Quantum Optics with Machine-Learning:
Introduction to Machine-Learning Enhanced Quantum State Tomography

Ray-Kuang Lee 李瑞光*
National Tsing Hua University (NTHU), Taiwan

Special Thank to

Dr. Kamal Kishor Choure

OPTICA
Formerly OSA

Phys. Rev. Lett. 128, 073604 (2022);
arXiv: 2111.08285 (2021).
Phys. Rev. Lett. 124, 171101 (2020);
Editors' Suggestion; Featured in Physics;

*http://mx.nthu.edu.tw/~rklee

TSING HUA UNIVERSITY

Outline

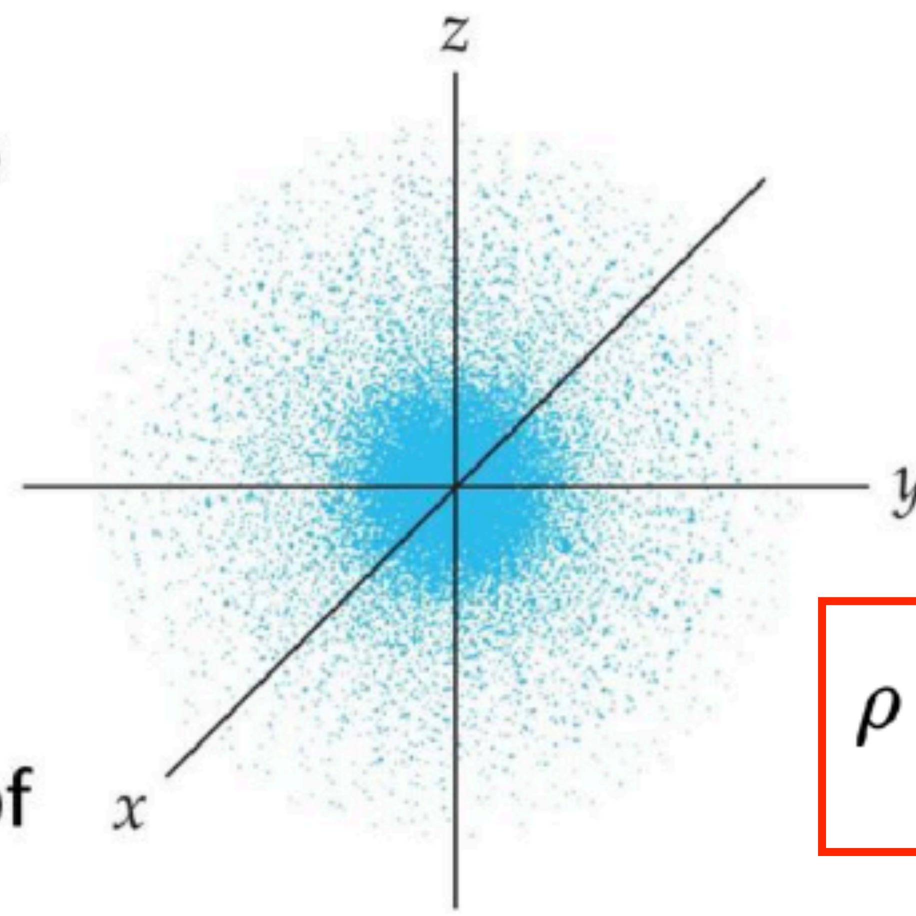
- Quantum Optics in Phase Space
- Quantum Noise Squeezing (SQZ)
- Machine-Learning enhanced Quantum State Tomography

- Applications with SQZ:
  - Optical Cat states
  - Quantum Photonic Chips
  - Error-Correction Code: GKP states
  - Quantum Random Number Generator
  - Gravitational Wave Detectors

Can We See Quantum ?

an Introduction to Quantum State Tomography

- The wave equation is designated with a lower case Greek *psi* (ψ).
- The square of the wave equation, ψ^2, gives a probability density map of where an electron has a certain statistical likelihood of being at any given instant in time.

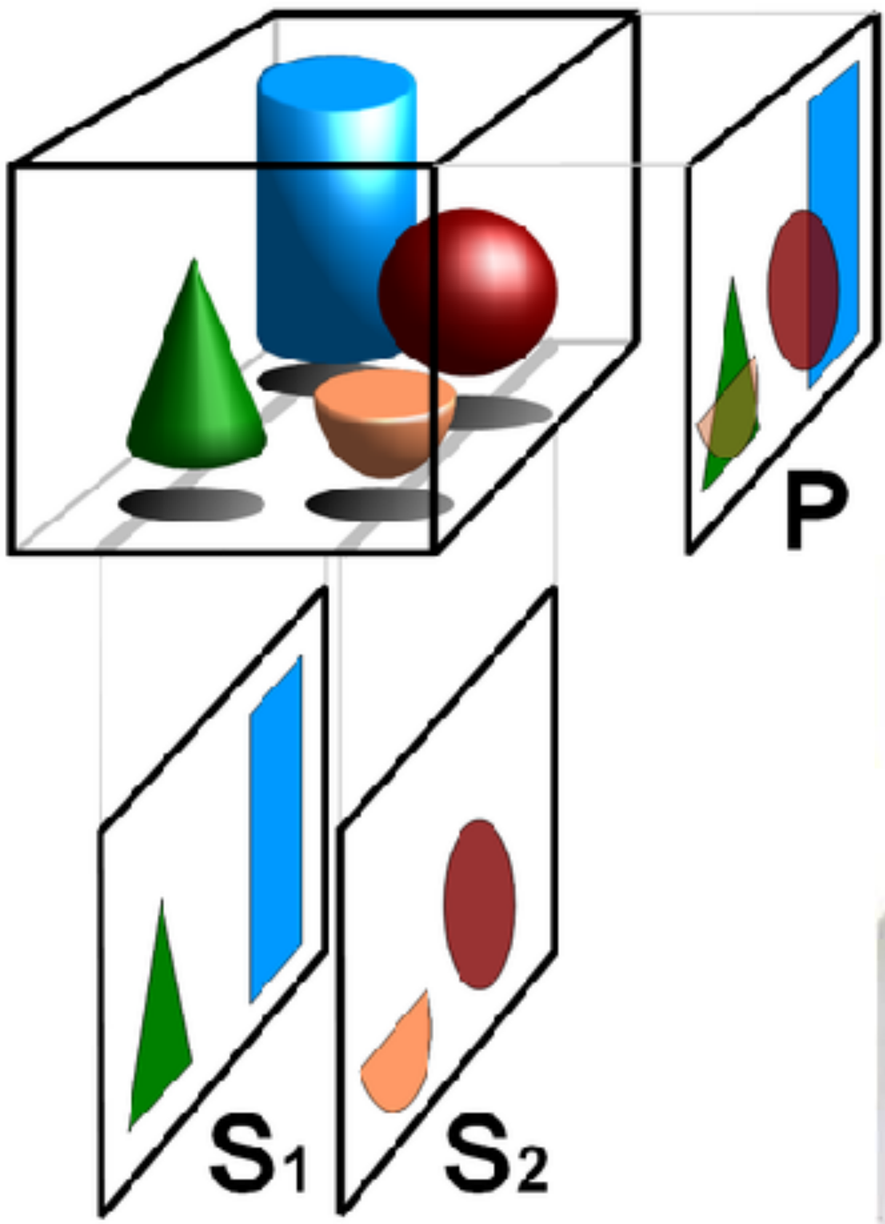


$$|\psi\rangle$$

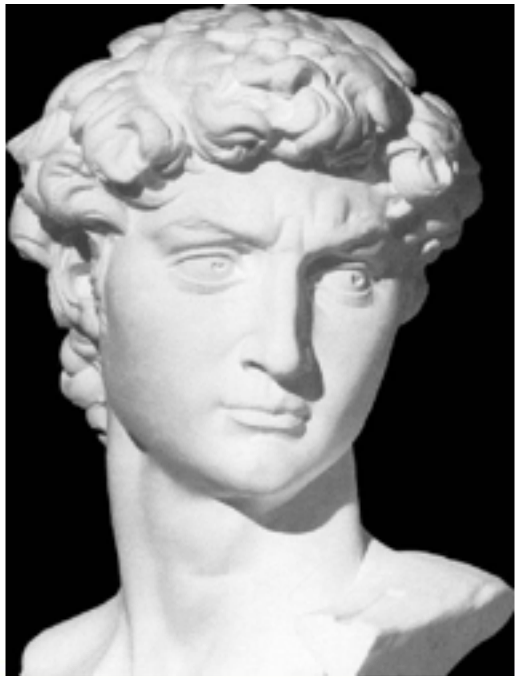
density matrix

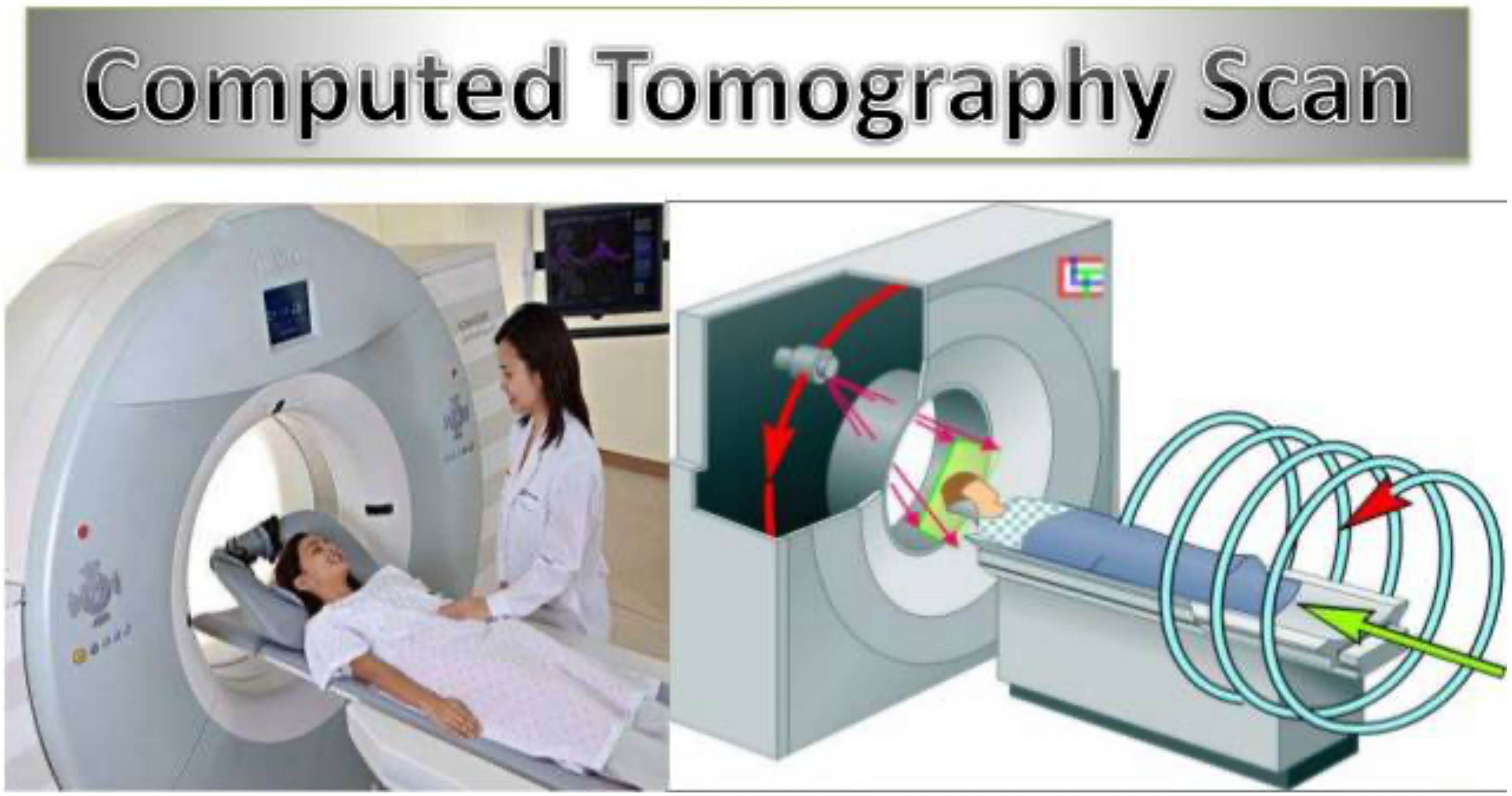
$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

(Computed) Tomography, CT



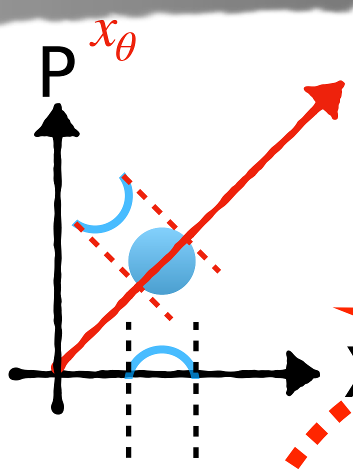
from Wiki

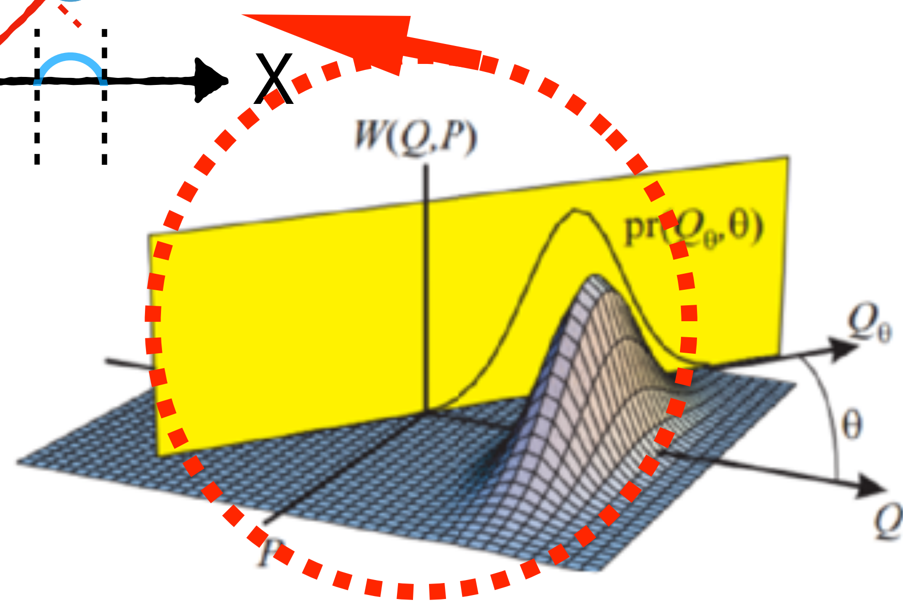


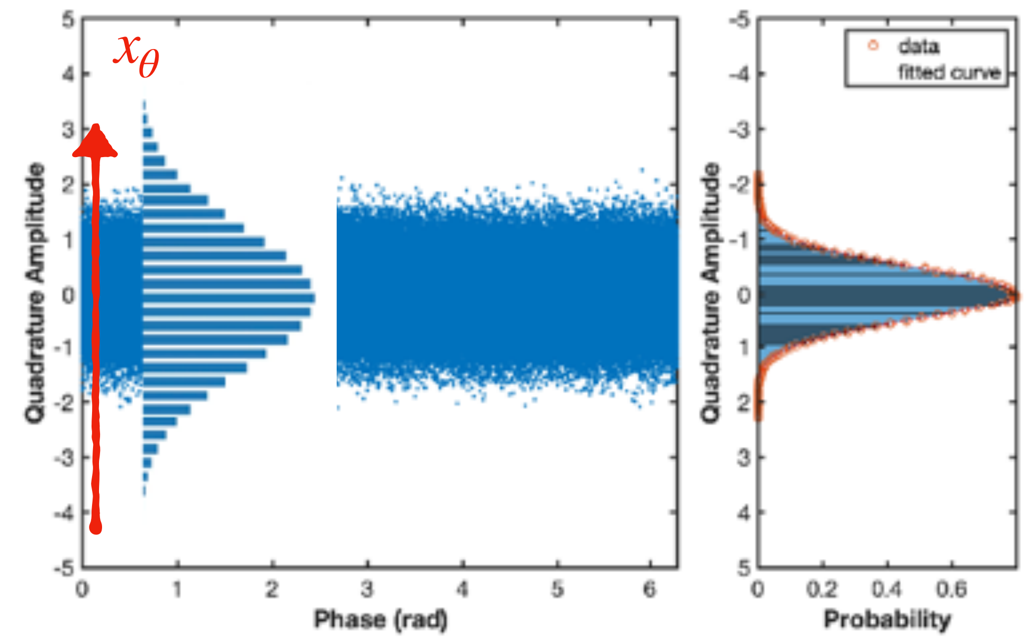


Quantum State Tomography

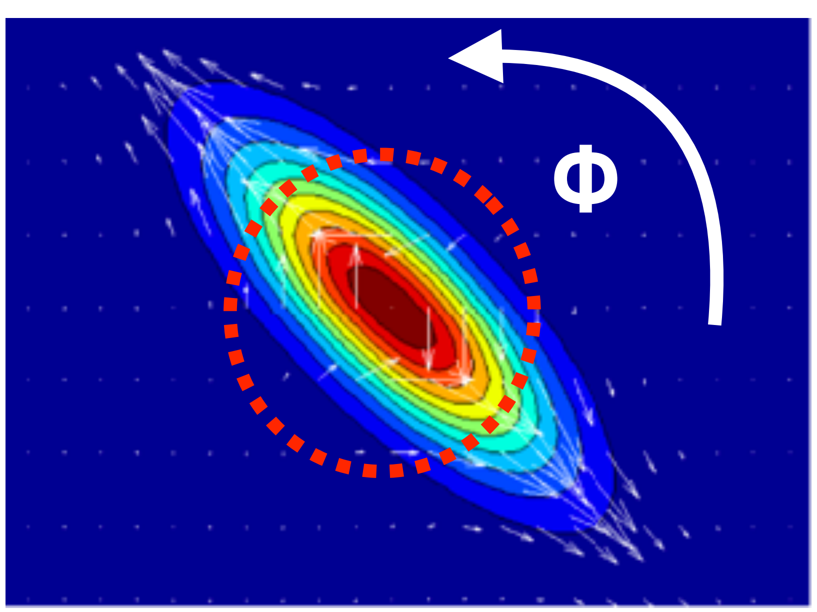
Wavefunction Reconstruction





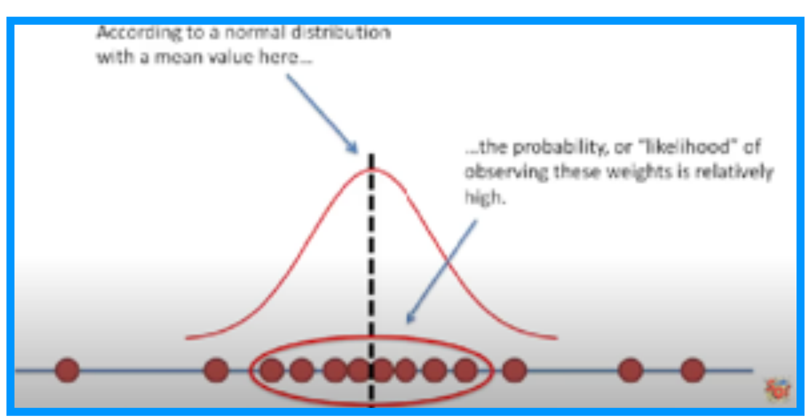


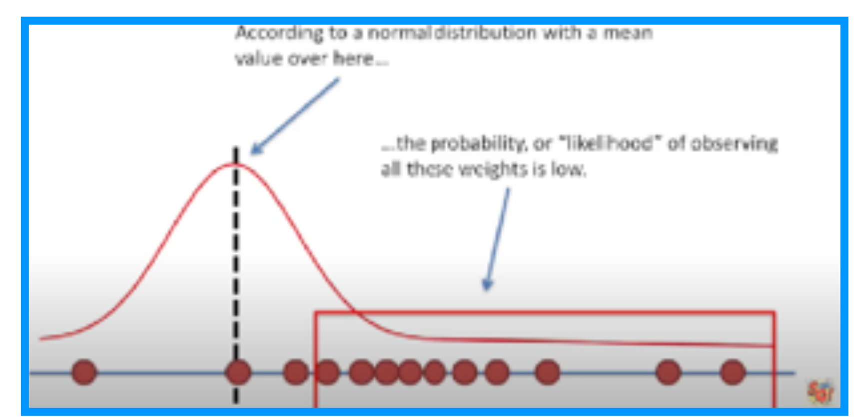
The goal of maximum likelihood estimation (MLE) is to find the optimal way to fit distribution to the data.



Define Likelihood as:

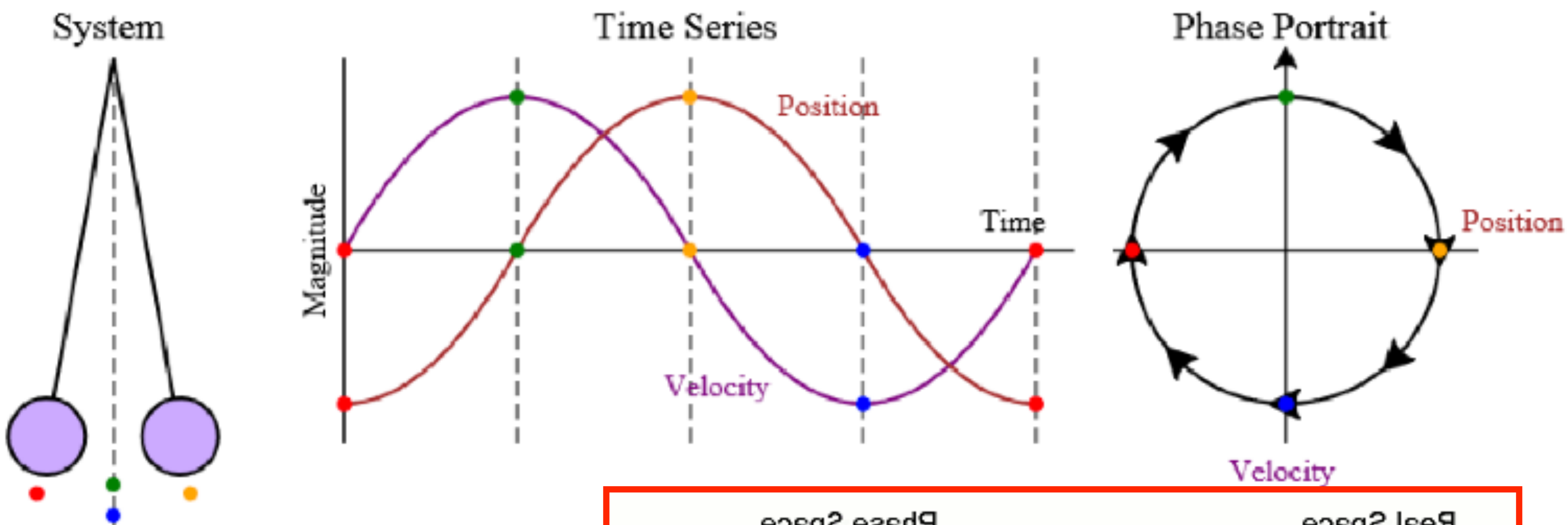
$$\mathcal{L} \equiv p_1^{f_1}\, p_2^{f_2}\, p_3^{f_3}\, p_4^{f_4}\, \ldots$$

