta\phi$ with l_0?

Universal entanglement behaviour!



$0 \leq \Delta\phi \lesssim 1 \rightarrow$ Gaussian approximation: $b \approx e^{-2(l_0\Delta\phi)^2}$

Concurrence: $C = \frac{1-b^2}{1+b^2} \approx \tanh\left(2l_0^2\Delta\phi^2\right)$

Conclusion and outlook

- ! Simple analytical formula for diffracted maximally-entangled qubits
- ! Multi-ring modes are more robust against diffraction

  G Sorelli, VN Shatokhin, FS Roux, A Buchleitner, *Phys. Rev. A 97 (1), 013849* (2018)
- ! Universal entanglement loss induced by angular uncertainty

  G Sorelli, VN Shatokhin, A Buchleitner, *Journal of Optics 22 (2), 024002* (2020)

- ? Generalization to more general/high-dimensional states
- ? Diffraction as preparation and not disturbance

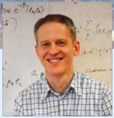
  G Puentes, *OSA Continuum 3, 1616-1632* (2020)

Thanks to my coworkers:

nmisa
National Metrology Institute of South Africa

Filippus Stef
Roux

Andreas
Buchleitner



Slava
Shatokhin

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Filippus Stef Roux

Andreas Buchleitner

Slava Shatokhin

Thank you for your attention!