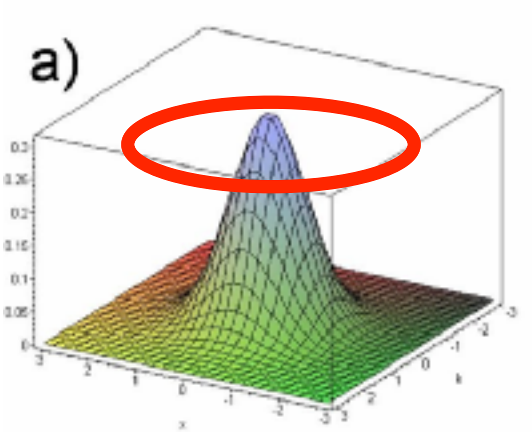


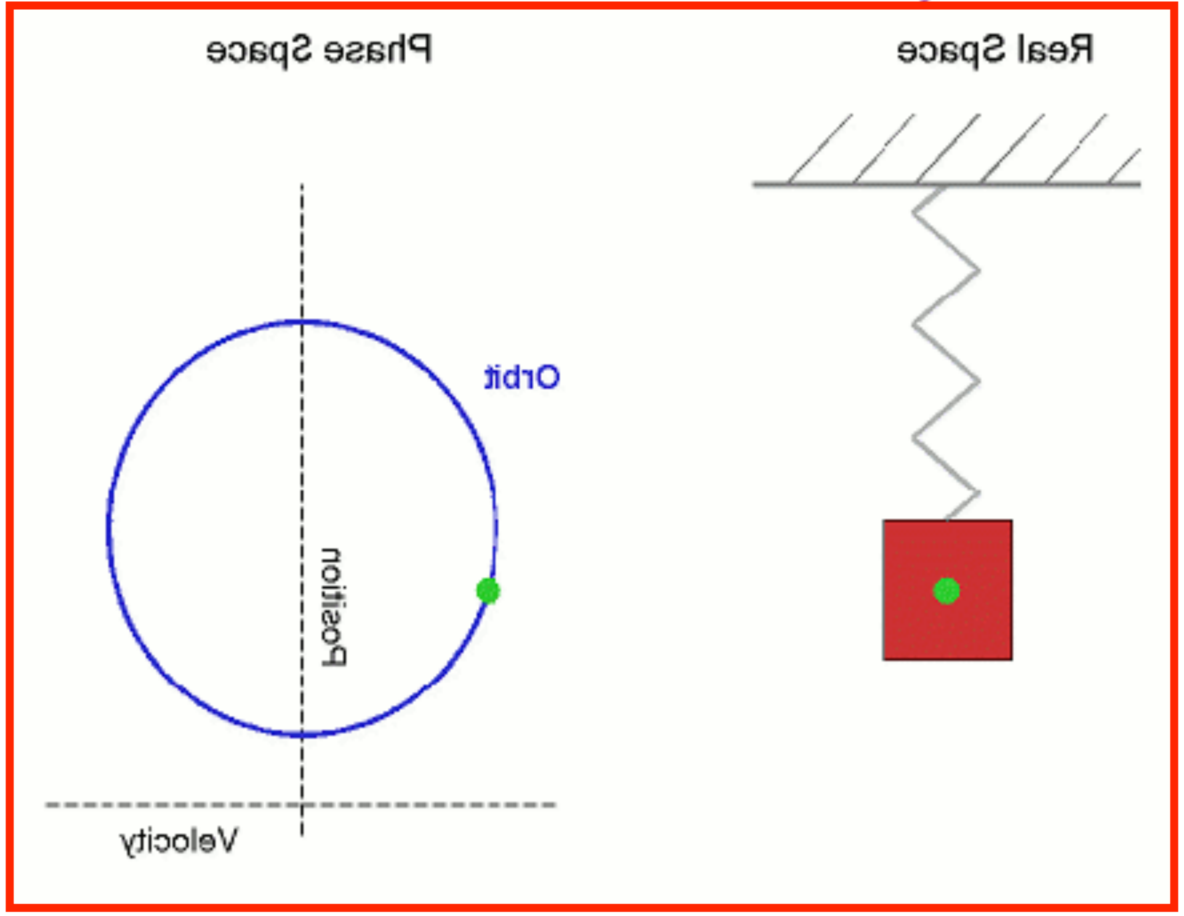


Phase space

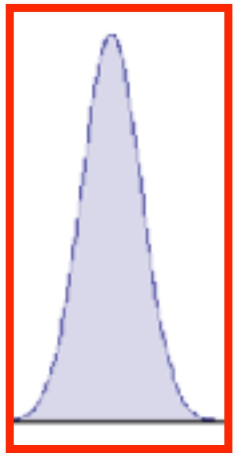
- Classical Mechanics



- 



wave-nature

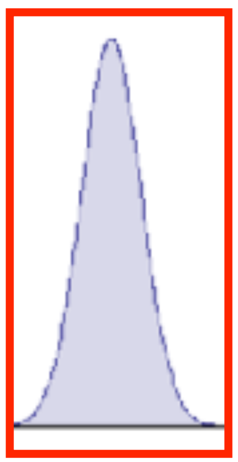


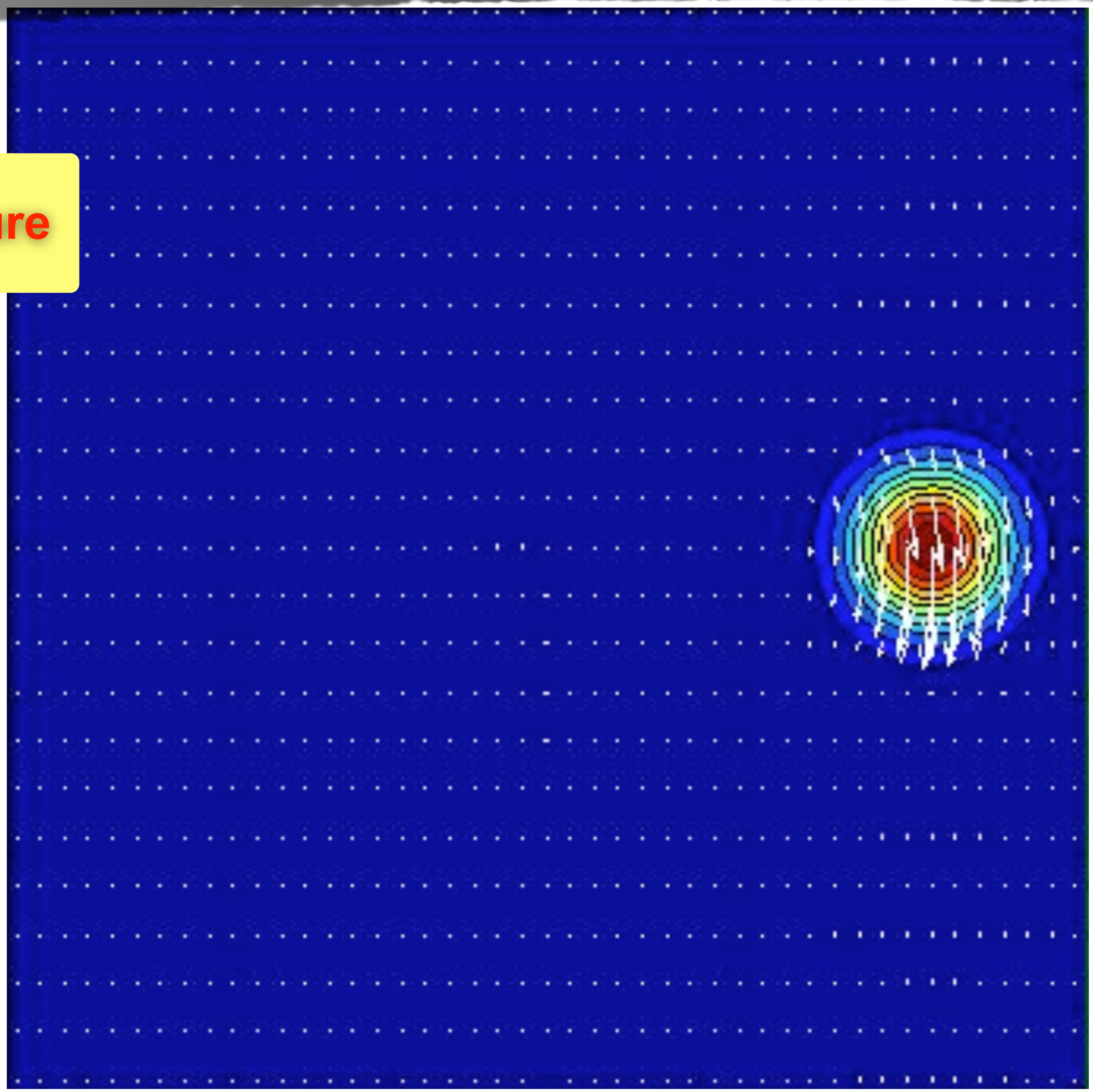
from Wiki

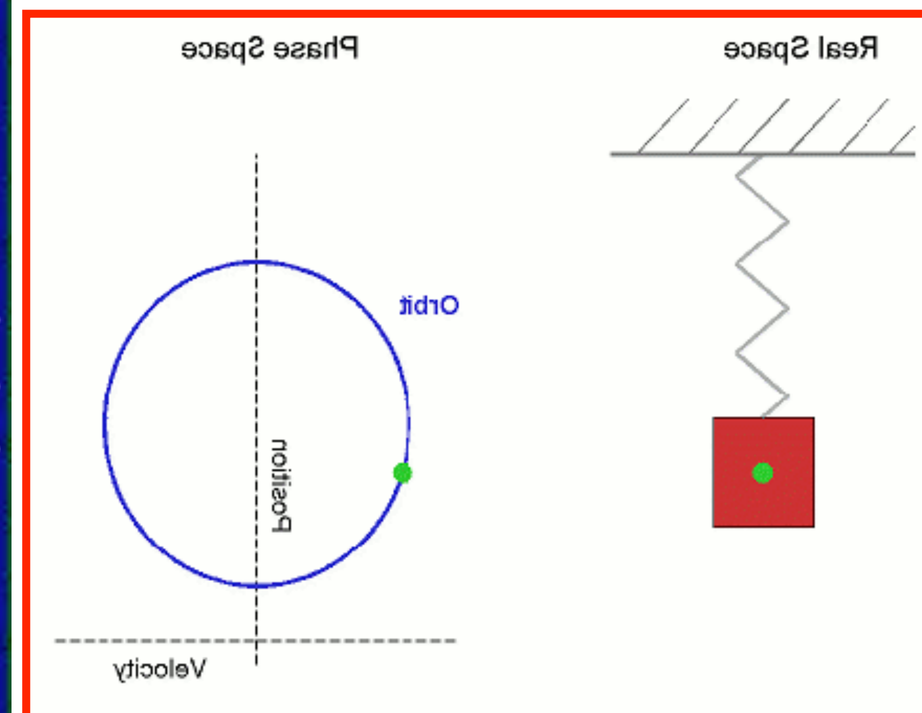
Coherent states

|α>

wave-nature



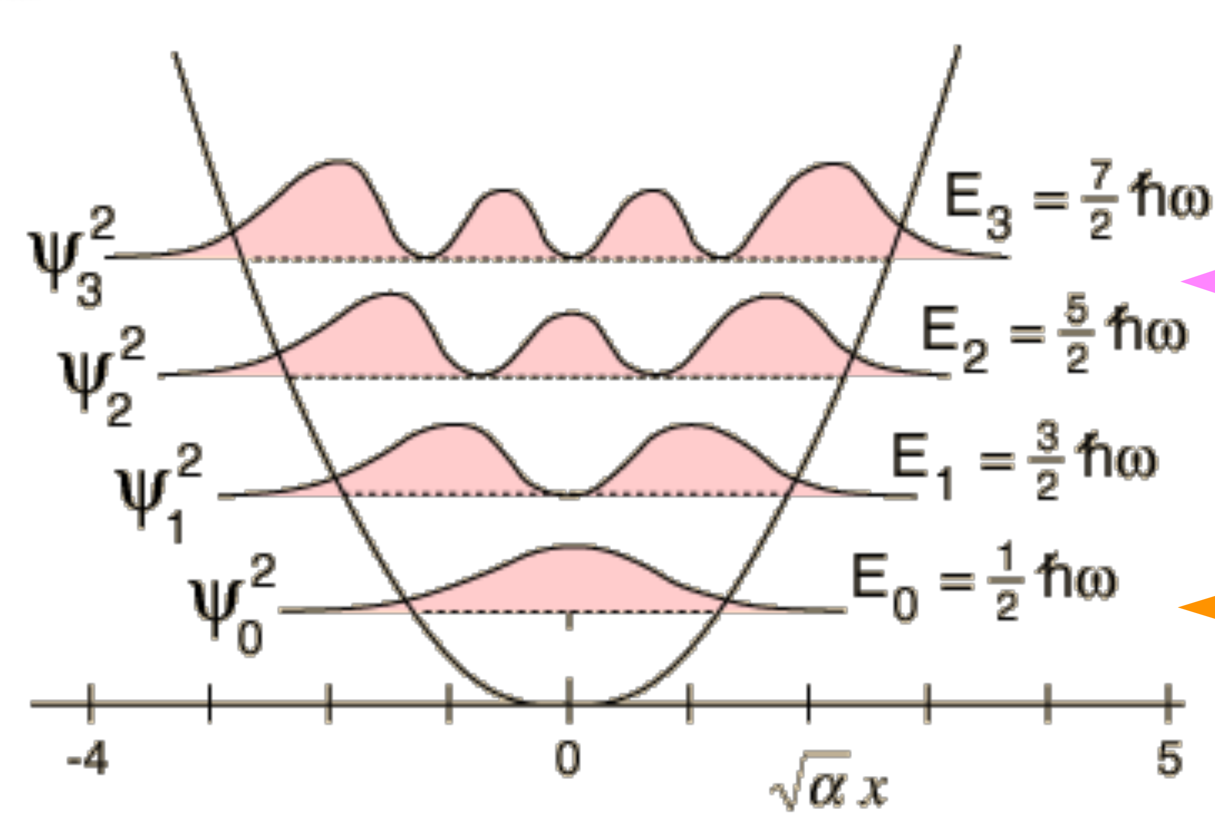




with Popo Yang

Popo Yang, Ivan F. Valtierra, Andrei B. Klimov, Shin-Tza Wu, RKL, Luis L. Sanchez-Soto, and Gerd Leuchs, Physica Scripta for the New Focus issue: Quantum Optics and Beyond - in honour of Wolfgang Schleich.

Quantum Simple Harmonic Oscillator (SHO)



Number (Fock) States |n>

Vacuum States |0>

$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2, \quad [\hat{x}, \hat{p}] = i\hbar.$$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}). \quad [\hat{a}, \hat{a}^\dagger] = 1,$$

$$\begin{aligned} \hat{N}|n\rangle &= n|n\rangle, \\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle, \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle, \\ E_n &= \hbar\omega(n + \frac{1}{2}). \end{aligned}$$

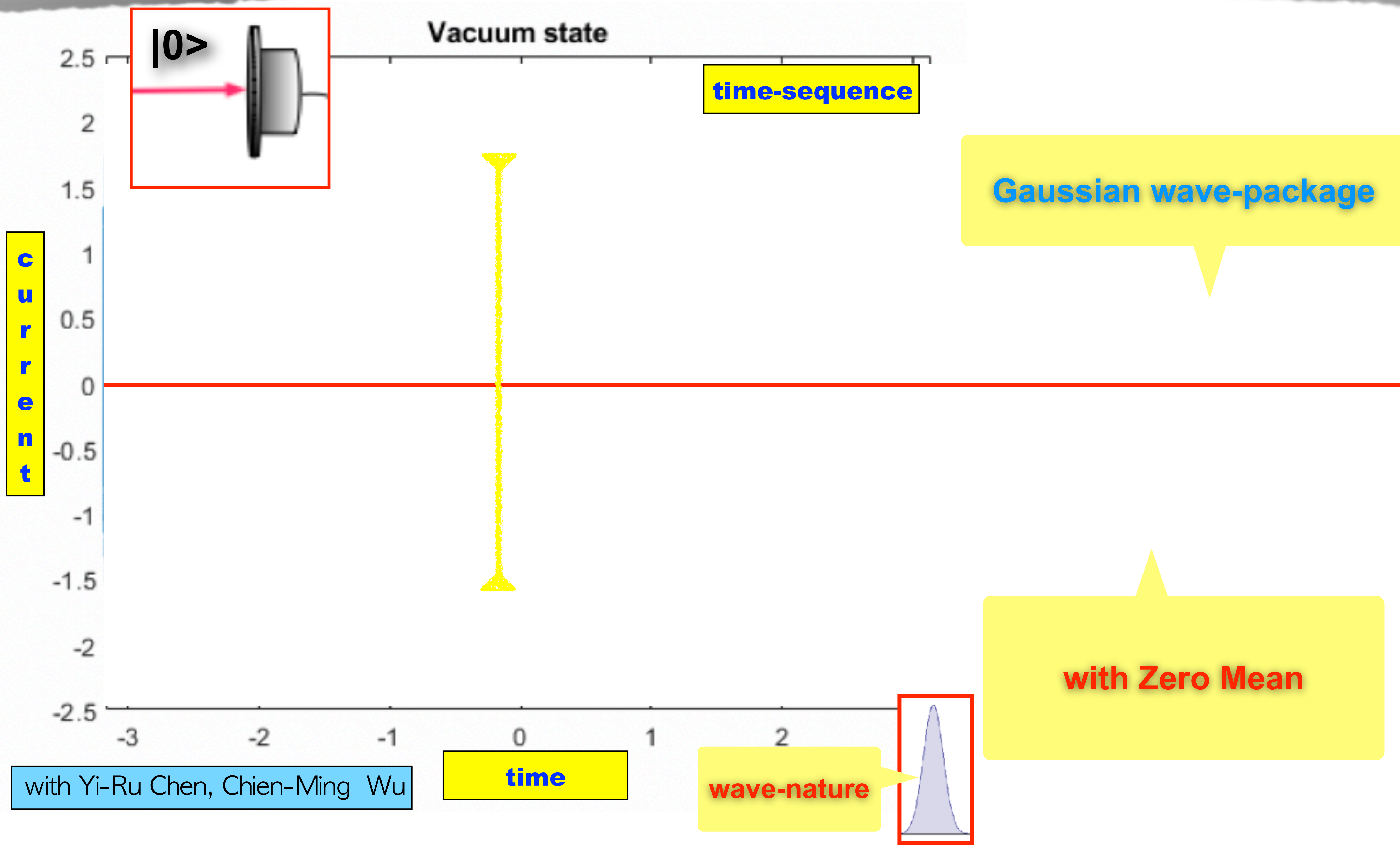
- Energy quantization
- Equally spacing in energy difference
- Zero-point energy $\neq 0$

$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \qquad \epsilon = 2n + 1, \qquad n = 0, 1, 2, 3\ldots$$

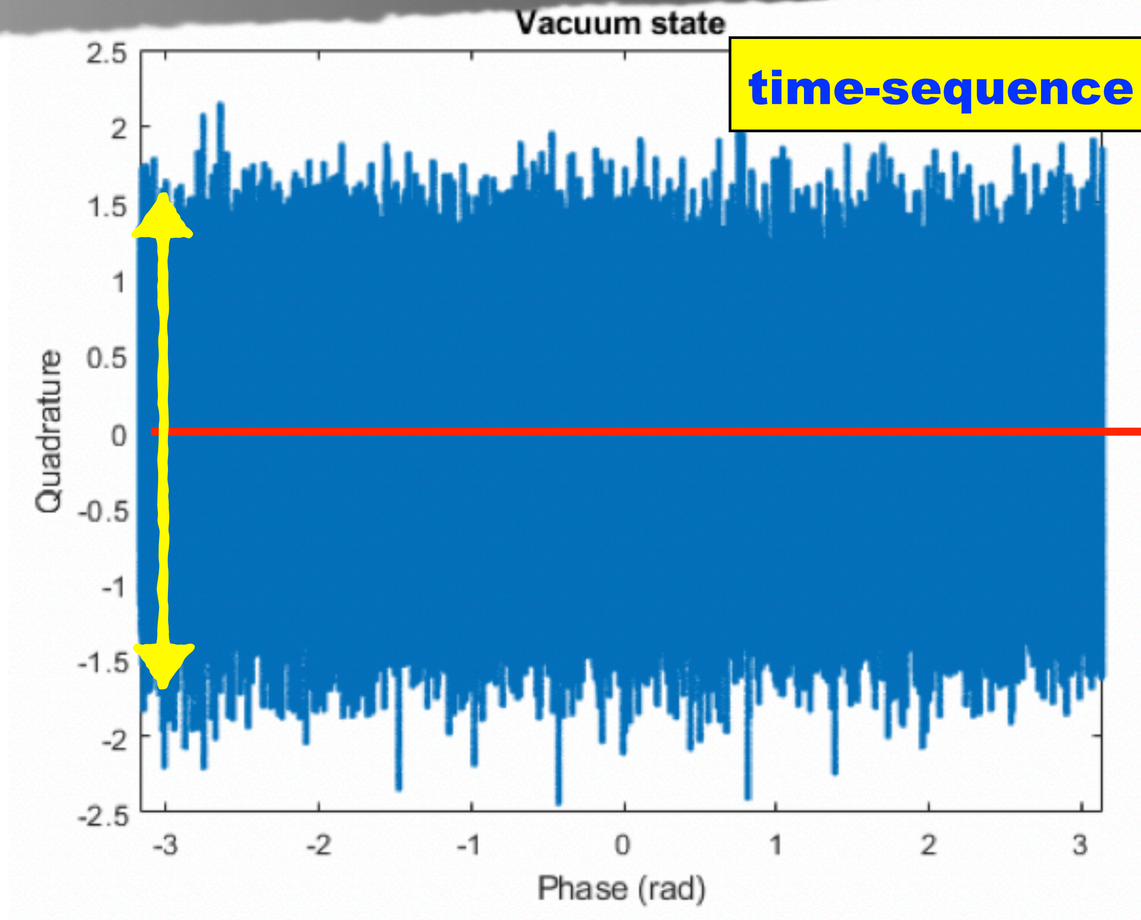
$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

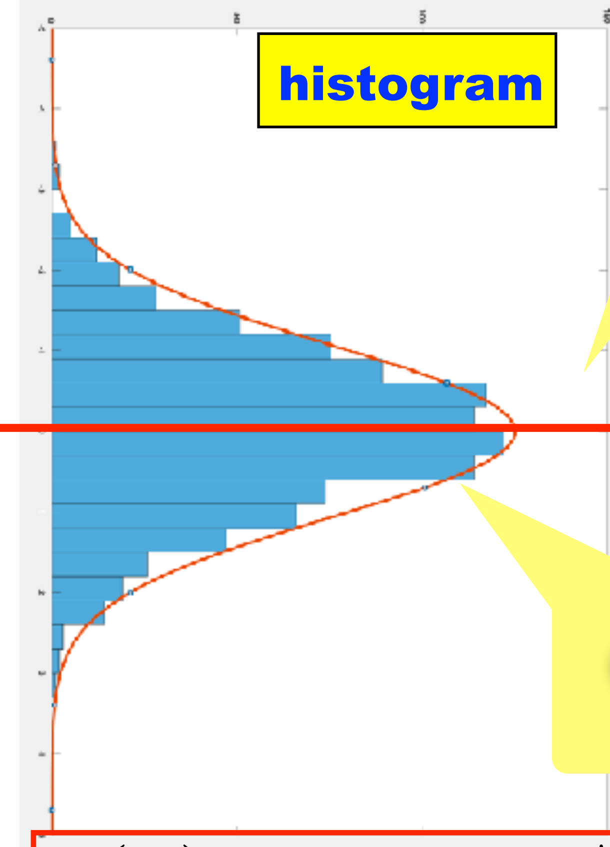
- **Energy Quantization**
- **Zero-Point Energy**

Vacuum State: |0>



Vacuum State: |0>





with Zero Mean

$$E_0 = \hbar\omega/2$$

Zero-Point Energy

Gaussian wave-package

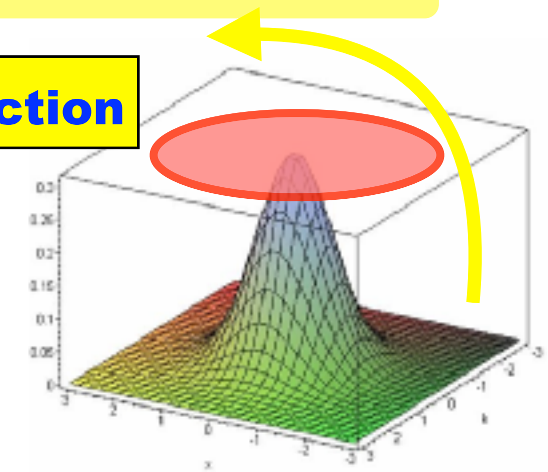
$$\Psi(x) = \langle x|0\rangle = C\,exp[-x^2/\Delta x^2]$$
$$\tilde{\Psi}(p) = \langle p|0\rangle = C\,exp[-\Delta x^2\,p^2]$$

Uncertainty-Relation

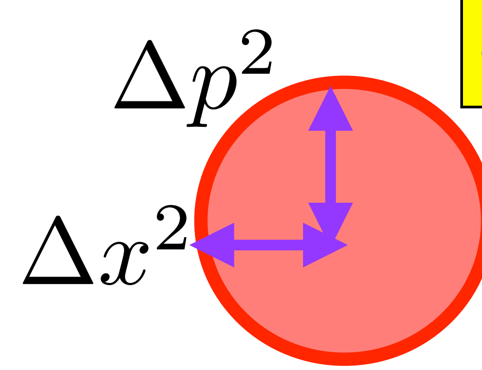
$$\Delta x^2 \times \Delta p^2 \geq \frac{\hbar^2}{4}$$

- Planck constant: $\hbar$
- Discrete Energy levels:
- Quantum states : $|\,\Psi\,\rangle$
- Wave-function
- Probability distribution
- Wave-Particle Duality
- Uncertainty Relation
- Vacuum fluctuation

Wave-function



circle



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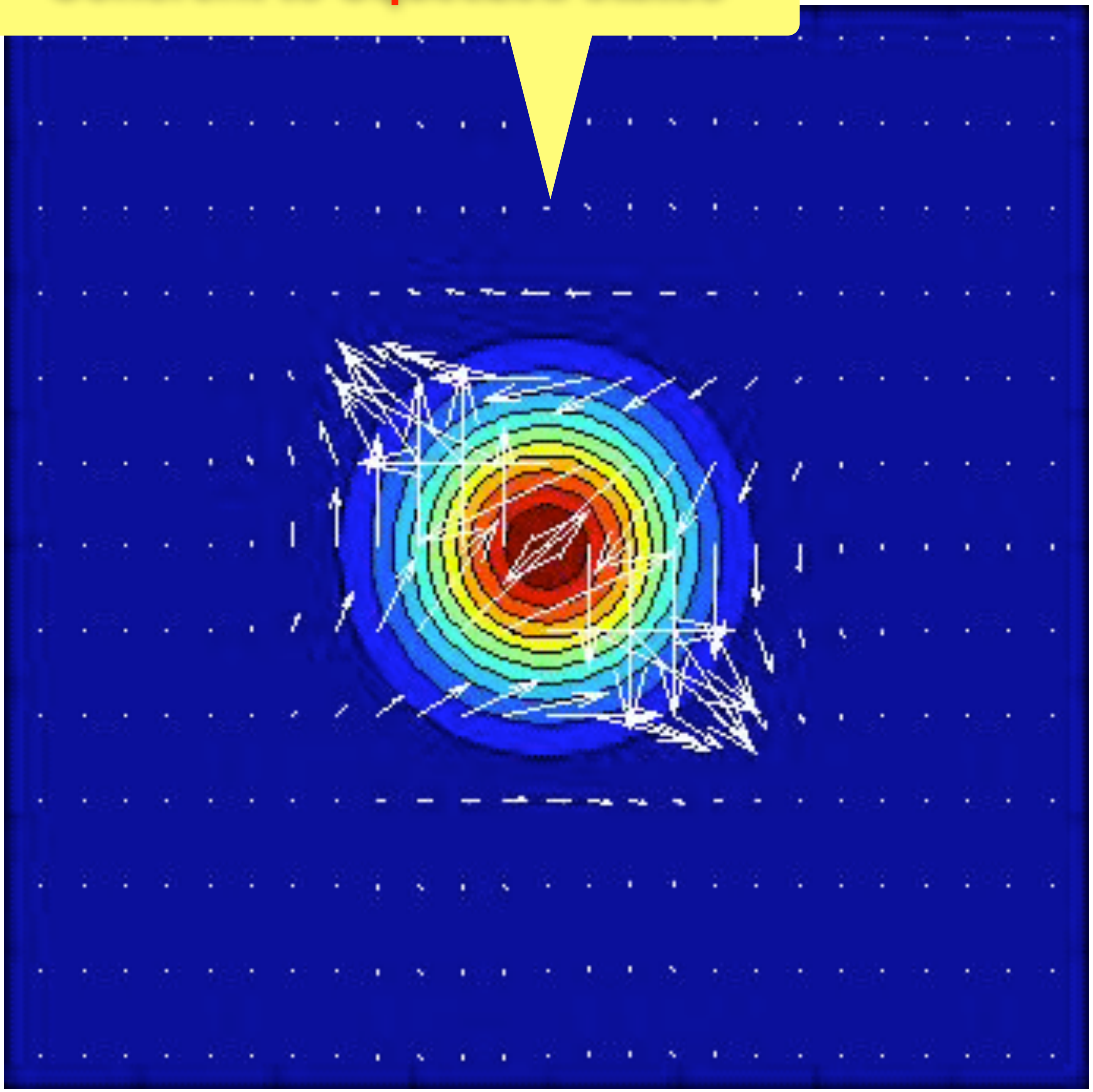
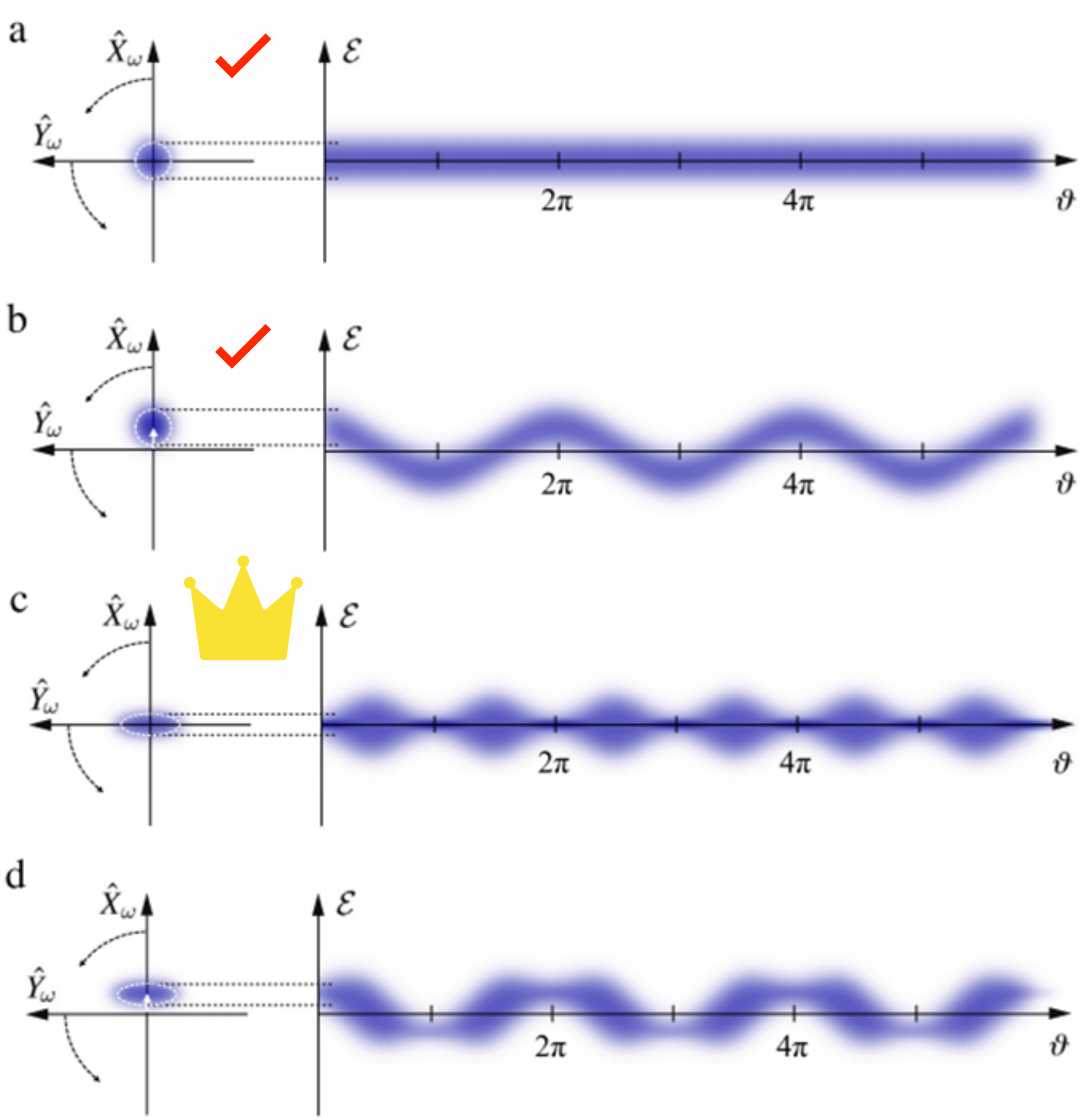
Squeezed States

|ξ>

Coherent to Squeezed states

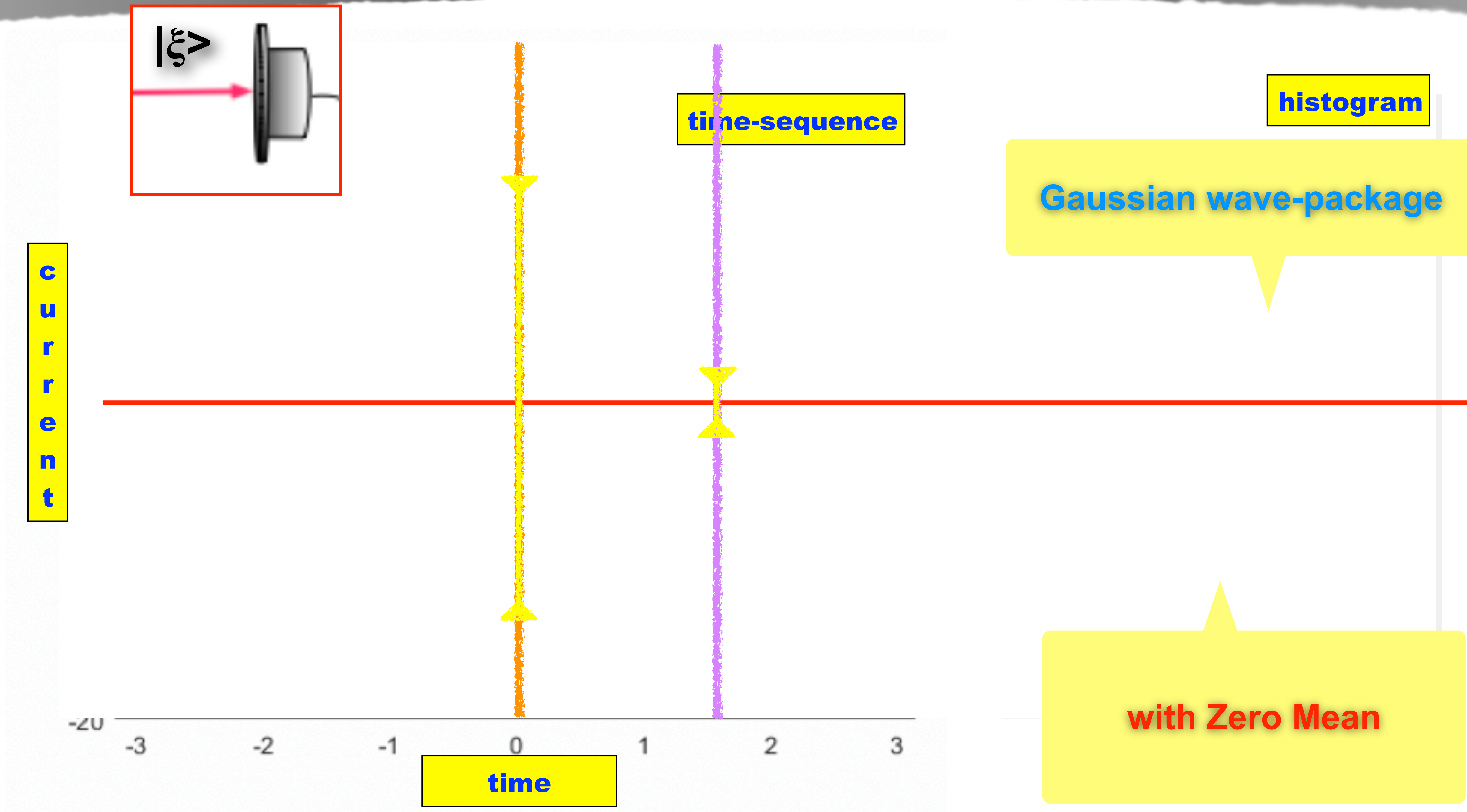




Courtesy:
Roman Schnabel (2017).

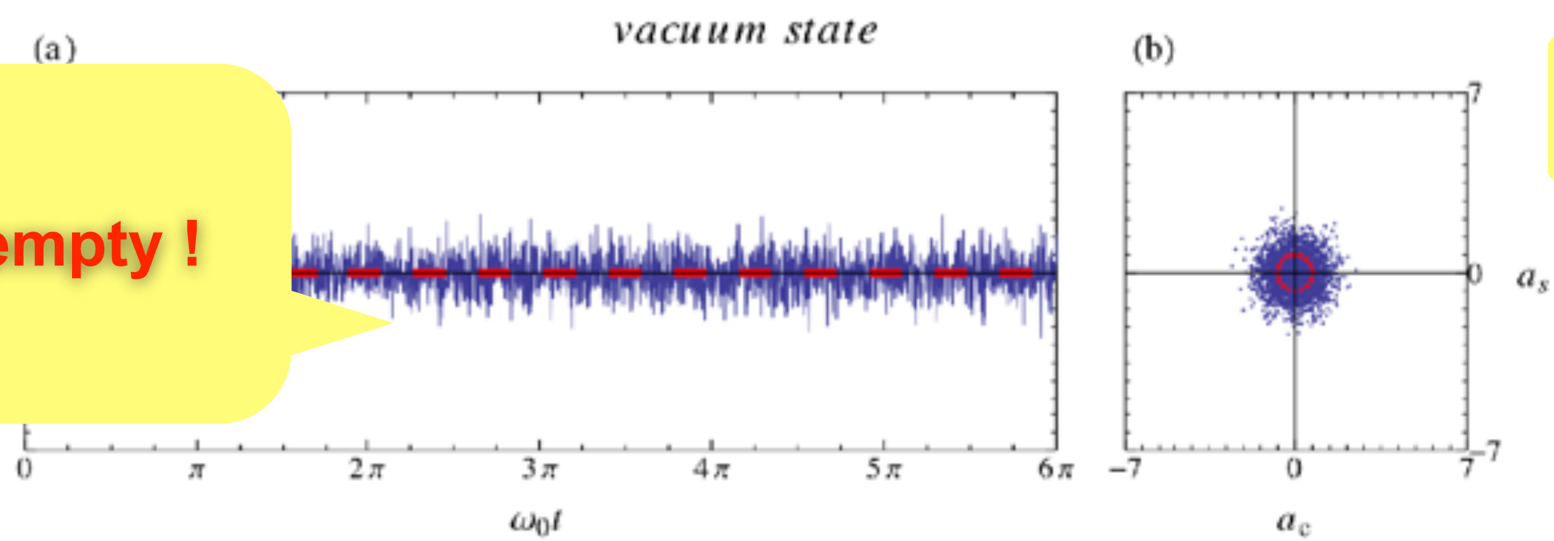
by Popo Yang

Squeezed Vacuum State: |ξ>

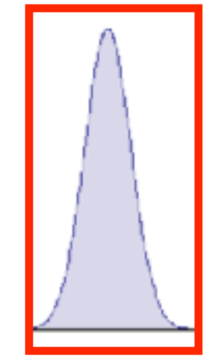


with Yi-Ru Chen, Chien-Ming Wu

Vacuum is NOT empty !



wave-nature



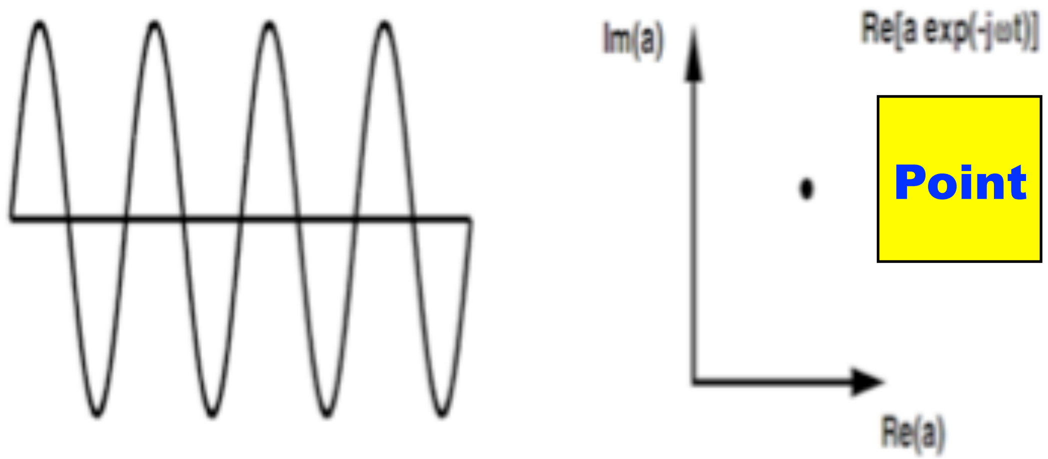
● coherent state

$$(\Delta\hat{X}_1)^2 = \frac{1}{4}, (\Delta\hat{X}_2)^2 = \frac{1}{4}, \quad \Delta\hat{X}_1 \Delta\hat{X}_2 = \frac{1}{4}$$

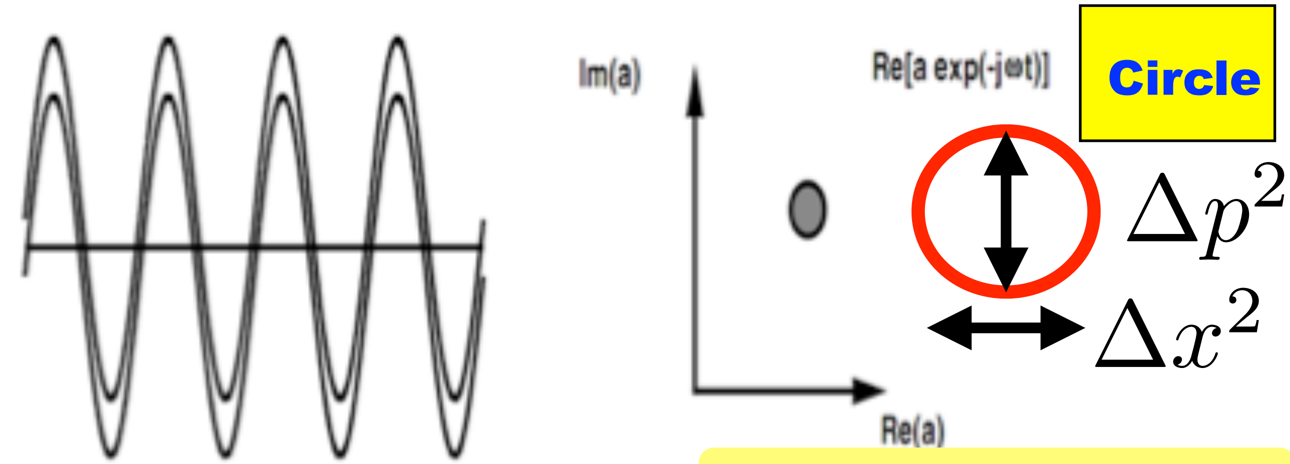
● squeezed state

$$(\Delta\hat{Y}_1)^2 = \frac{1}{4}e^{-2r}, (\Delta\hat{Y}_2)^2 = \frac{1}{4}e^{2r}, \Delta\hat{Y}_1 \Delta\hat{Y}_2 = \frac{1}{4}$$

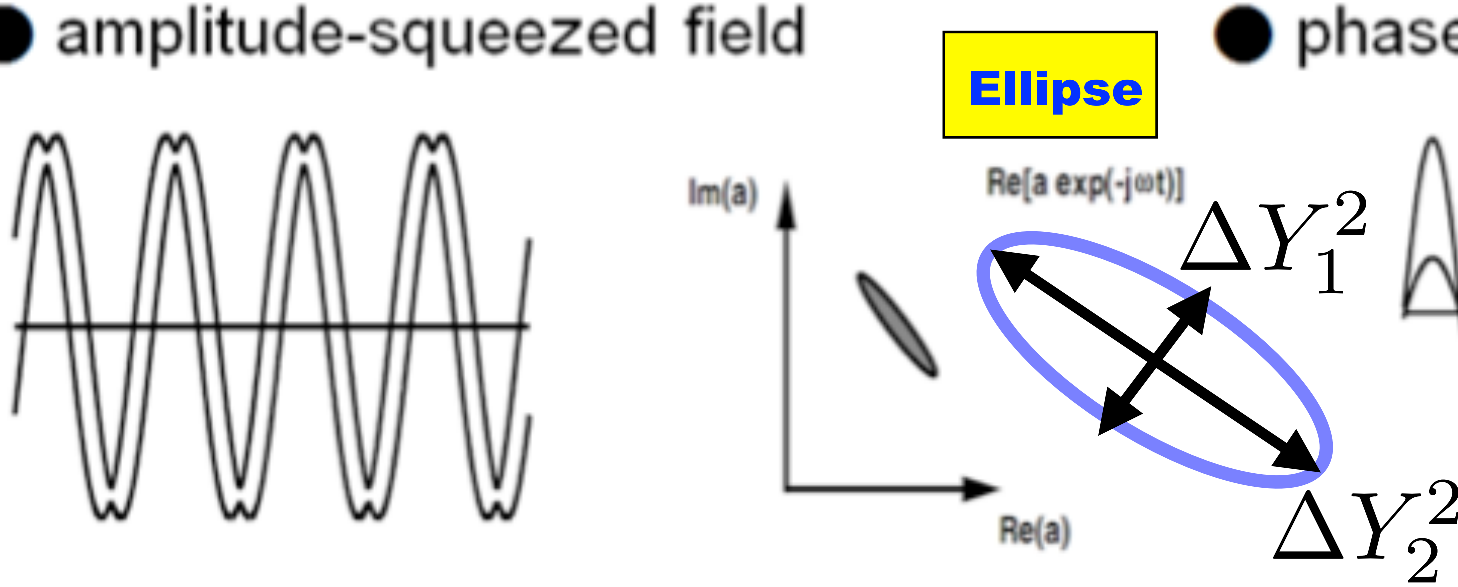
● classical field



● coherent field

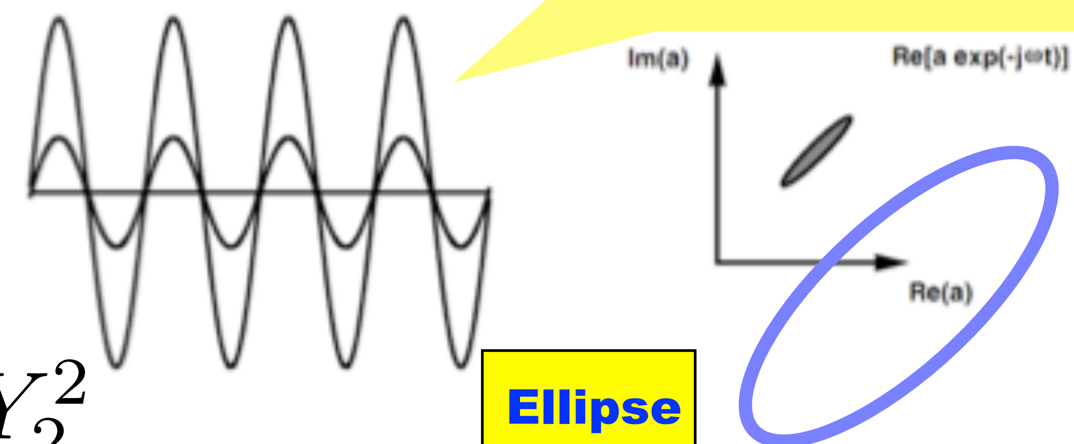


● amplitude-squeezed field



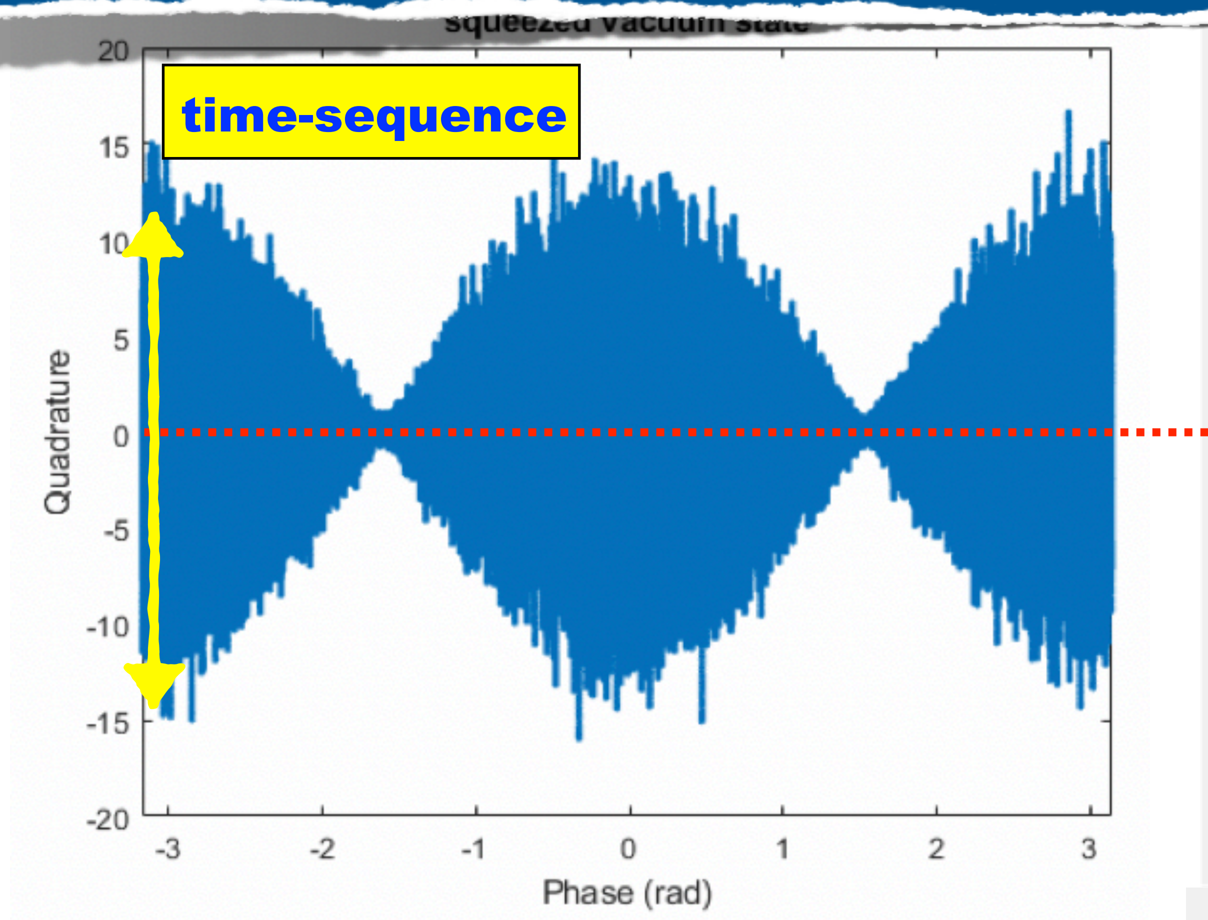
● phase-squeezed field

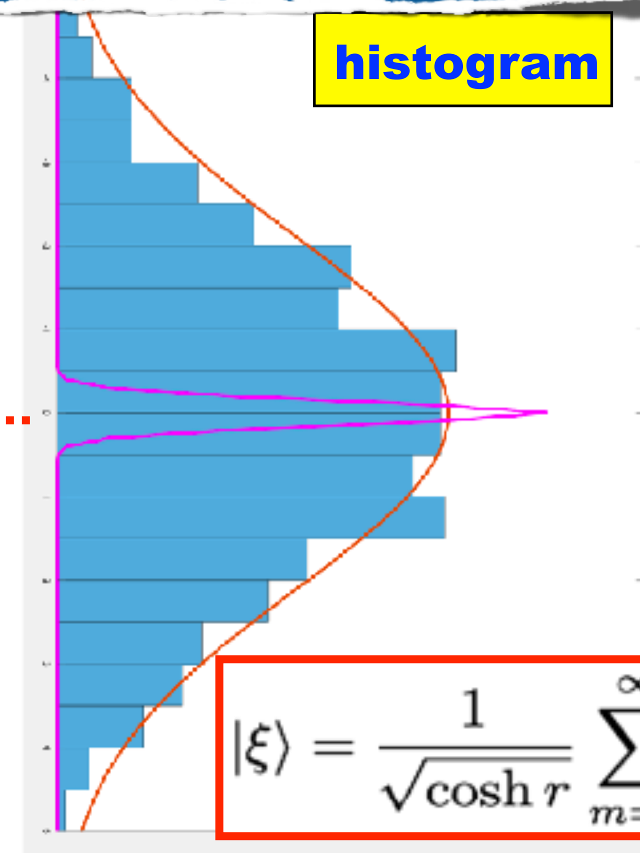
Non-classical states

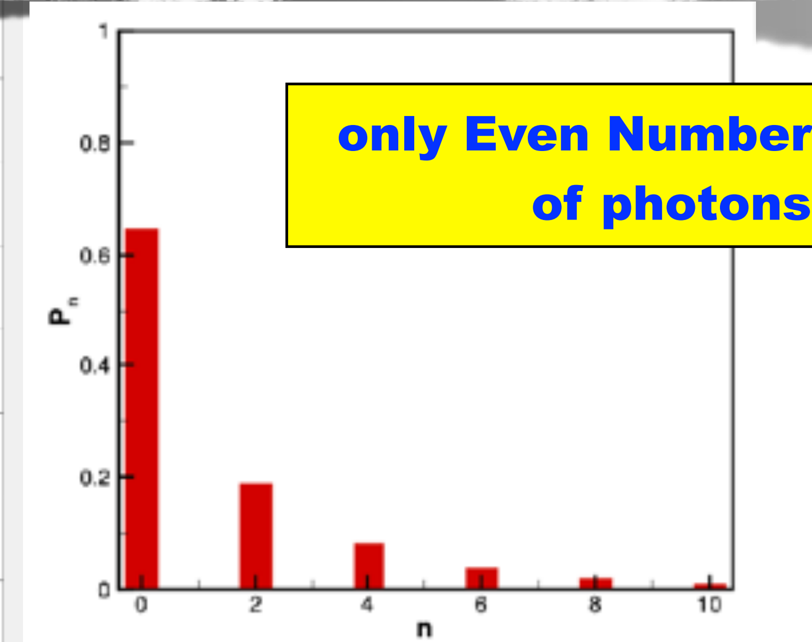


Squeezed States

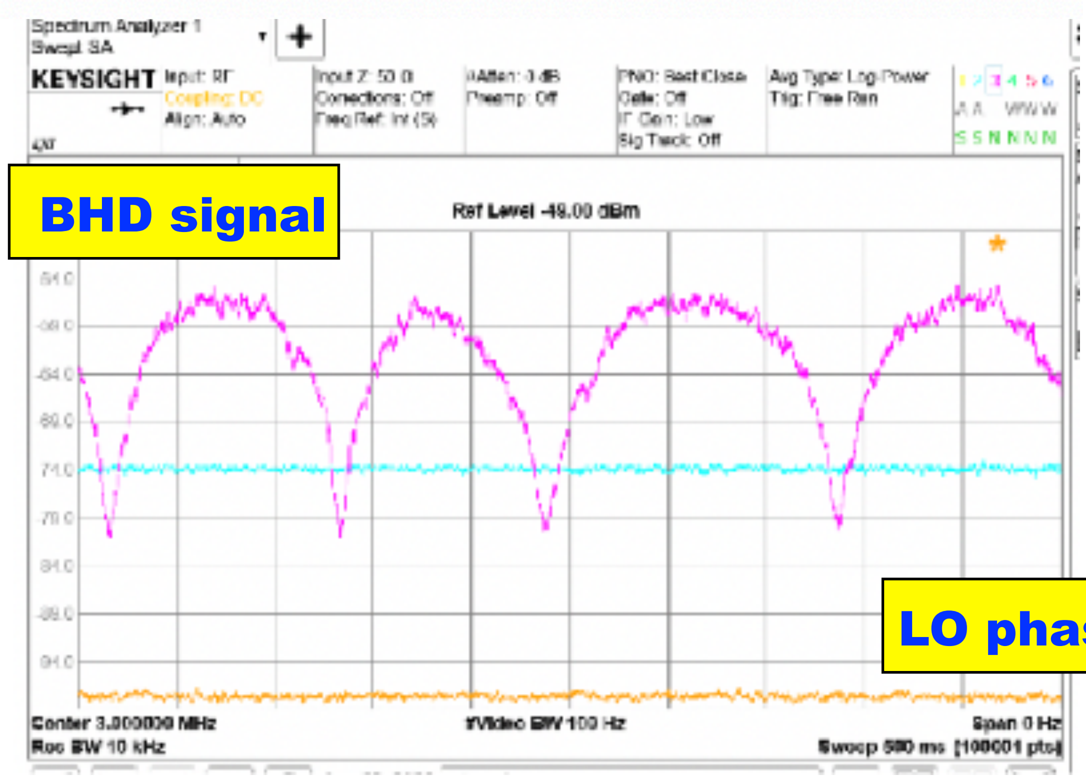
|ξ>

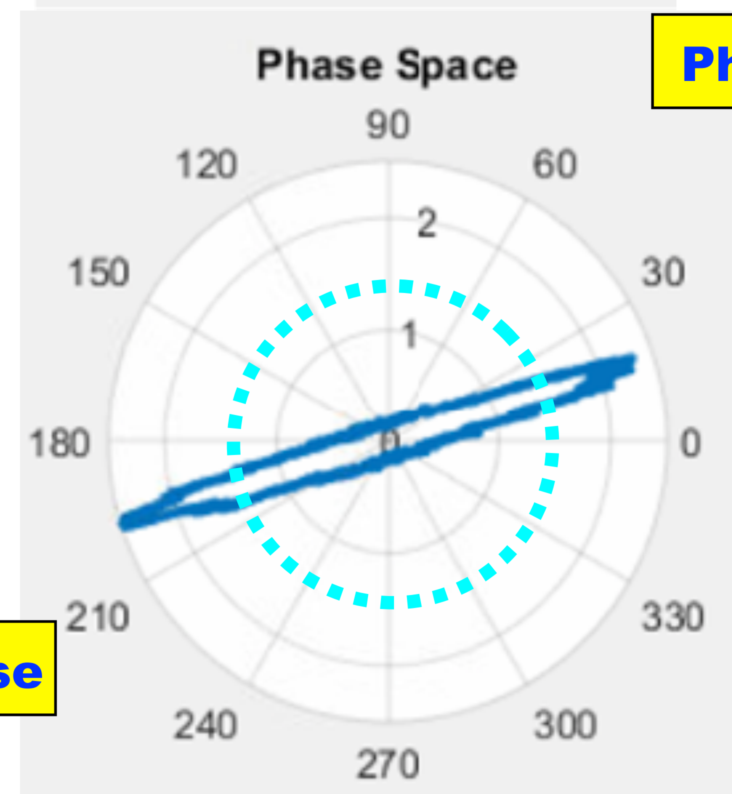


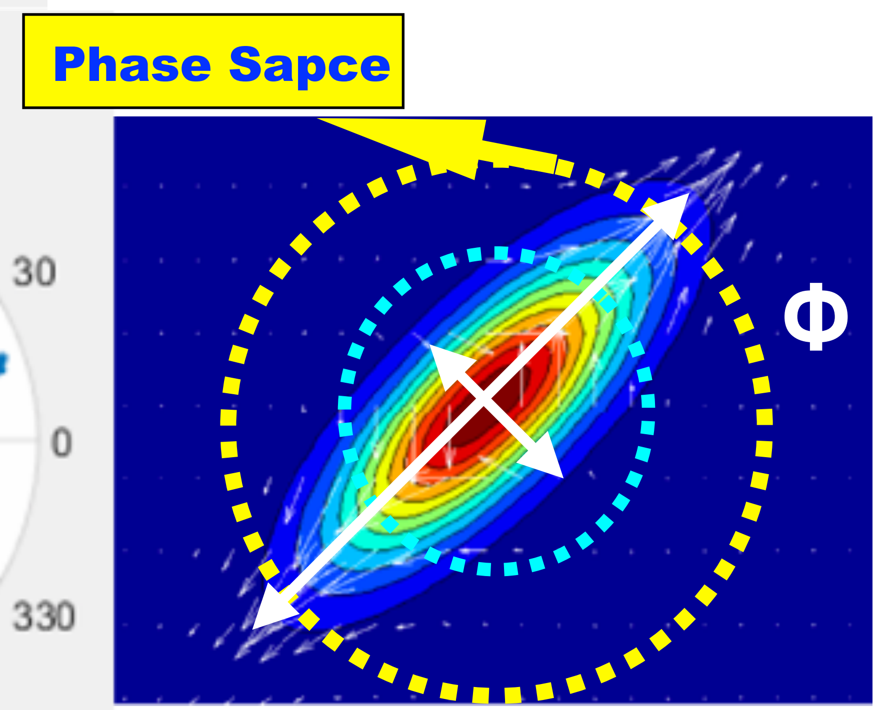




$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle$$

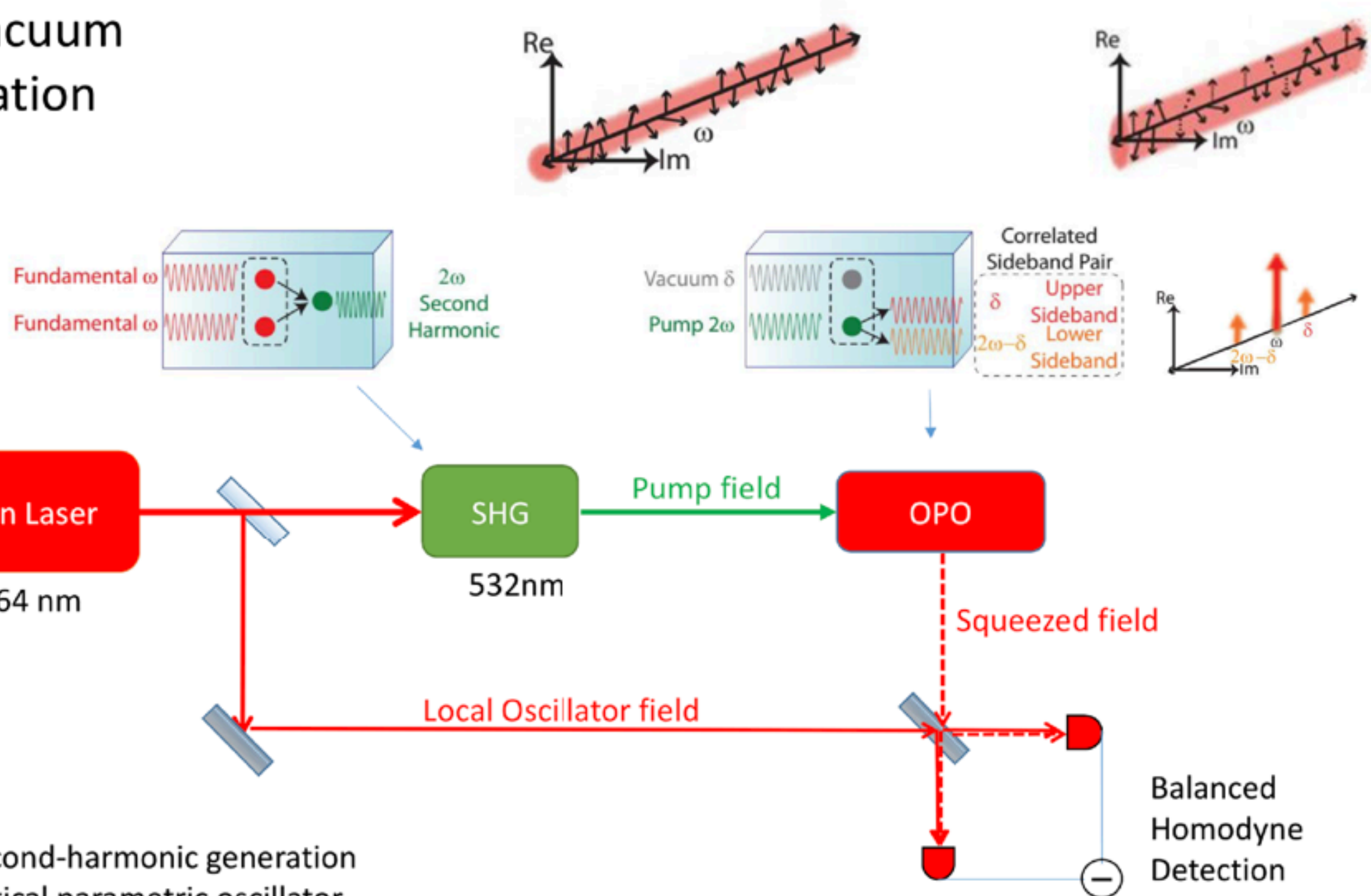






Optical Parametric Oscillator, OPO

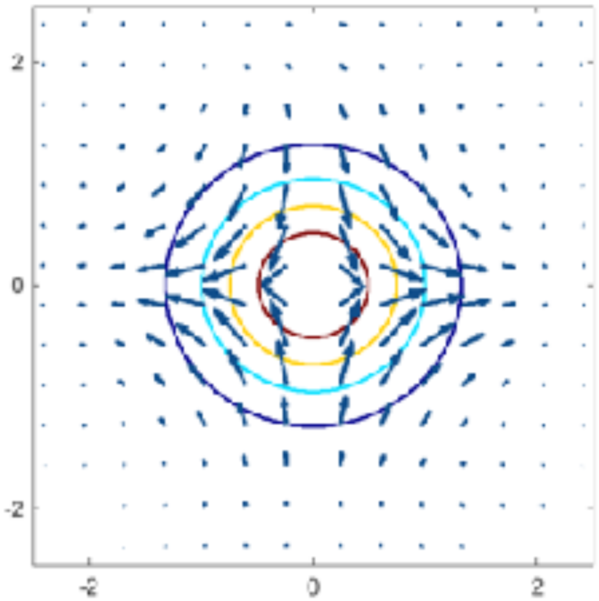
Squeezed Vacuum
State Generation

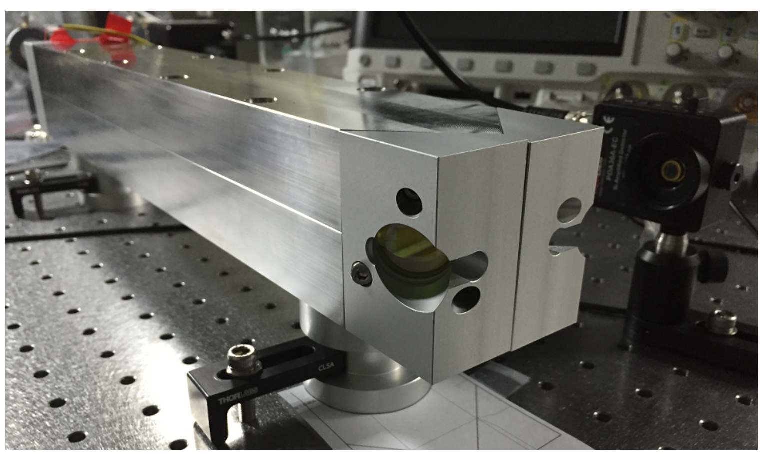


SHG : Second-harmonic generation
OPO: Optical parametric oscillator

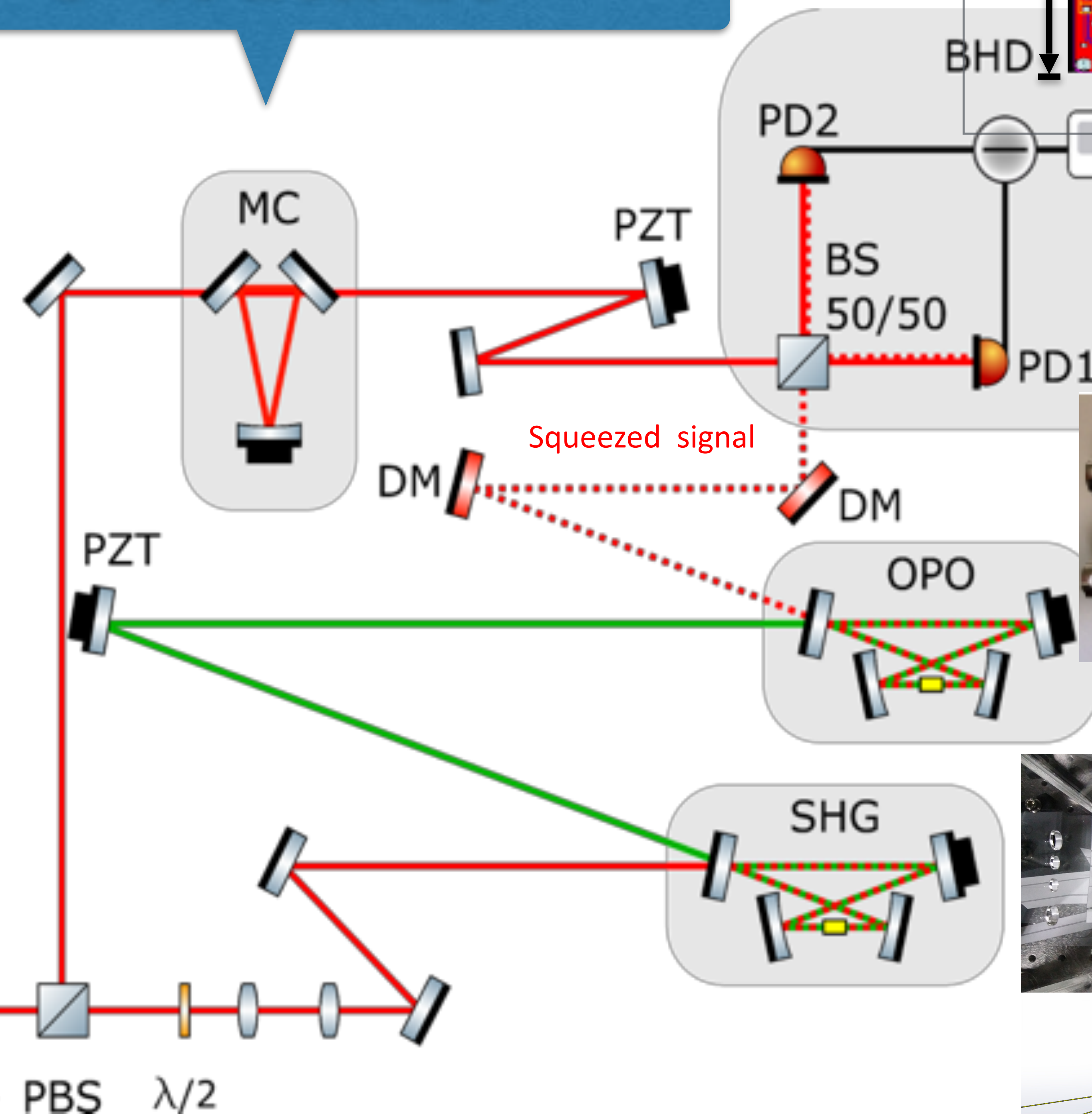
1st Squeezer @ Taiwan

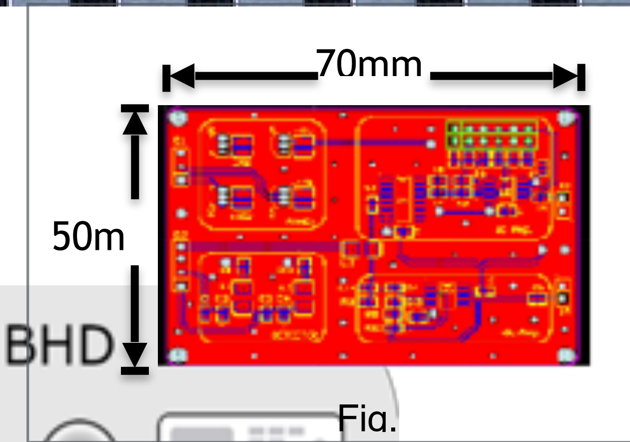
Exp. Reconstruction

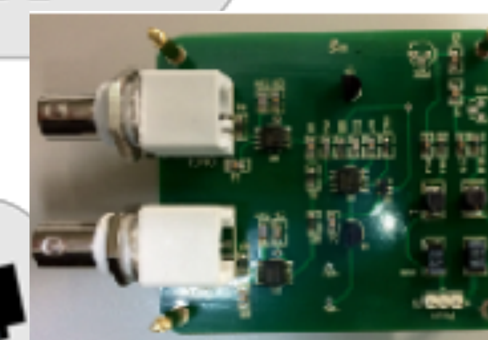


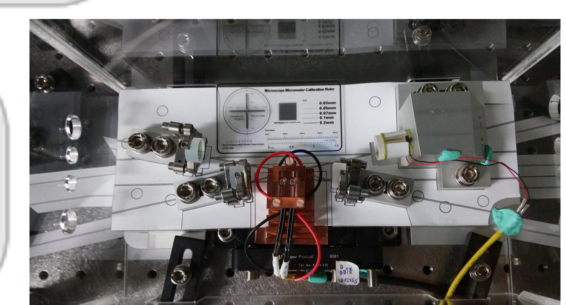


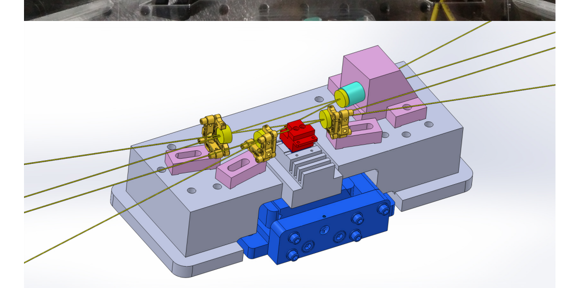
1064 nm, 2W, BW < 1 kHz





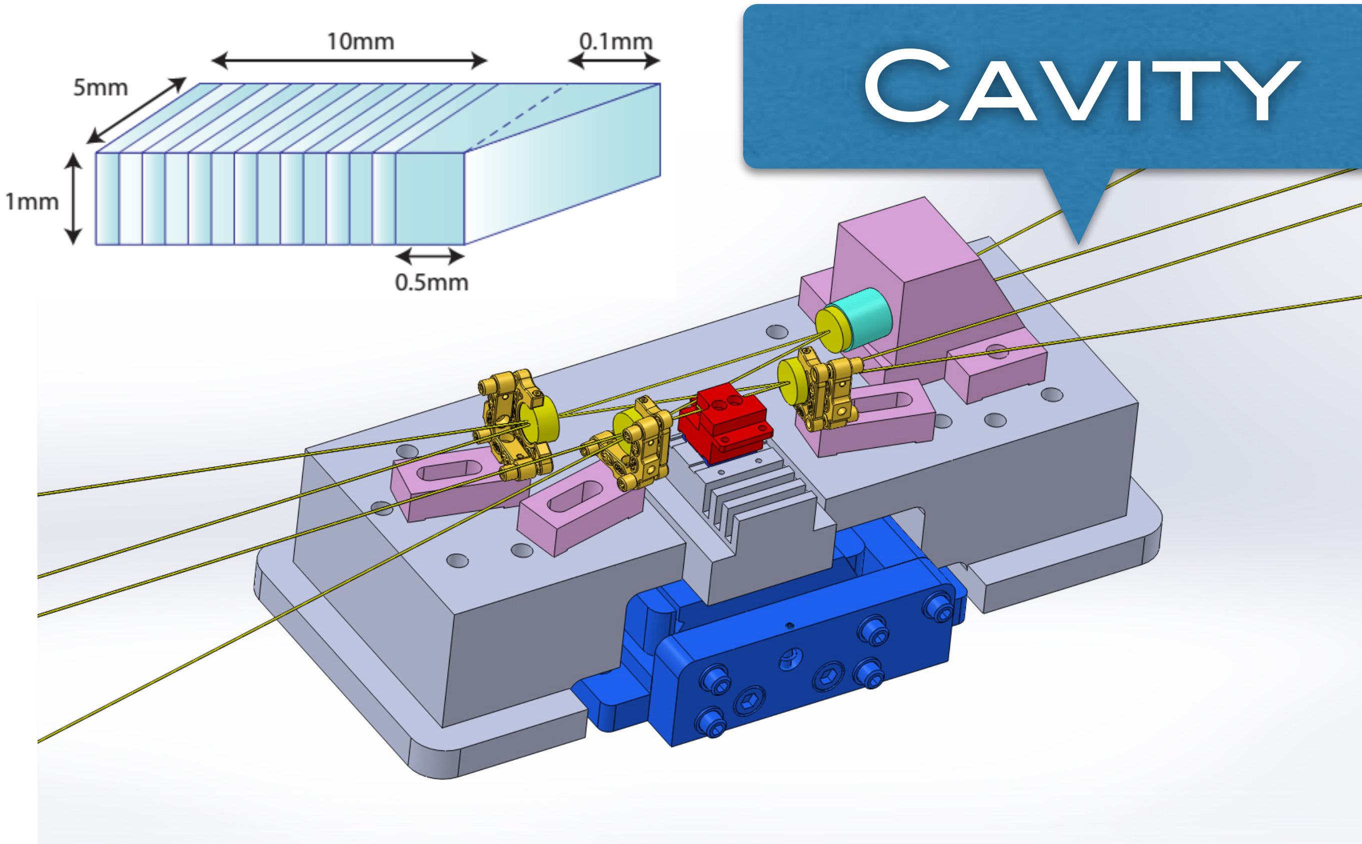


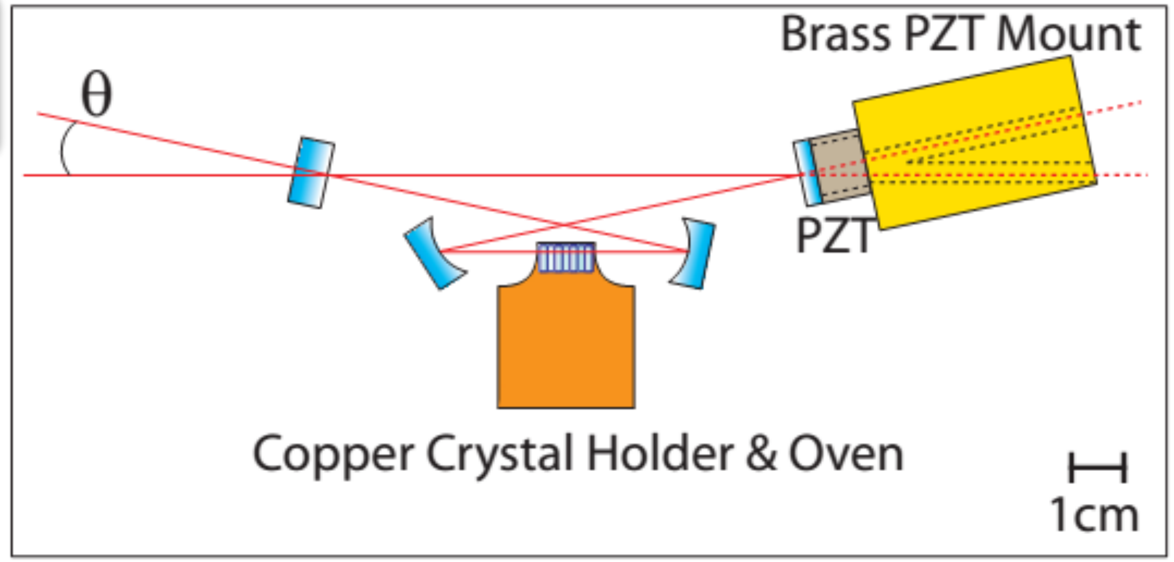


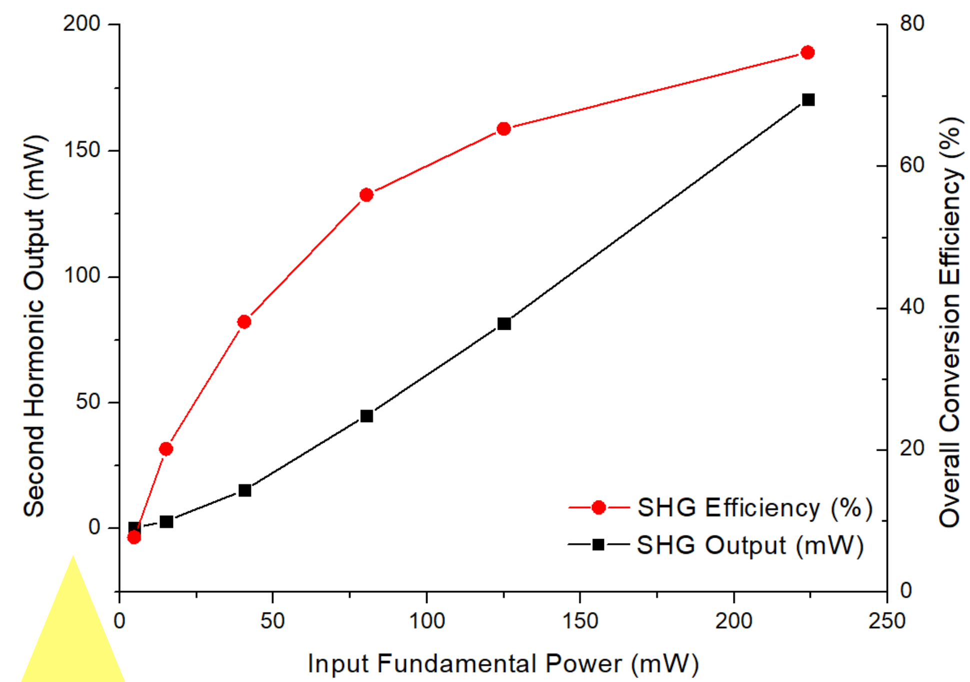


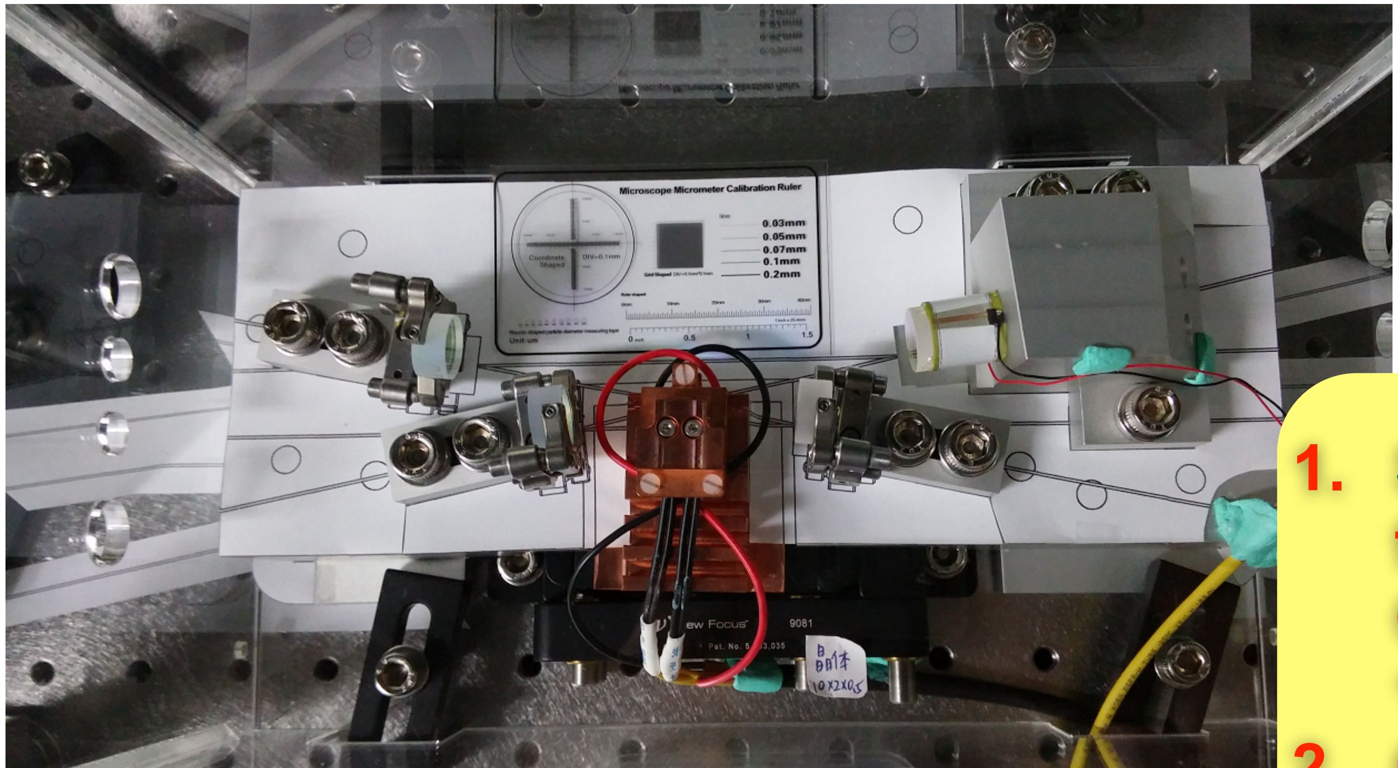
Yi-Ru Chen et al., arXiv: 2111.08285 (2021).

CAVITY







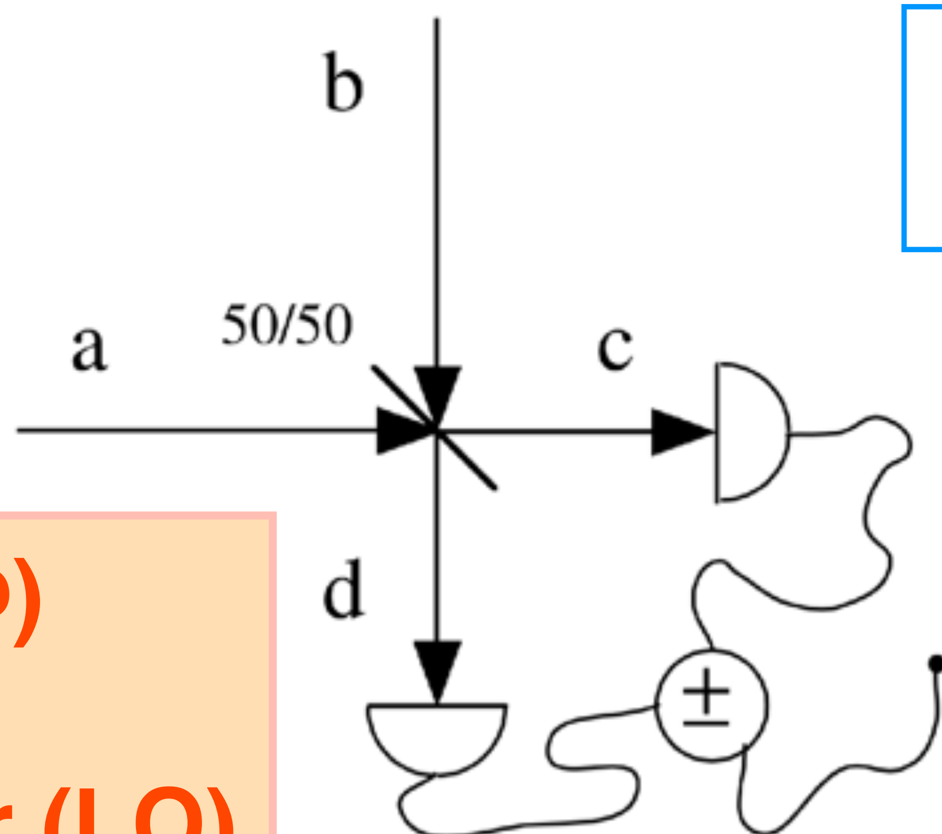


1. SHG : PPLN+Mg 1mm*3mm*10mm Bulk, from HC Photonics (Taiwan company) Conversancy efficiency~ 78 % (at 224mW 1064 nm input)
2. OPO: PPKTP 1mm*5mm*10.5

Balanced Homodyne Detector, BHD

|0> or |ξ>

$$\tilde{a} = \alpha + \delta\tilde{a}(\omega)$$
$$\tilde{b} = \beta + \delta\tilde{b}(\omega)$$



$$\tilde{c} = \sqrt{1-\xi}\,\tilde{a} - \sqrt{\xi}\,\tilde{b}$$
$$\tilde{d} = \sqrt{\xi}\,\tilde{a} + \sqrt{1-\xi}\,\tilde{b}$$

|α> *exp(i Φ)

Local Oscillator (LO)

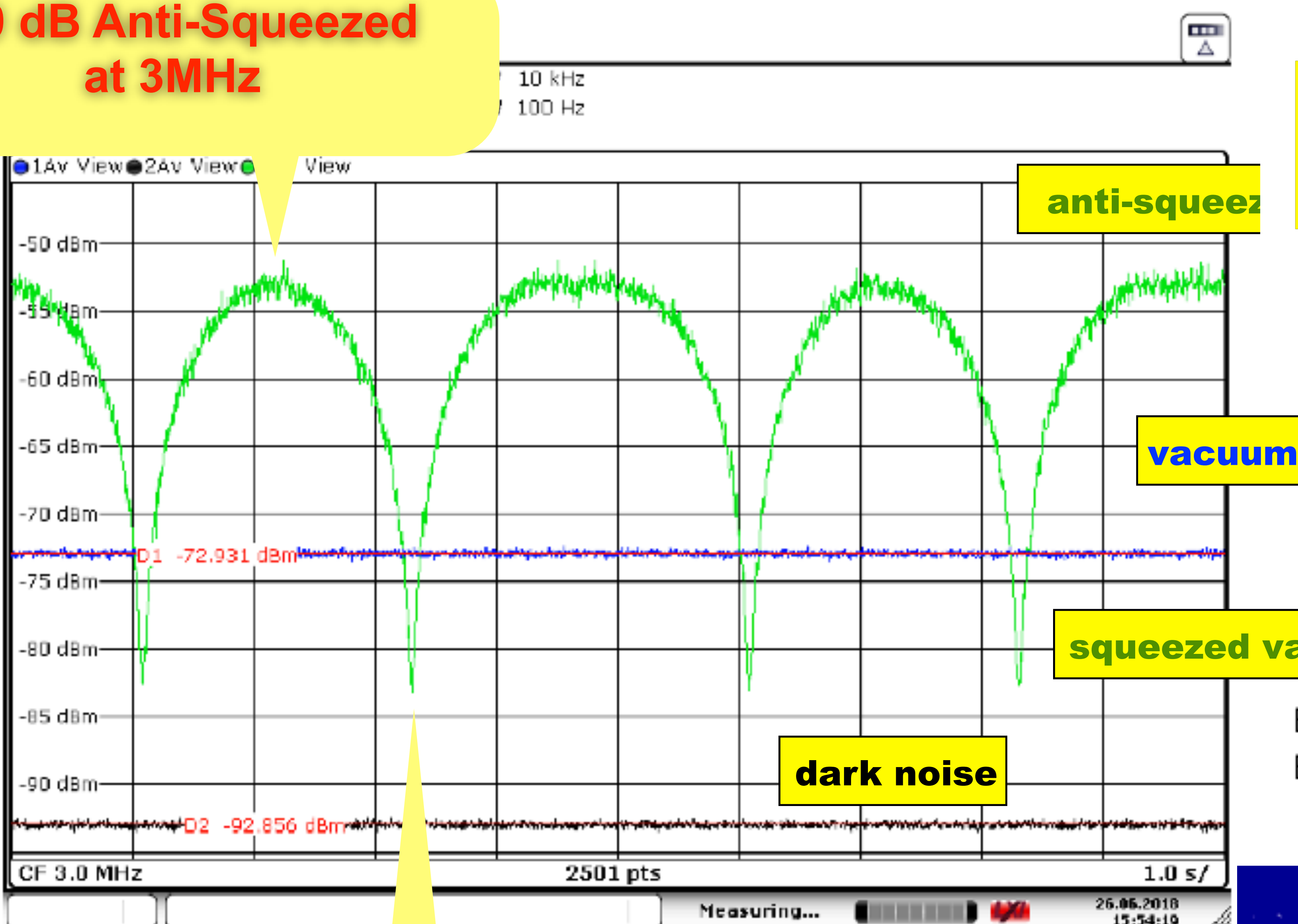
$$\tilde{n}^{-} = 0 + \alpha\,\delta\tilde{X}_1{}^{b}$$

- **Bias the output signal with Local Oscillator (LO), which is a strong Classical field.**
- **Clearance (>30 dB): away from the dark noises**
- **CMRR (>80 dB): Common-Mode Rejection Ratio (the balanced)**
- **Phase of quadrature is referred to LO**

OPO 532nm incident power: 96mW
MC output(LO beam)= 14.5mW

specially ordered InGaAs Photodiodes
Laser Components GmbH ϕ=500μm QE≥99%

+20 dB Anti-Squeezed at 3MHz



10dB = 10-fold
20dB = 100-fold
noise reduction

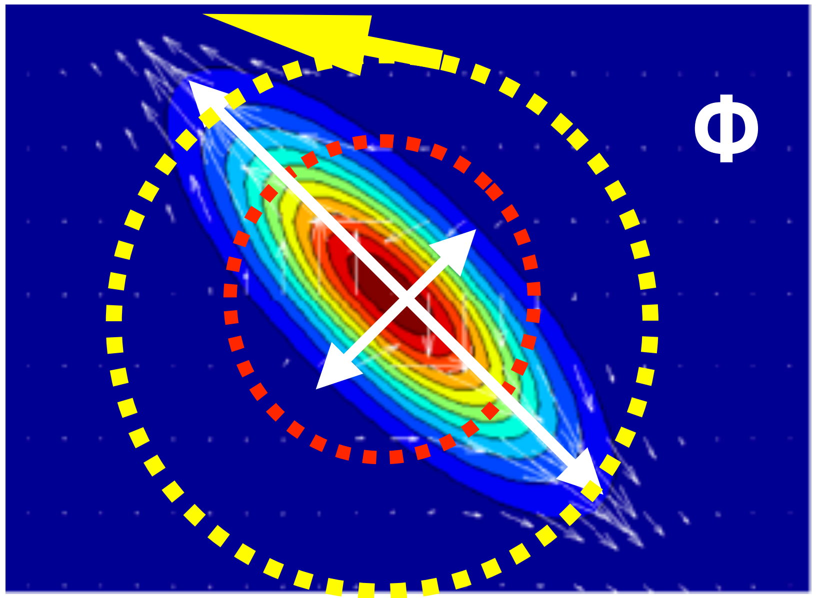
Clearance= 20dB

Zero Span mode at 3MHz
RBW=10kHz
VBW=100Hz

Blue Line: 14.5mW Vacuum noise
Black Line: Dark noise

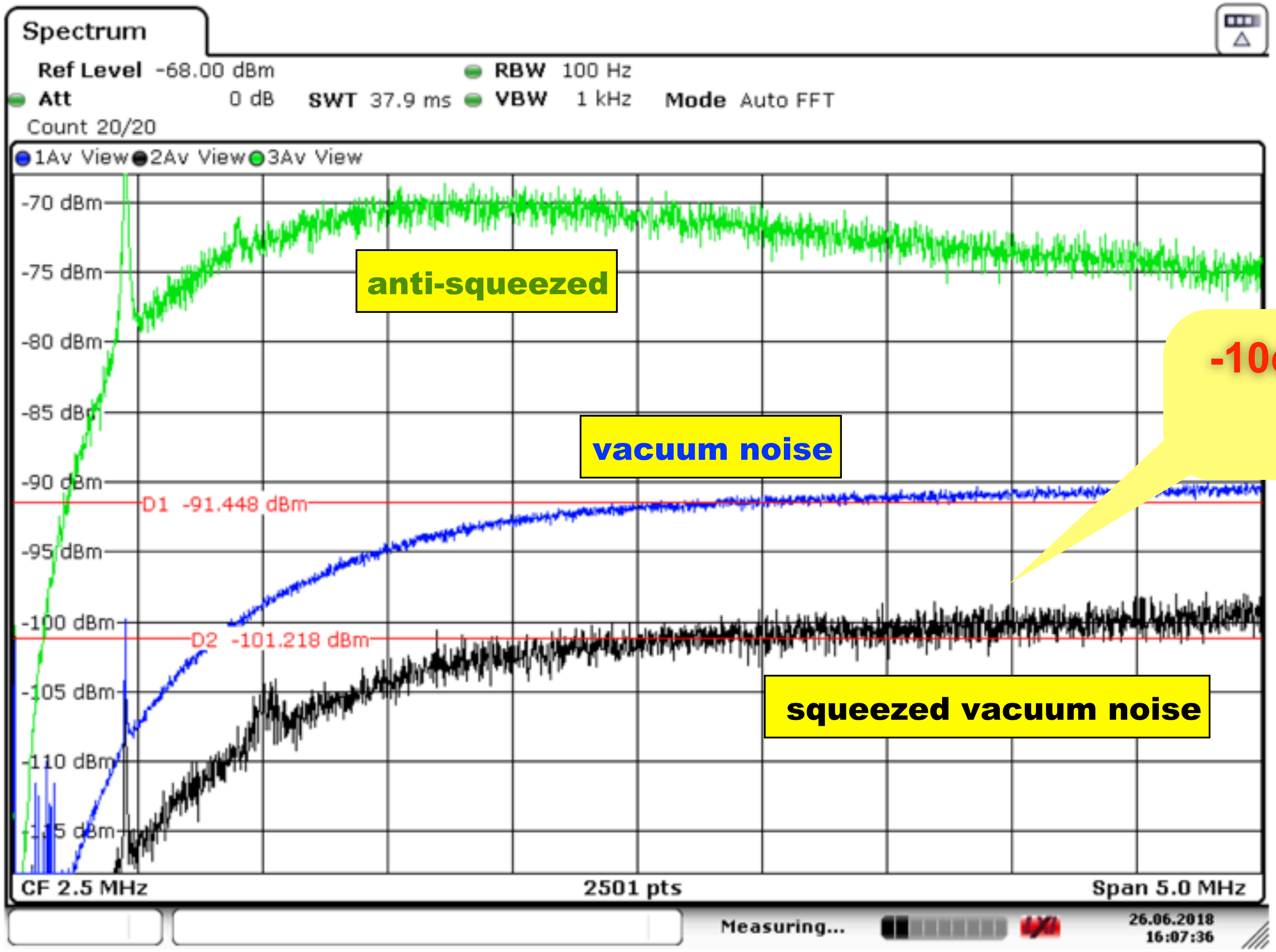
-10 dB Squeezed vacuum at 3MHz

Squeezing angle, Φ



by Chien-Ming Wu

Date: June 26th, 2018



Squeezed at 3MHz
-9.76dB

Blue Line: 14.5mW Vacuum noise
Black Line: Squeezed signal
Green Line: Anti-squeezed signal

by Chien-Ming Wu (吳建明博士)

Chien-Ming Wu, et al., “Detection of 10 dB vacuum noise squeezing at 1064 nm by balanced homodyne detectors with a common mode rejection ratio more than 80 dB,” Conference on Lasers and Electro-Optics (CLEO), JTu2A.38 (2019).

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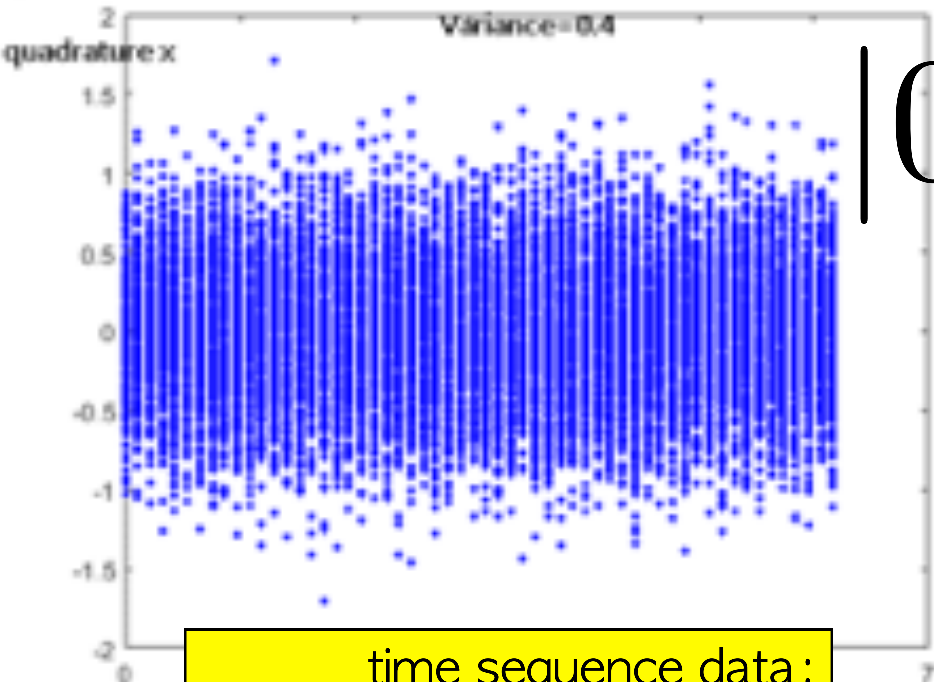
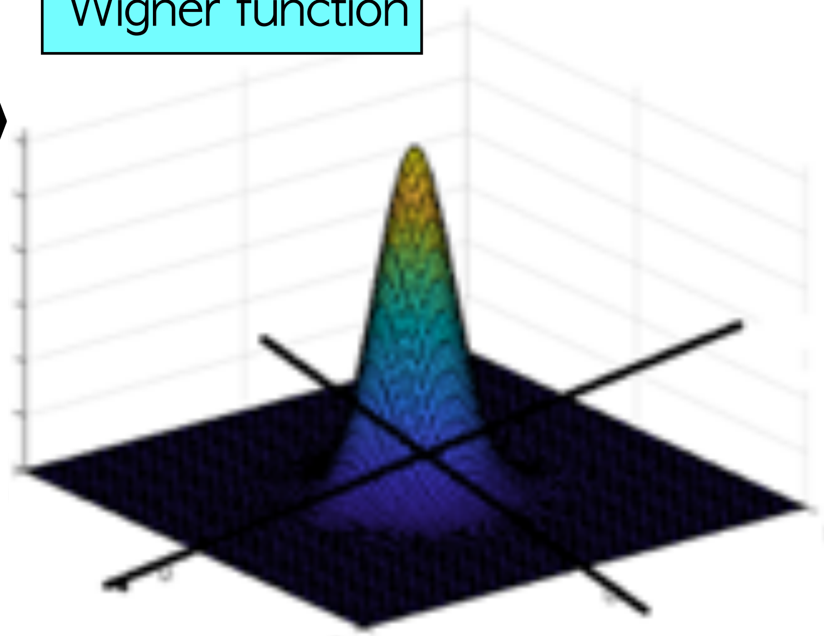
Quantum State Tomography

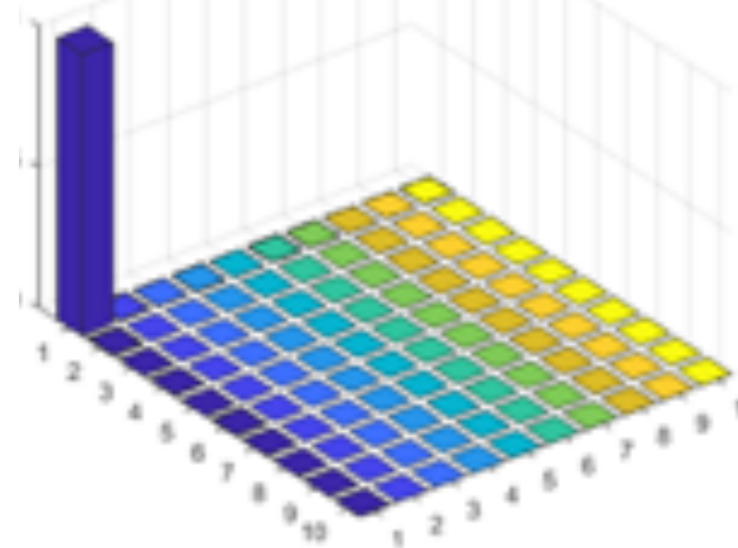


$|0\rangle$

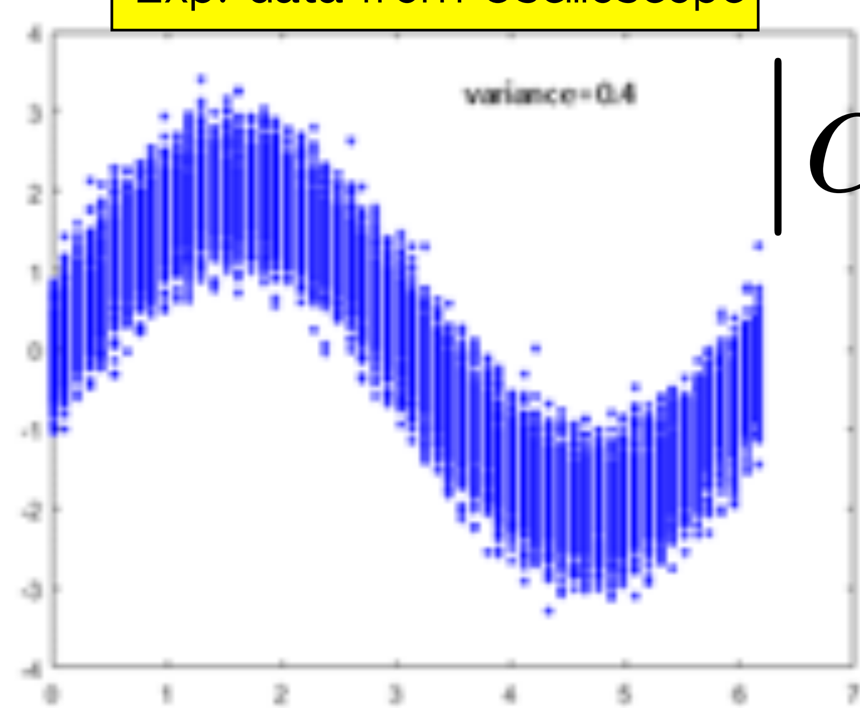
Wigner function



Density matrix in number basis

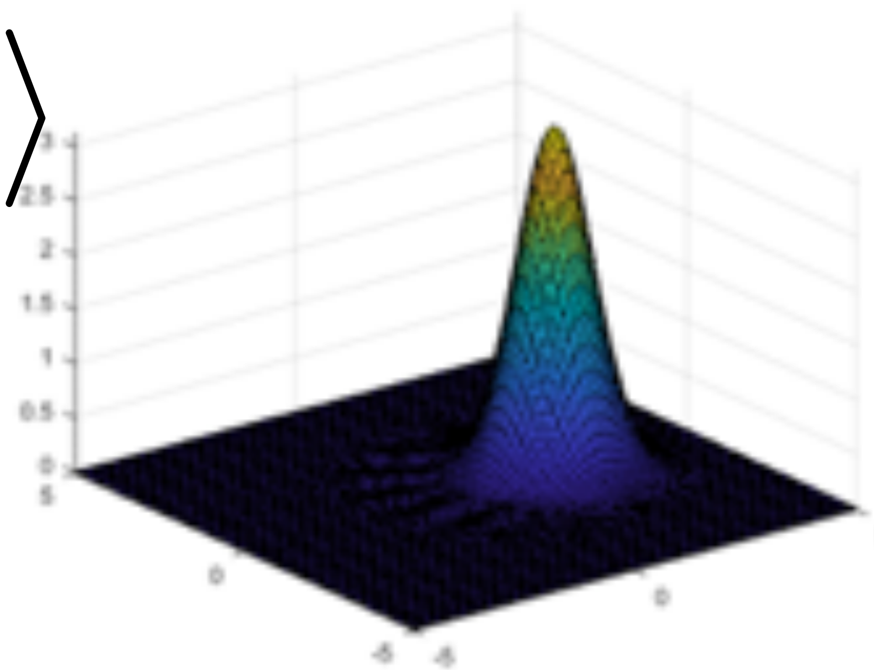


time sequence data:
Exp. data from Oscilloscope

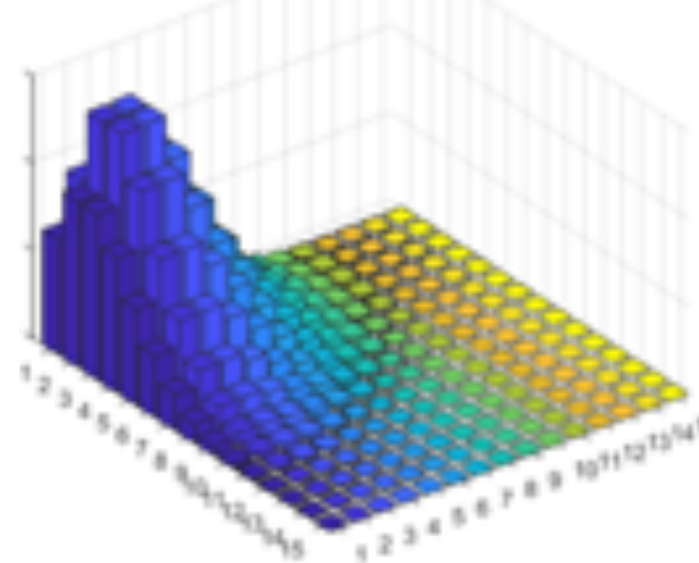


$|\alpha\rangle$

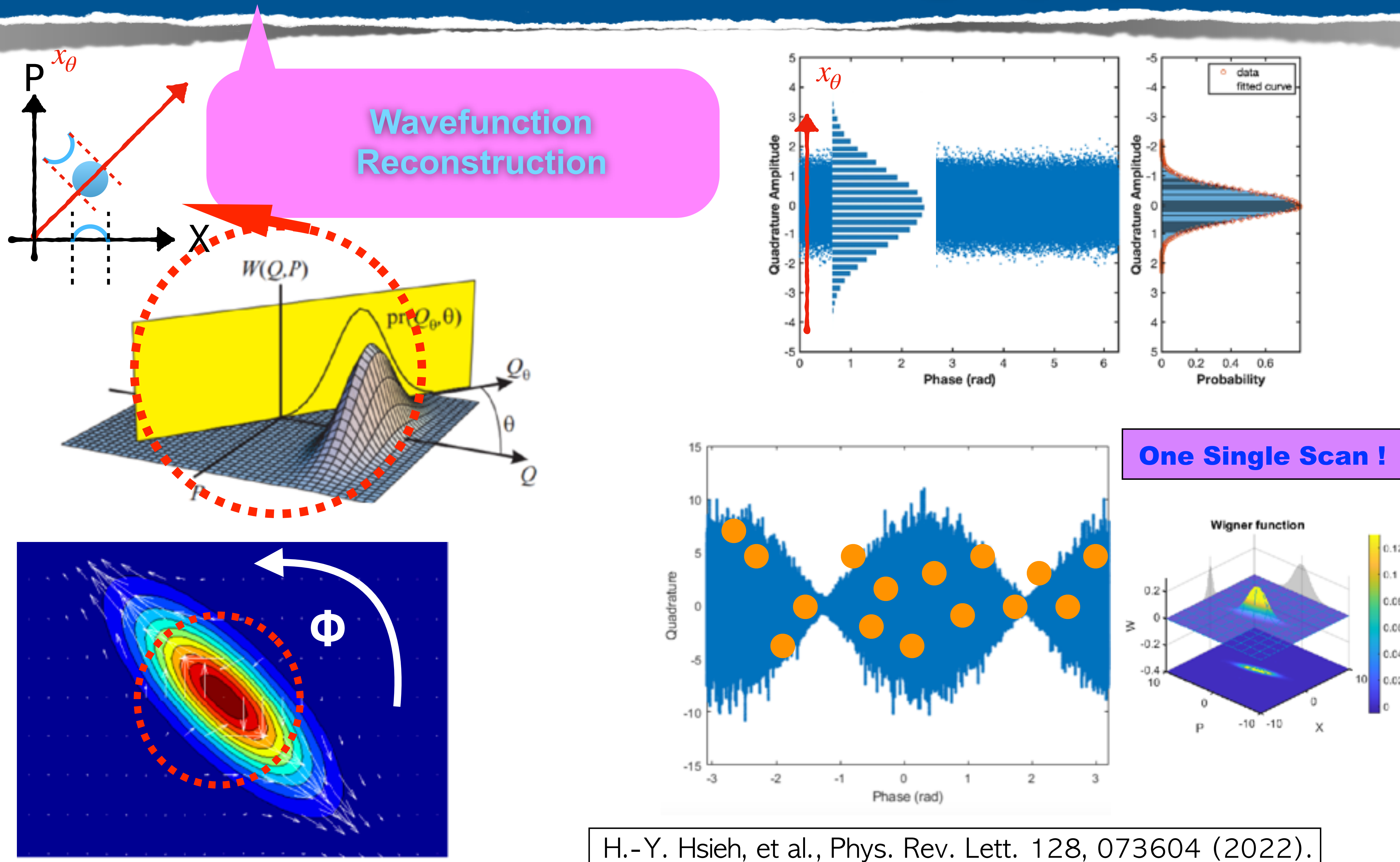
with Max. Likelihood estimation



$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

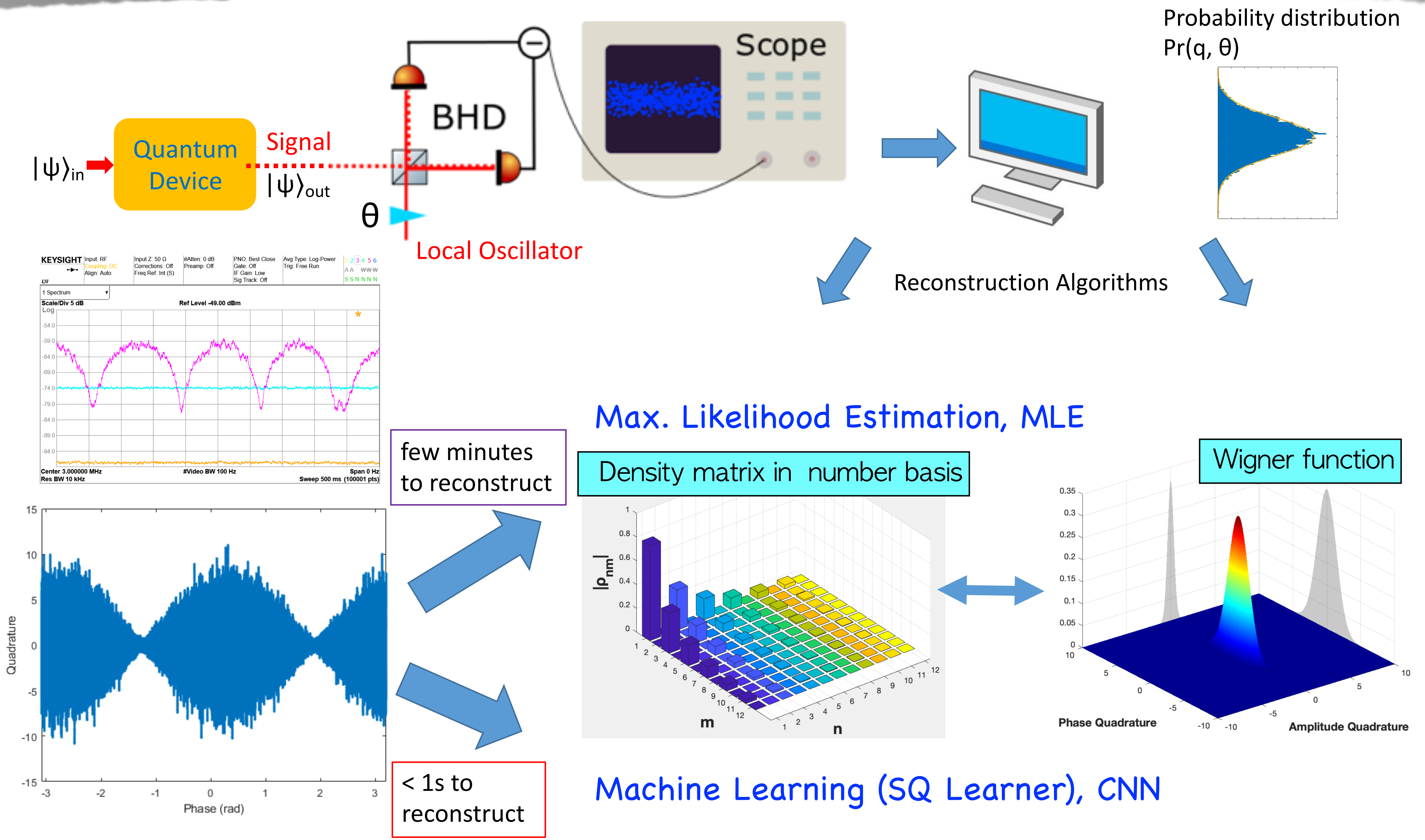


Quantum State Tomography

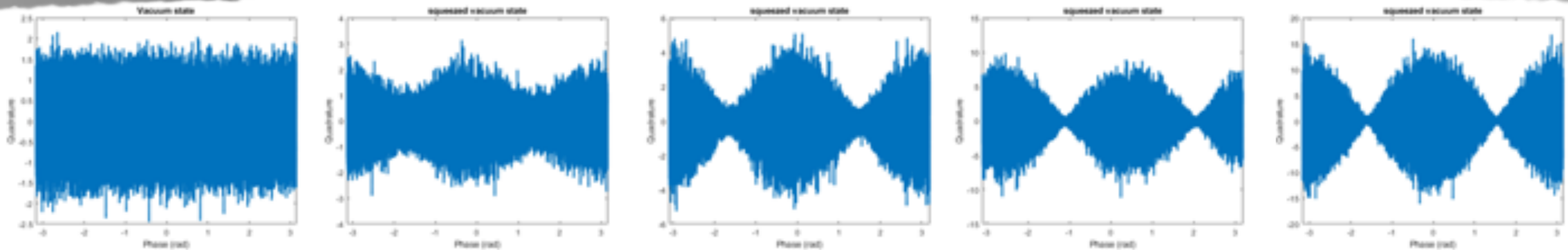


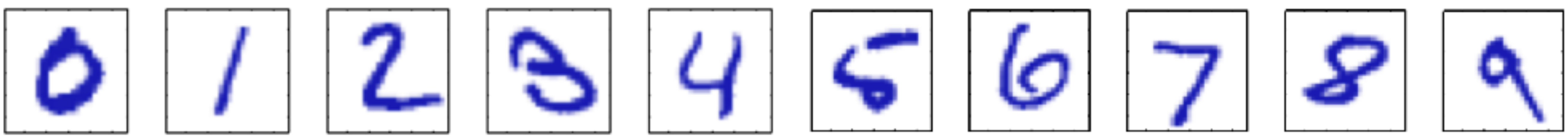
H.-Y. Hsieh, et al., Phys. Rev. Lett. 128, 073604 (2022).

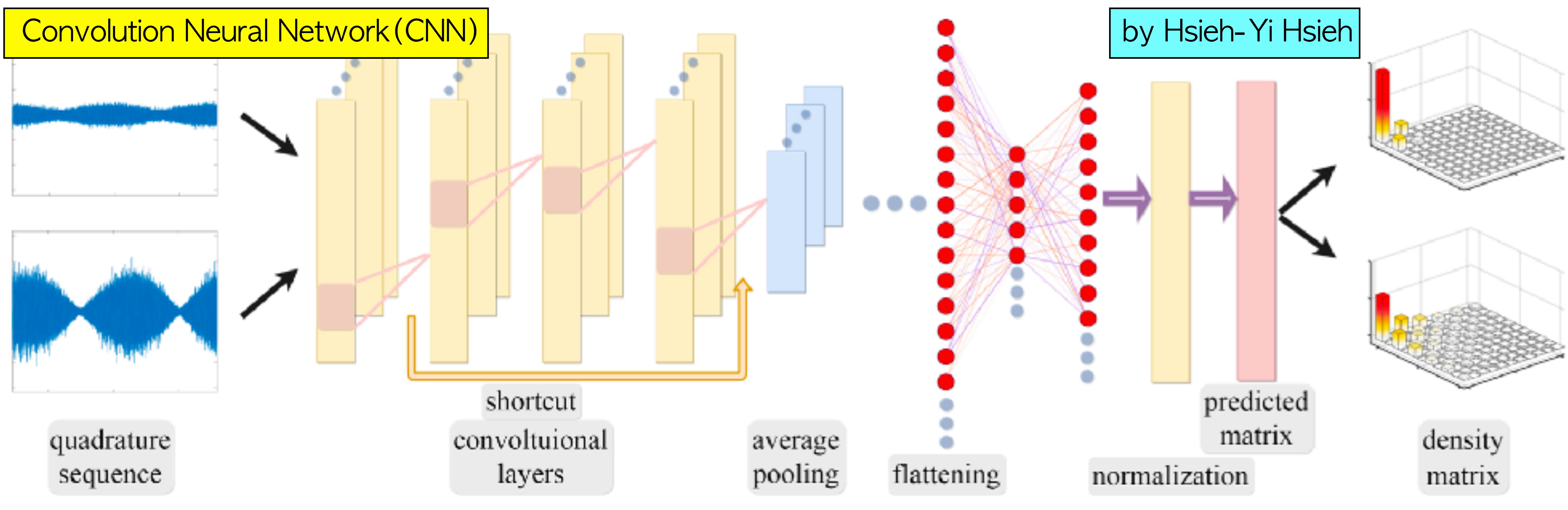
Machine Learning (SQ Learner) vs MLE



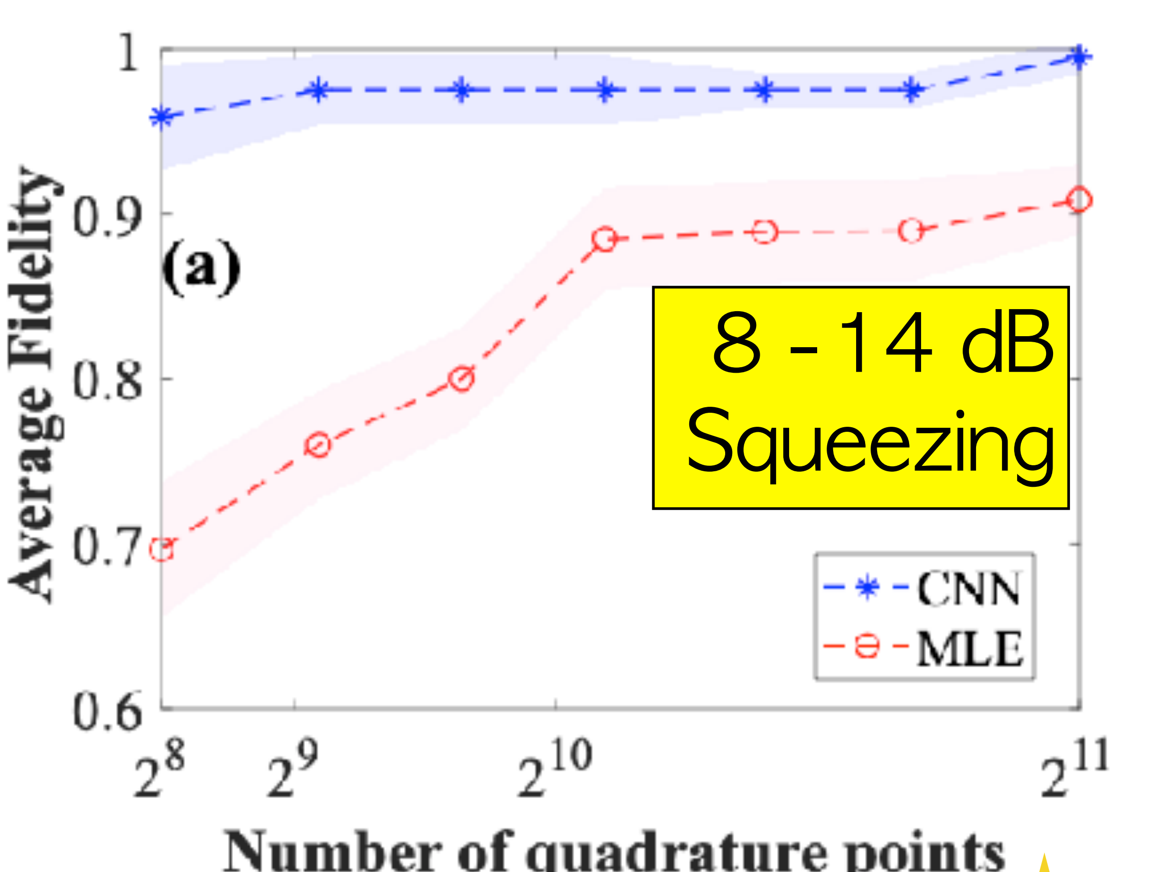
Pattern Recognition & Machine Learning







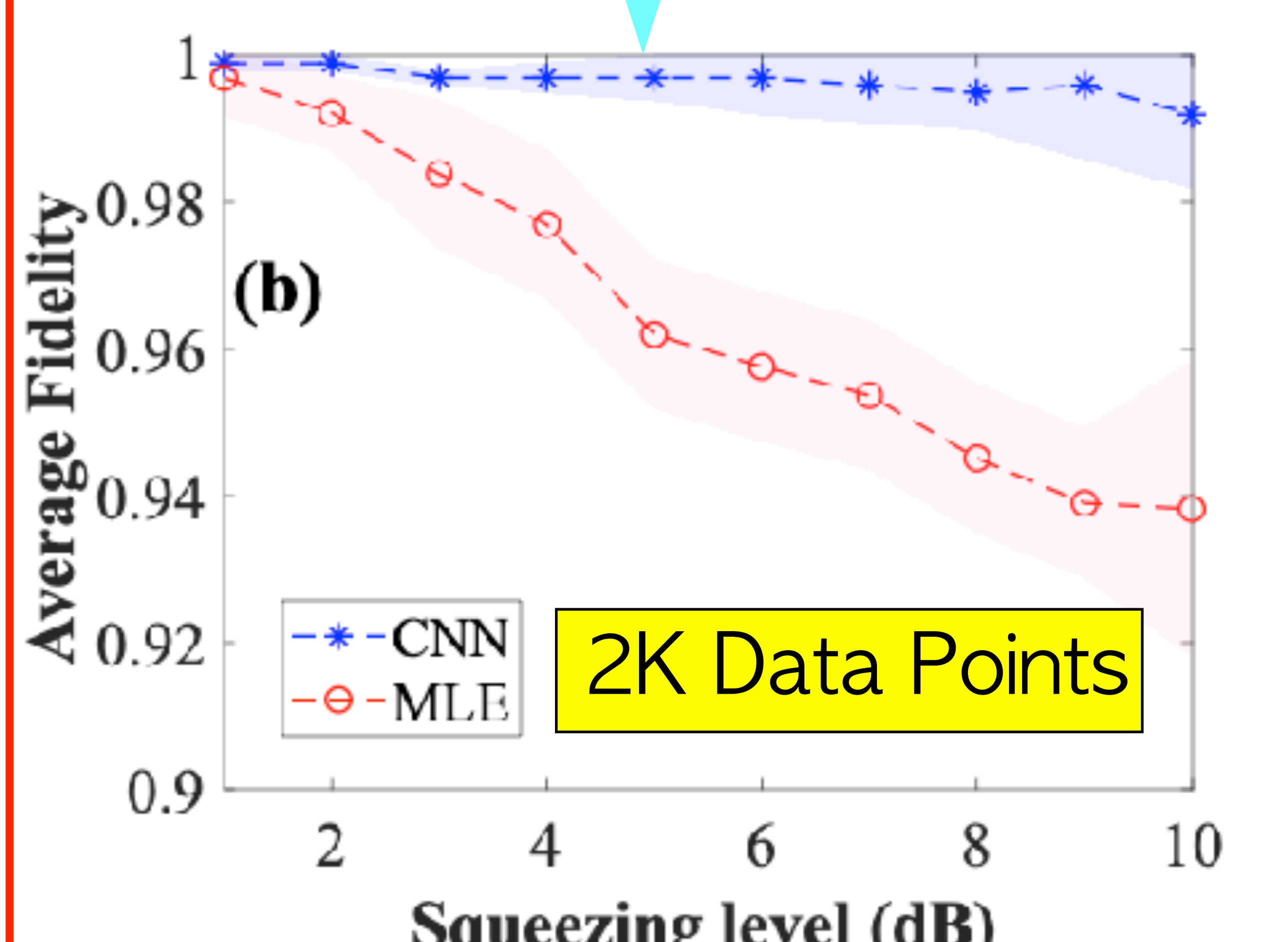
Machine Learning (SQ Learner) vs MLE

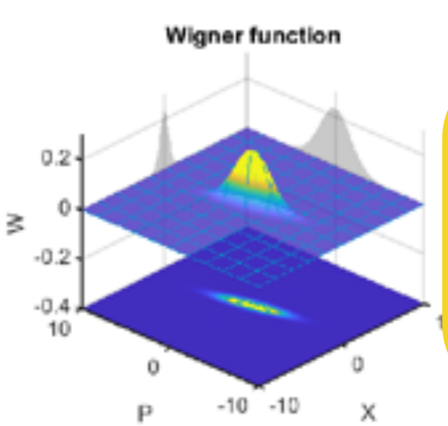


Fidelity:

$$F(\rho,\sigma) \equiv [\mathrm{Tr}\{\sqrt{\sqrt{\rho},\sigma\sqrt{\rho}}\}]^2$$

in less than one Second
Real-Time Reconstruction





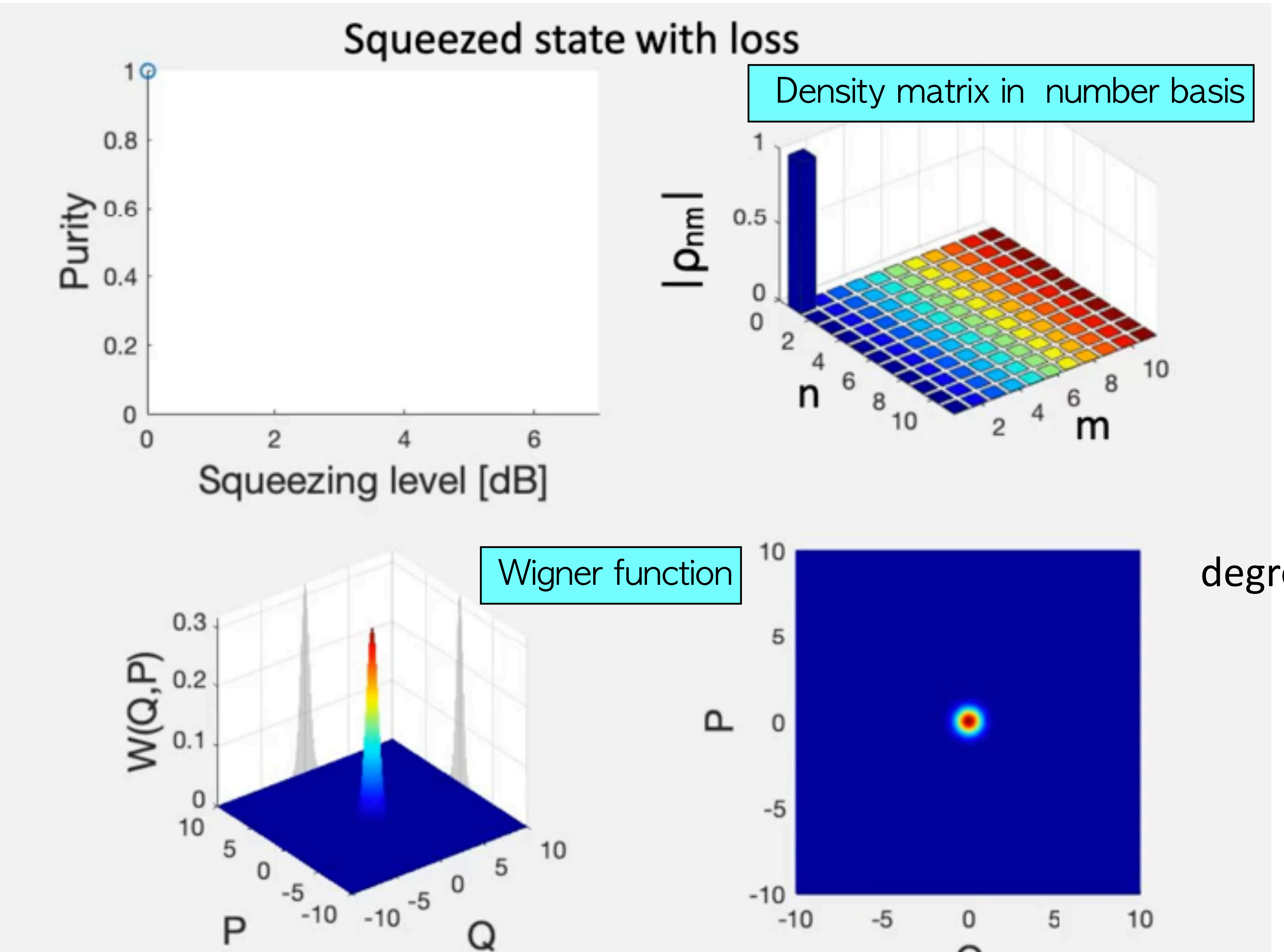
at Least several Hours to reconstruct wavefunction

Hsieh-Yi Hsieh, Yi-Ru Chen, et al.,
Phys. Rev. Lett. 128, 073604 (2022).

Applications of real-time tomography in squeezed state:

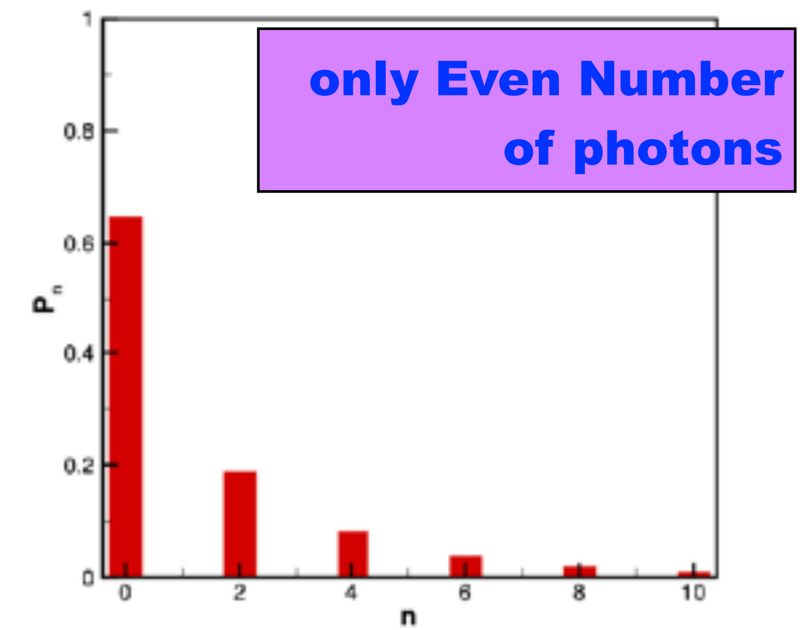
- Monitor the purity of a quantum state in real-time, and reveal the dynamics.
- The purity of a normalized quantum state is a scalar defined as:

$$\gamma \equiv \mathrm{tr}(\rho^2) \ , \ 0 < \gamma \leq 1$$

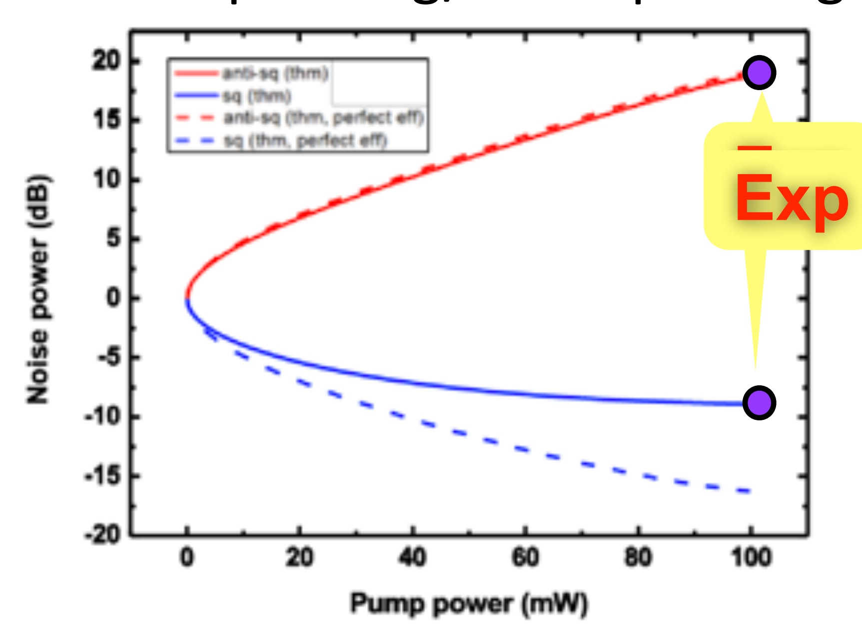


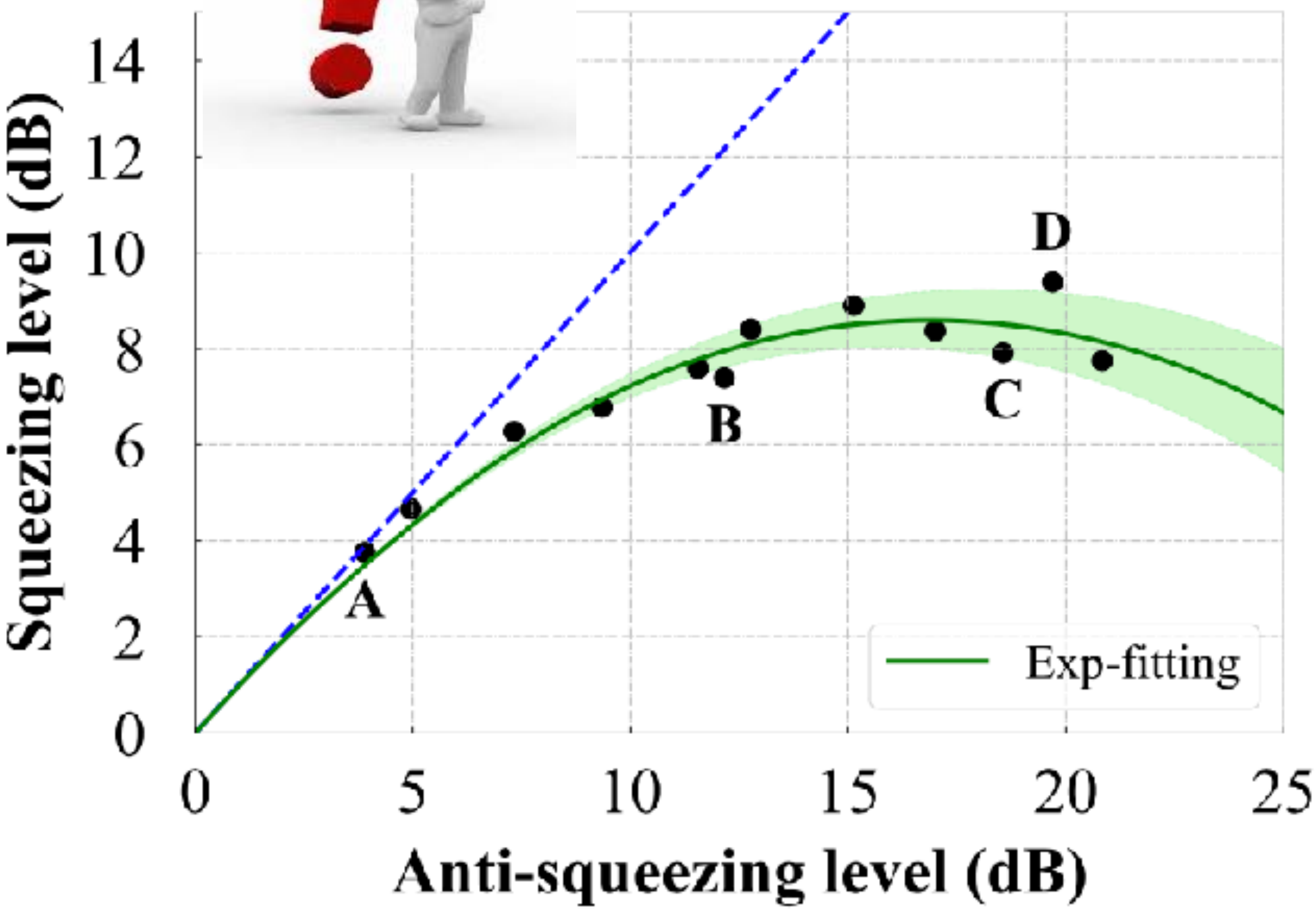
Hsieh-Yi Hsieh, Yi-Ru Chen, et al., Phys. Rev. Lett. 128, 073604 (2022).

γ =1 for pure squeezed state



degrees of squeezing/anti-squeezing

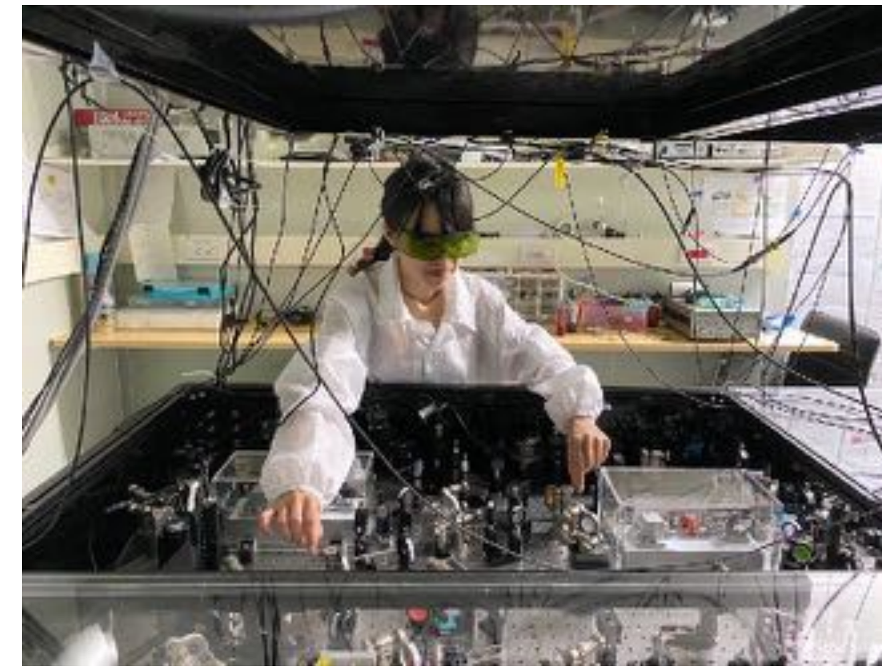


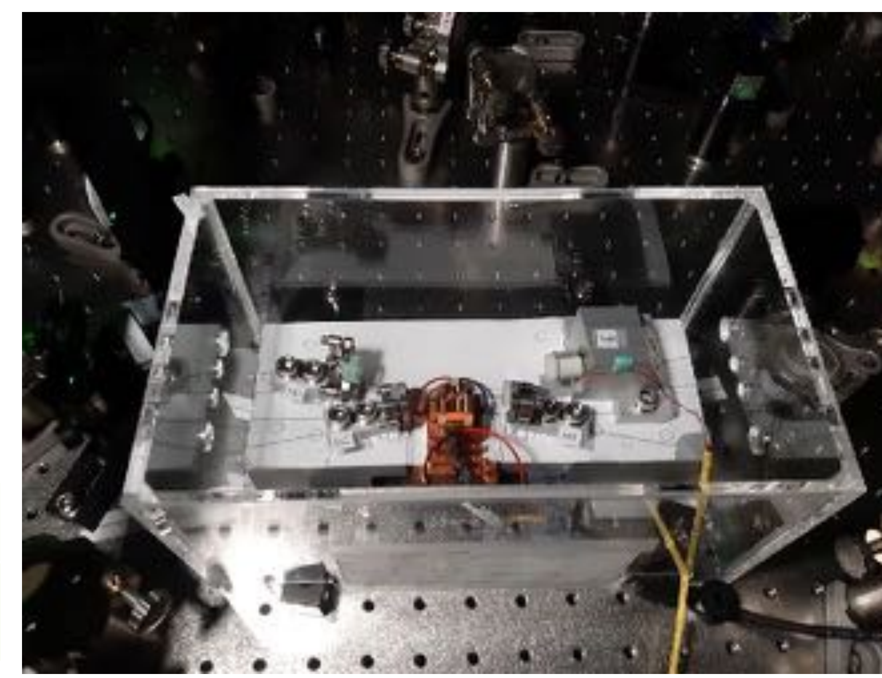


Threshold Power	120	mW
Escape efficiency (estimated)	0.976	

Estimation:

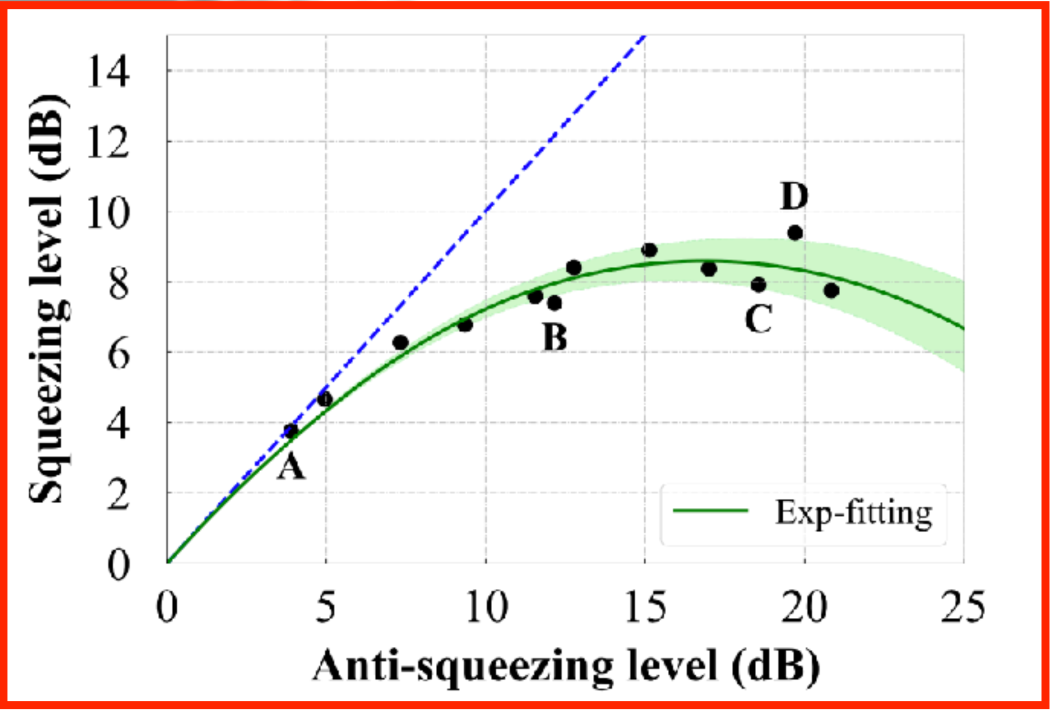
16.2 dB Squeezing

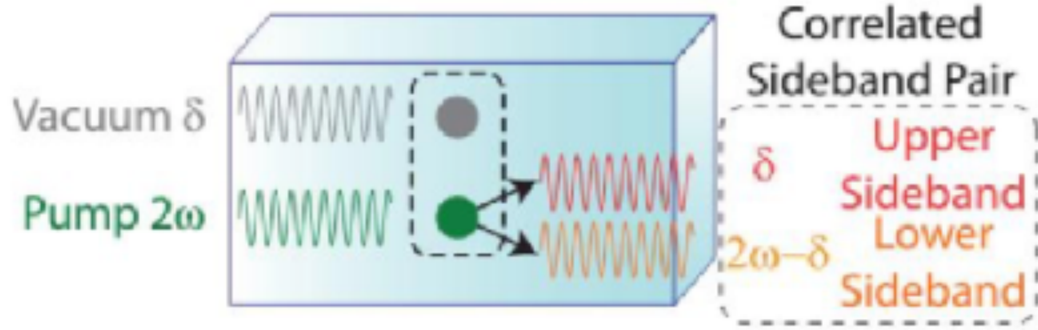




by
Yi-Ru Chen
Chien-Ming Wu

Degradation: Loss and Phase noise



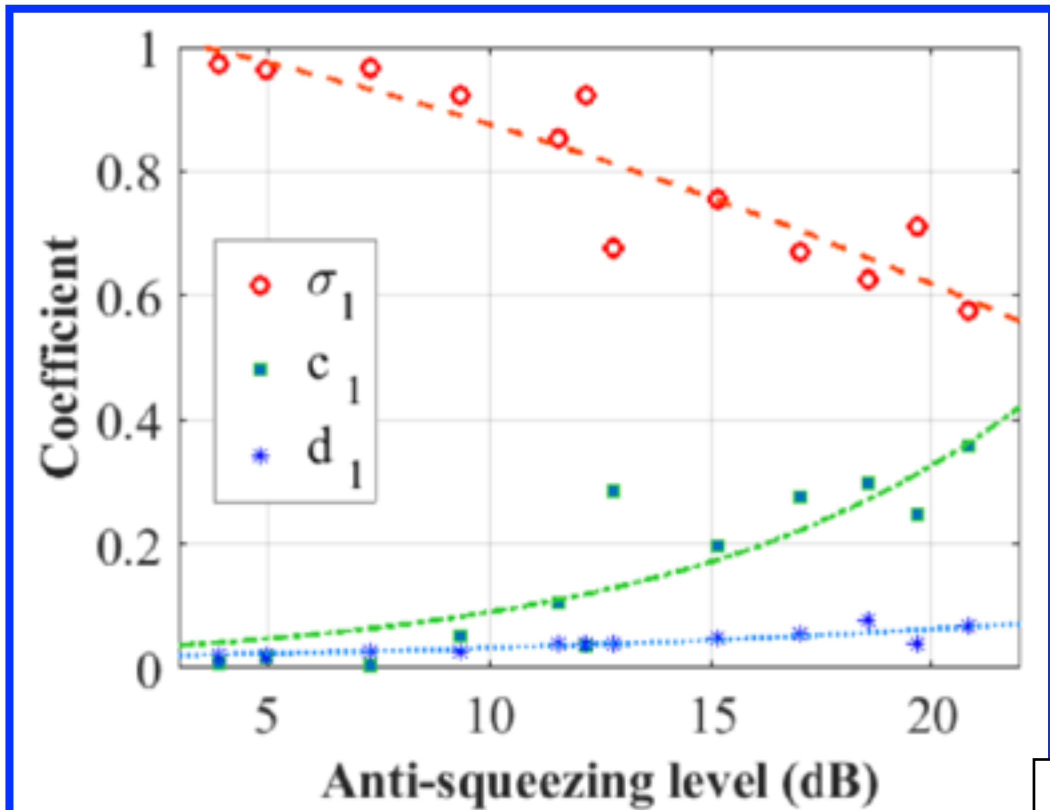


Loss: L

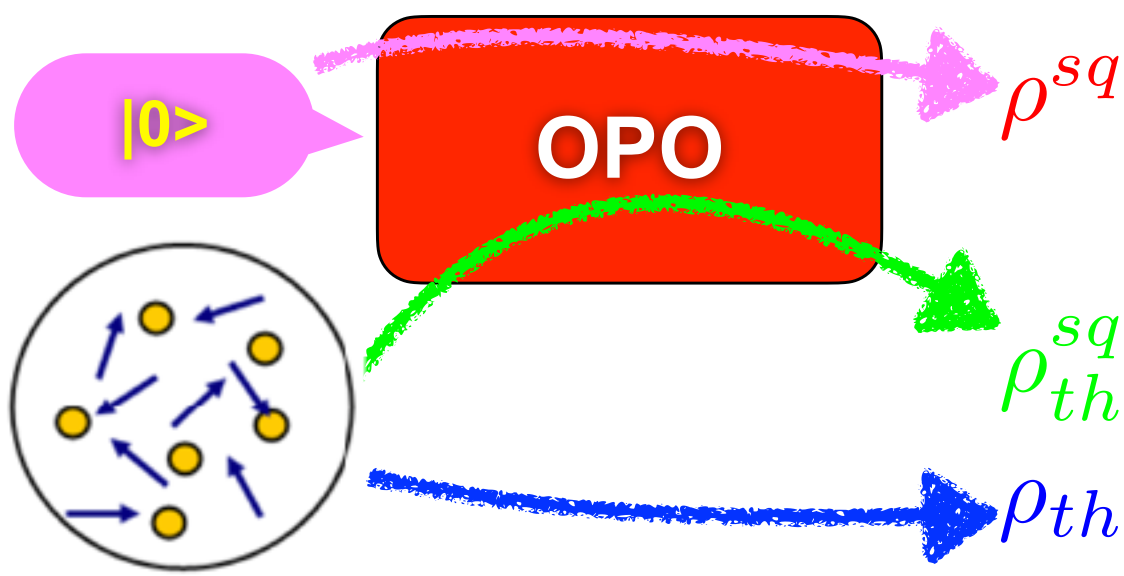
Phase noise: θ

$$V^{\mathrm{sq}} = (1-L)[V^{\mathrm{sq}}_{\mathrm{id}} \times \cos^2\theta + V^{\mathrm{as}}_{\mathrm{id}} \times \sin^2\theta] + L,$$

$$V^{\mathrm{as}} = (1-L)[V^{\mathrm{as}}_{\mathrm{id}} \times \cos^2\theta + V^{\mathrm{sq}}_{\mathrm{id}} \times \sin^2\theta] + L,$$



$$\rho = \sigma_1\, \rho^{sq} + c_1\, \rho^{sq}_{th} + d_1\, \rho_{th}$$



H.-Y. Hsieh, et al., Phys. Rev. Lett. 128, 073604 (2022).

Yes !

accelerated with Machine Learning !

Can We See Quantum ?

in Real-Time !

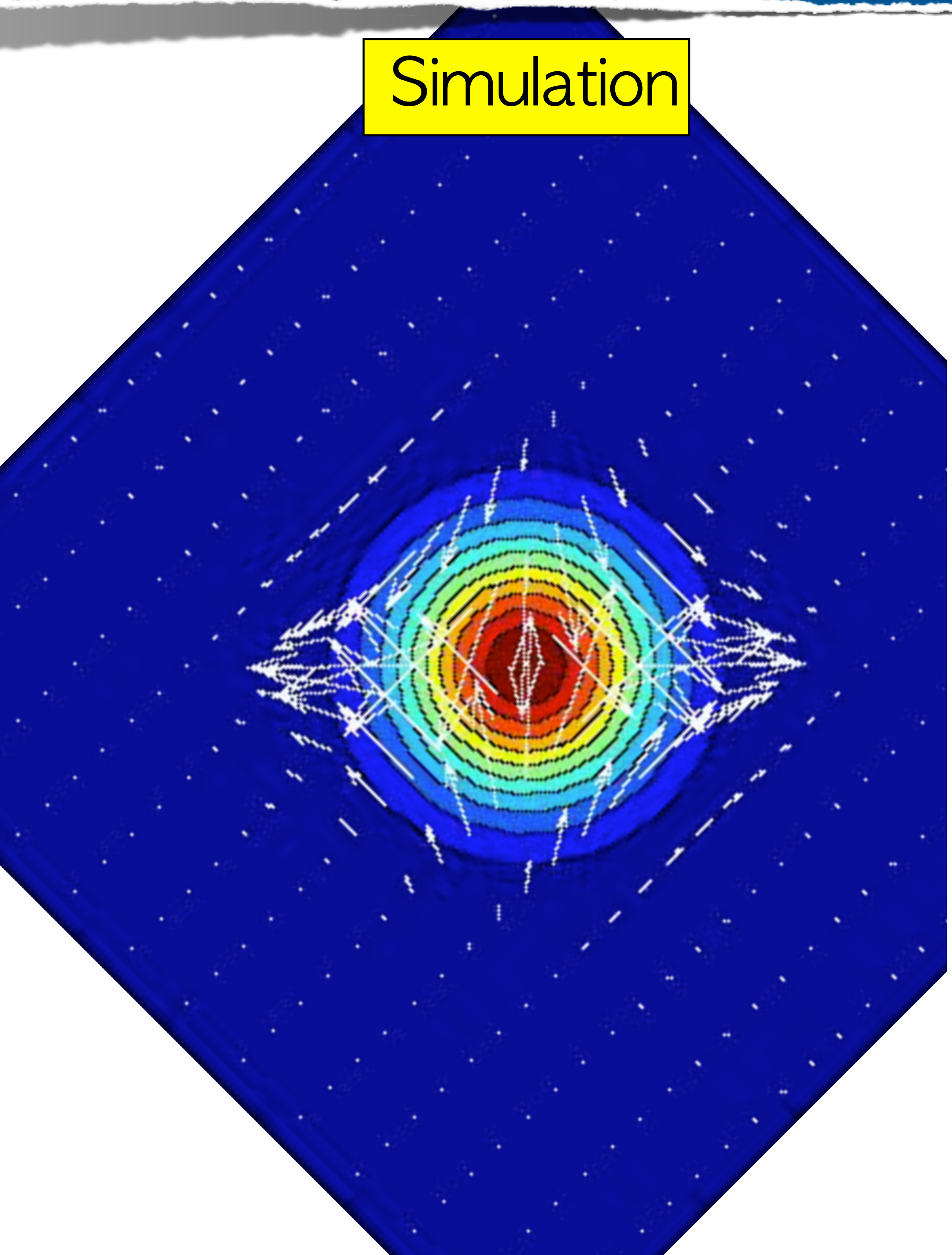
Phase space: Wigner Flow (Current)

- The time evolution of Wigner distribution can be cast in the form of a flow field $J(x,p;t)$ describes the flow of Wigner's quasiprobability density
  $J_x = \frac{p}{m} W(x,p,t)$
  $J_p = \int d\xi e^{\frac{i\xi p}{\hbar}} \Psi^*(x+\frac{\xi}{2},t)\Psi(x-\frac{\xi}{2},t)[\frac{V(x-\frac{\xi}{2})-V(x)}{\xi} - \frac{V^*(x+\frac{\xi}{2})-V^*(x)}{\xi}]$
- Continuity equation for Hermitian Hamiltonian
  $\frac{\partial}{\partial t} W(x,p;t) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial p} J_p = 0$
- Continuity equation for Hermitian non-Hamiltonian
  $\frac{\partial}{\partial t} W(x,p,t) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial p} J_p = \frac{i}{\hbar}[V^*(x,t) - V(x,t)]W(x,p,t)$

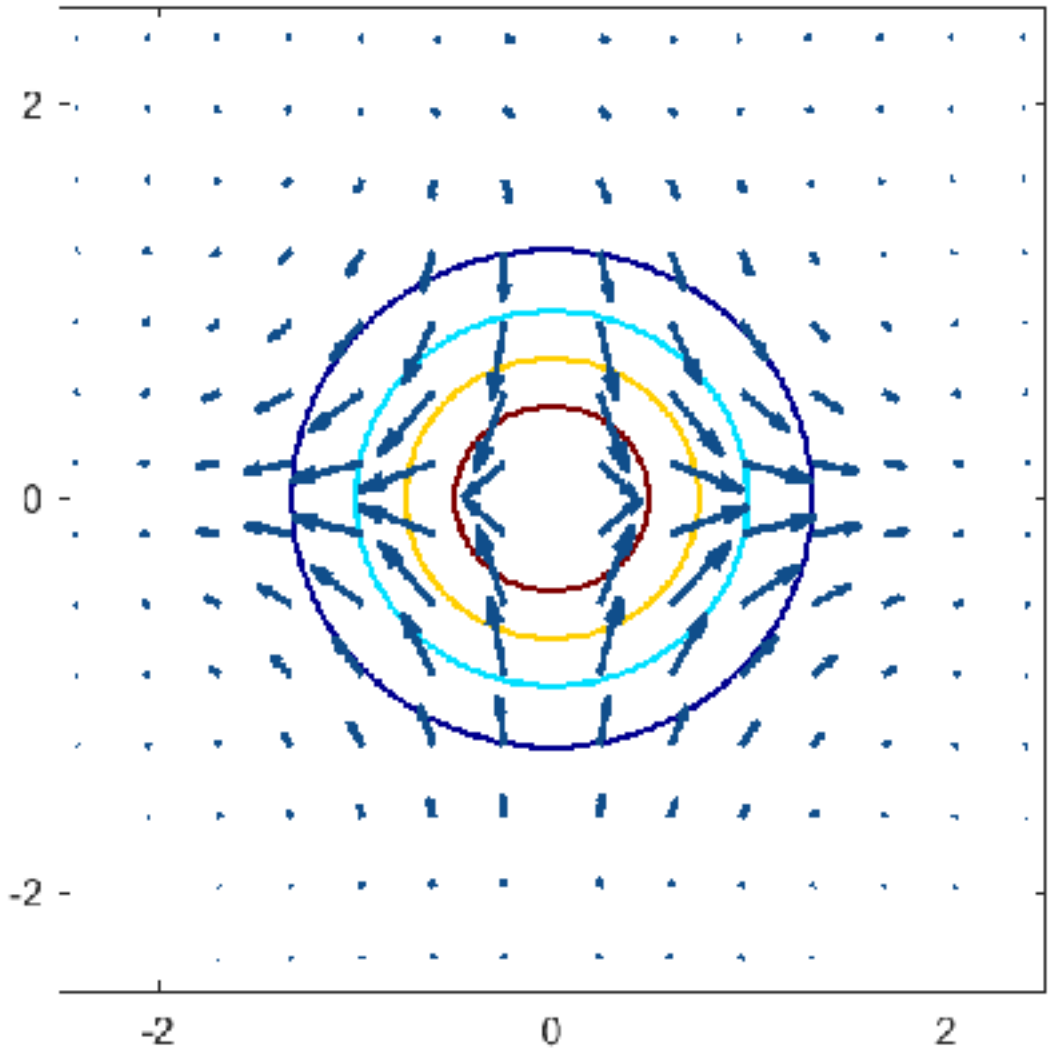
O. Steuernagel, D. Kakofengitis, and G. Ritter, Phys. Rev. Lett. 110, 030401 (2013).

Wigner Flow (Current)

Simulation

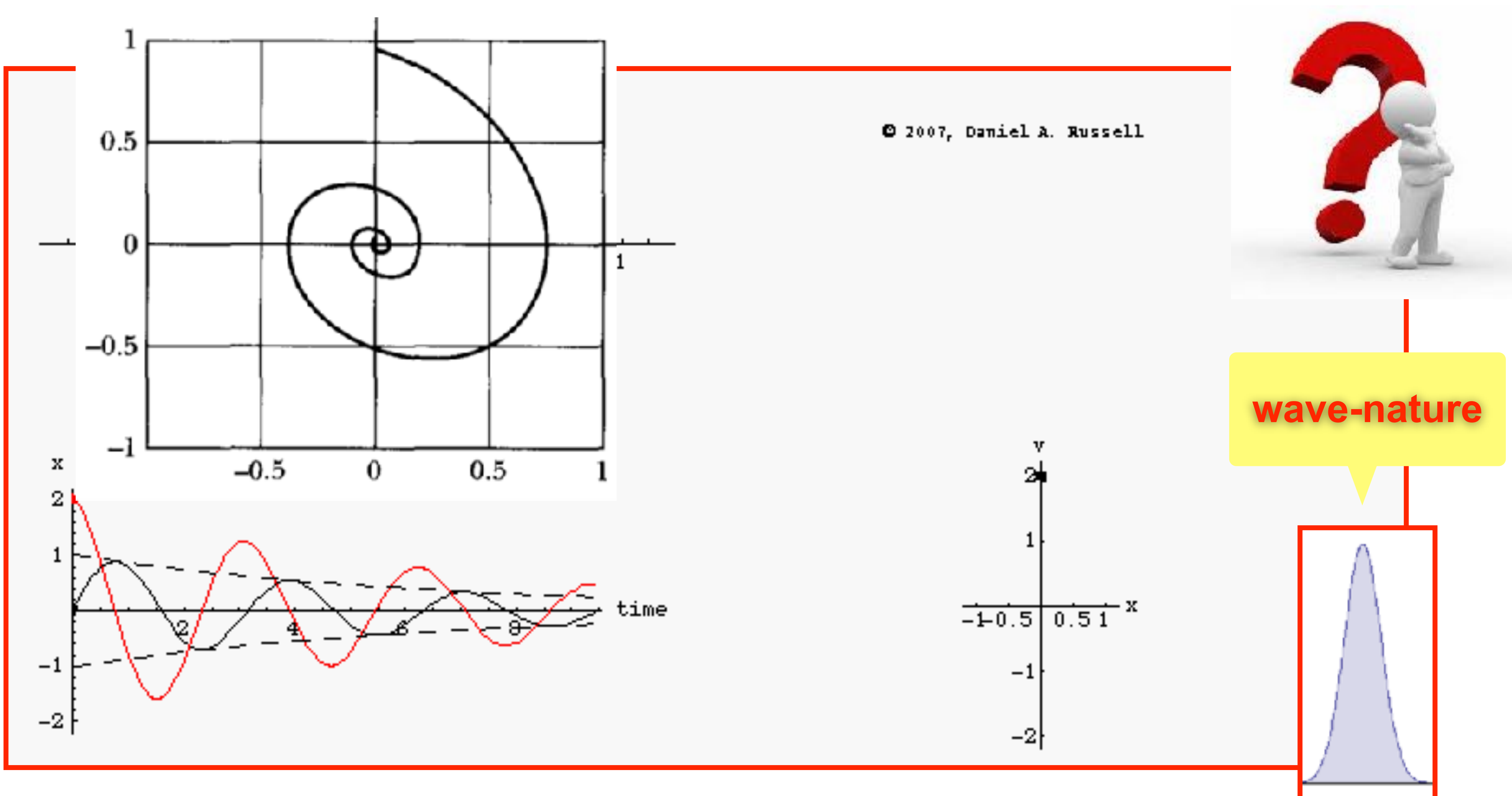


Exp. Reconstruction



Yi-Ru Chen et al., arXiv: 2111.08285 (2021).

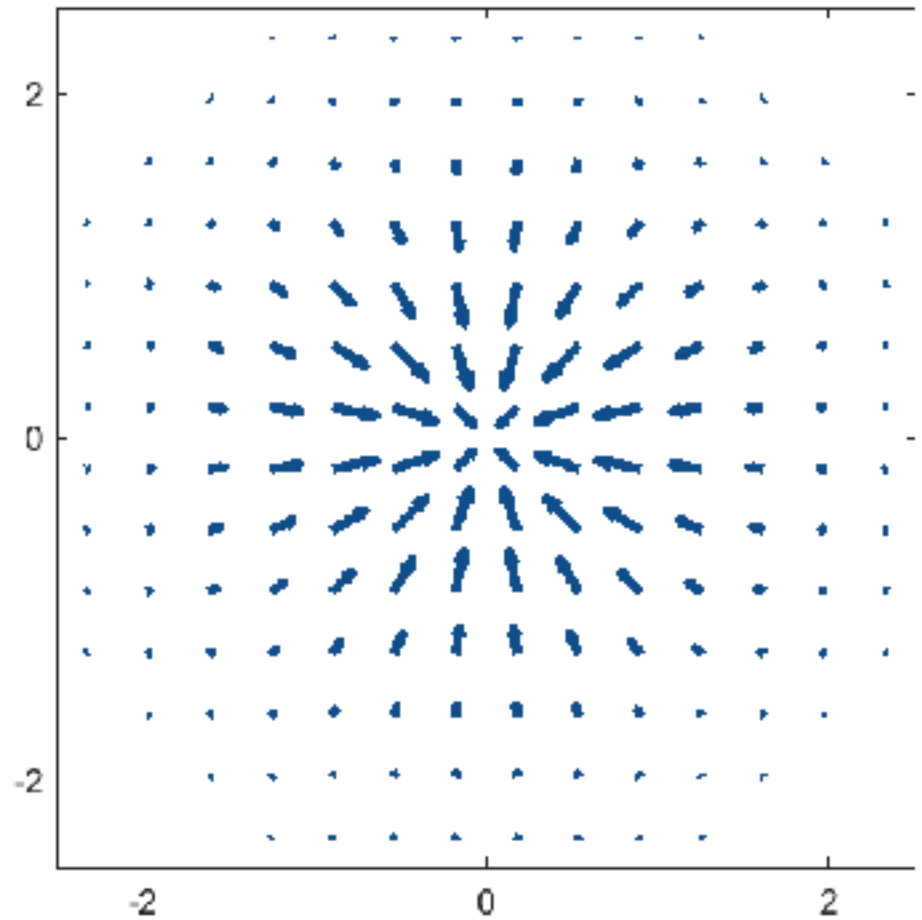
Damped SHO



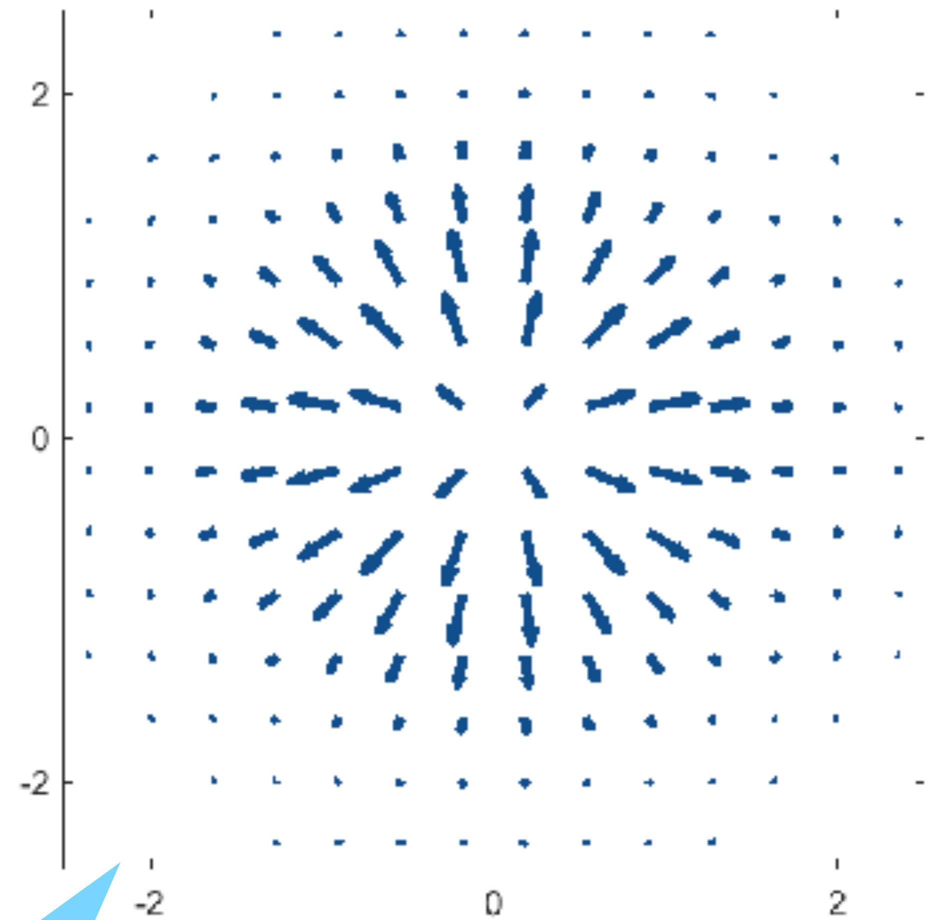
https://www.acs.psu.edu/drussell/Demos/phase-diagram/phase-diagram.html

Push-and-Pull:

Damping Flow



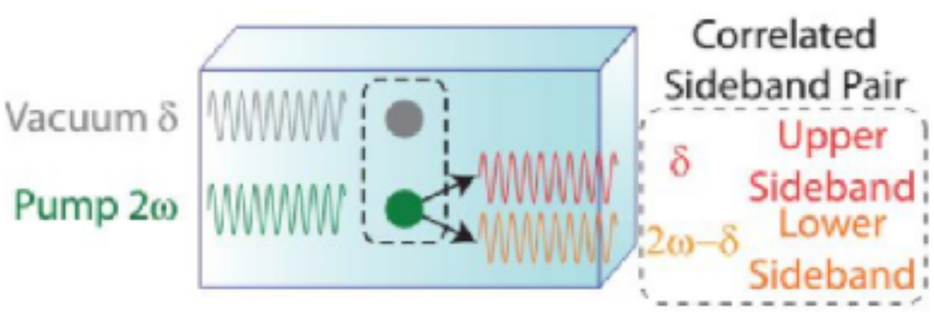
Diffusive Flow



Diffusive Current due to the Wave Nature!

Yi-Ru Chen et al., arXiv: 2111.08285 (2021).

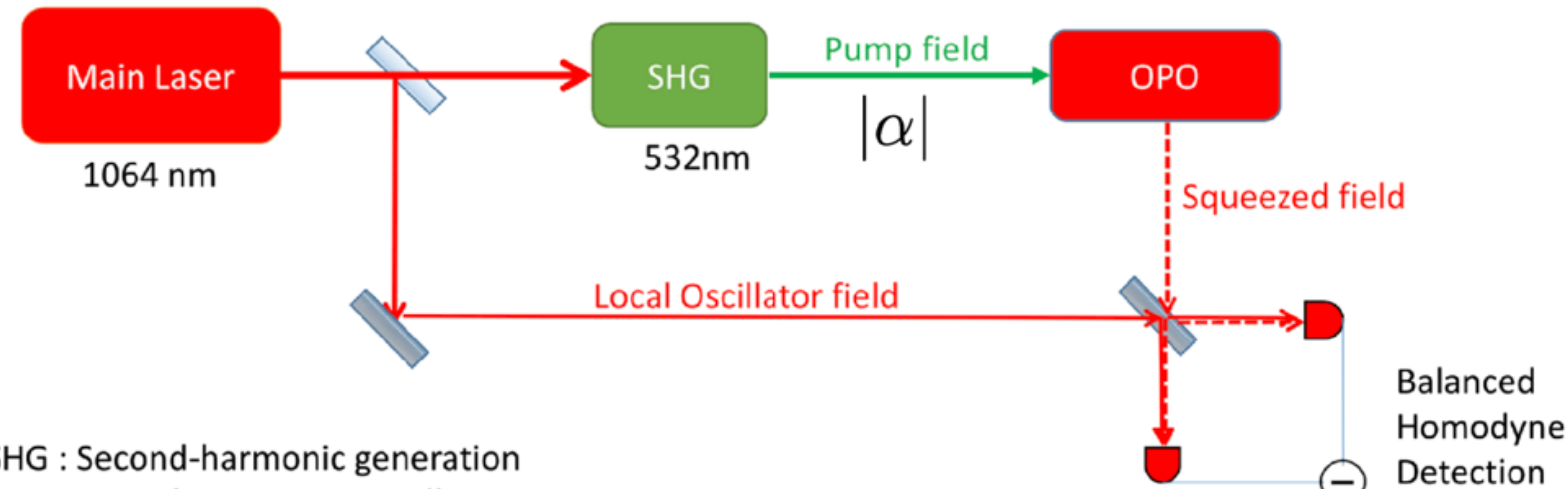
OPO: effective time (via Pump)



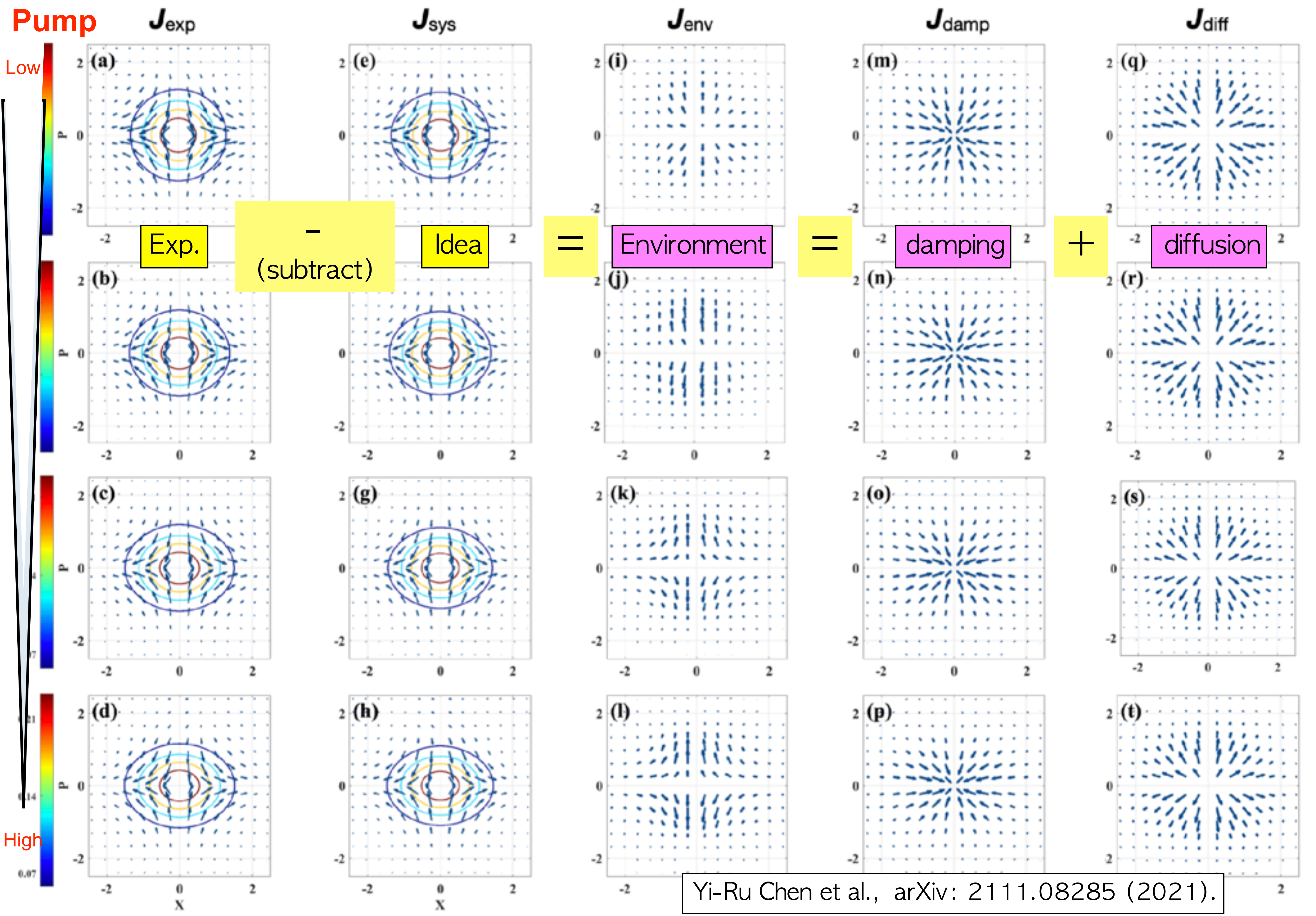
$$\hat{H} = \frac{i\hbar\chi^{(2)}}{2}(|\alpha|\hat{a}^2 - |\alpha|\hat{a}^{\dagger 2}),$$

$$\hat{U}(t) = \exp[\frac{-i\hat{H}t}{\hbar}] = \exp[\frac{\chi^{(2)}|\alpha|t}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})],$$

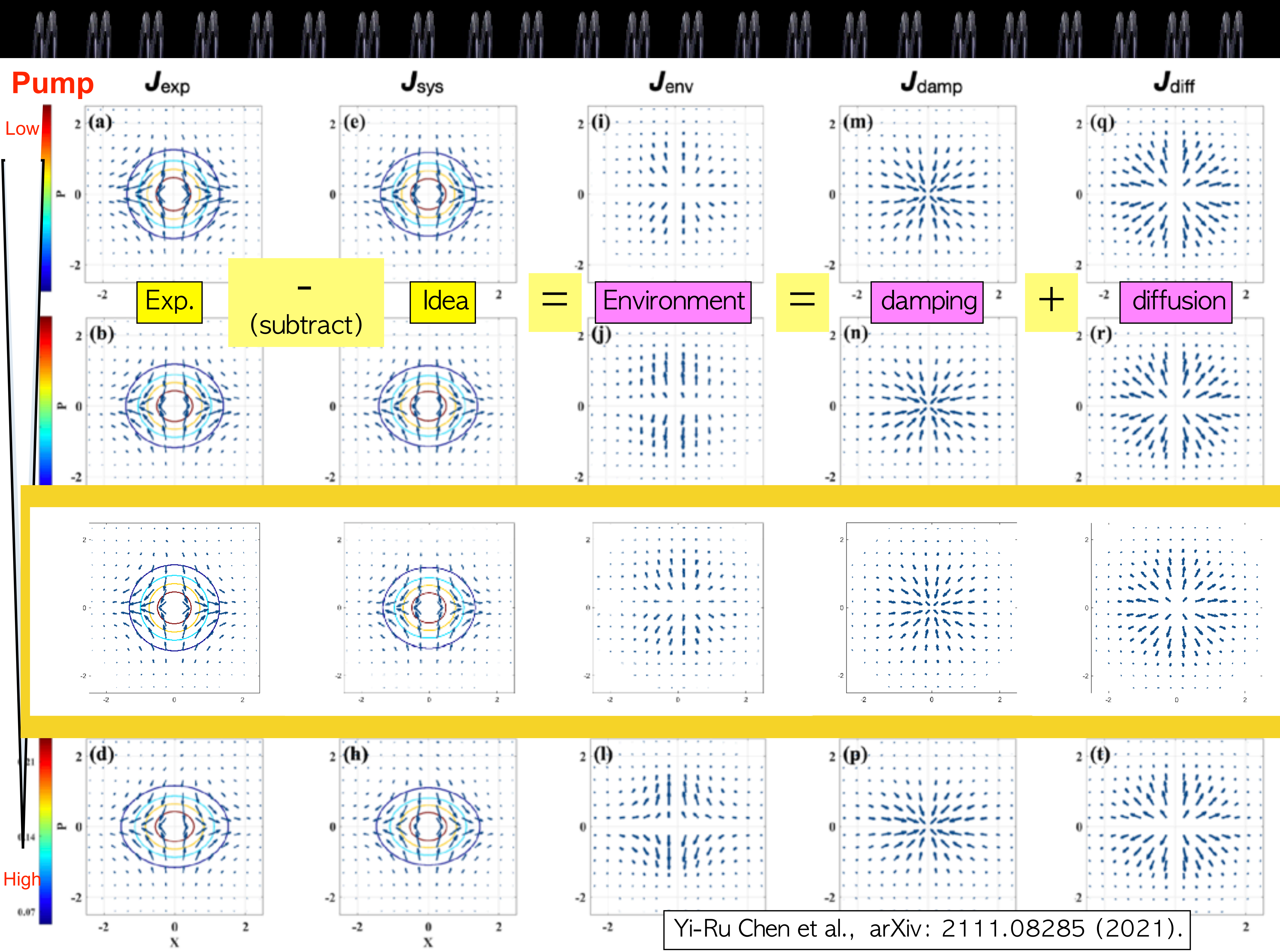
$$\tau_{\text{eff}} \propto \chi^{(2)}|\alpha| \equiv |\xi|,$$



SHG : Second-harmonic generation
OPO: Optical parametric oscillator



Yi-Ru Chen et al., arXiv: 2111.08285 (2021).

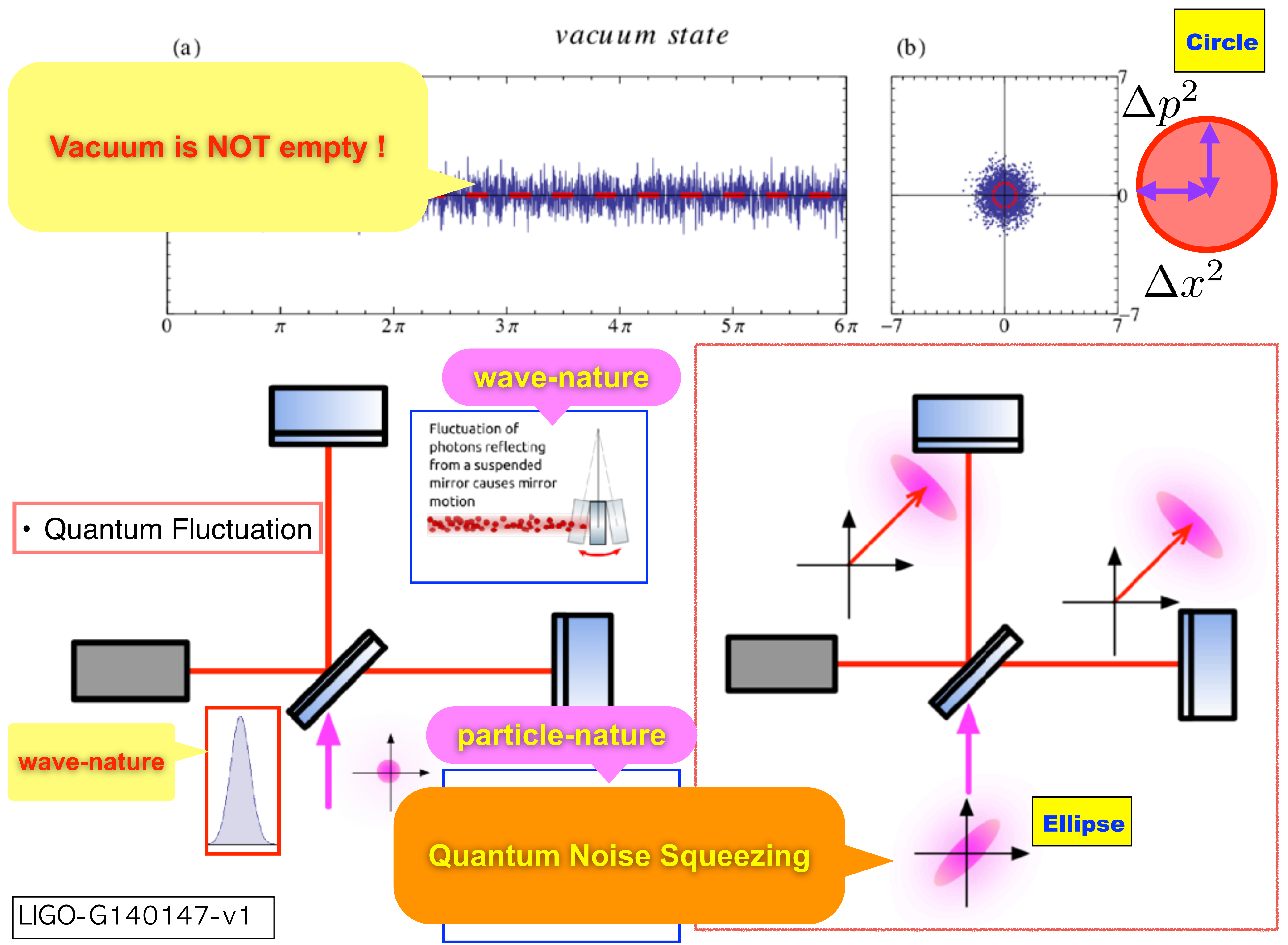


Yi-Ru Chen et al., arXiv: 2111.08285 (2021).

Outline

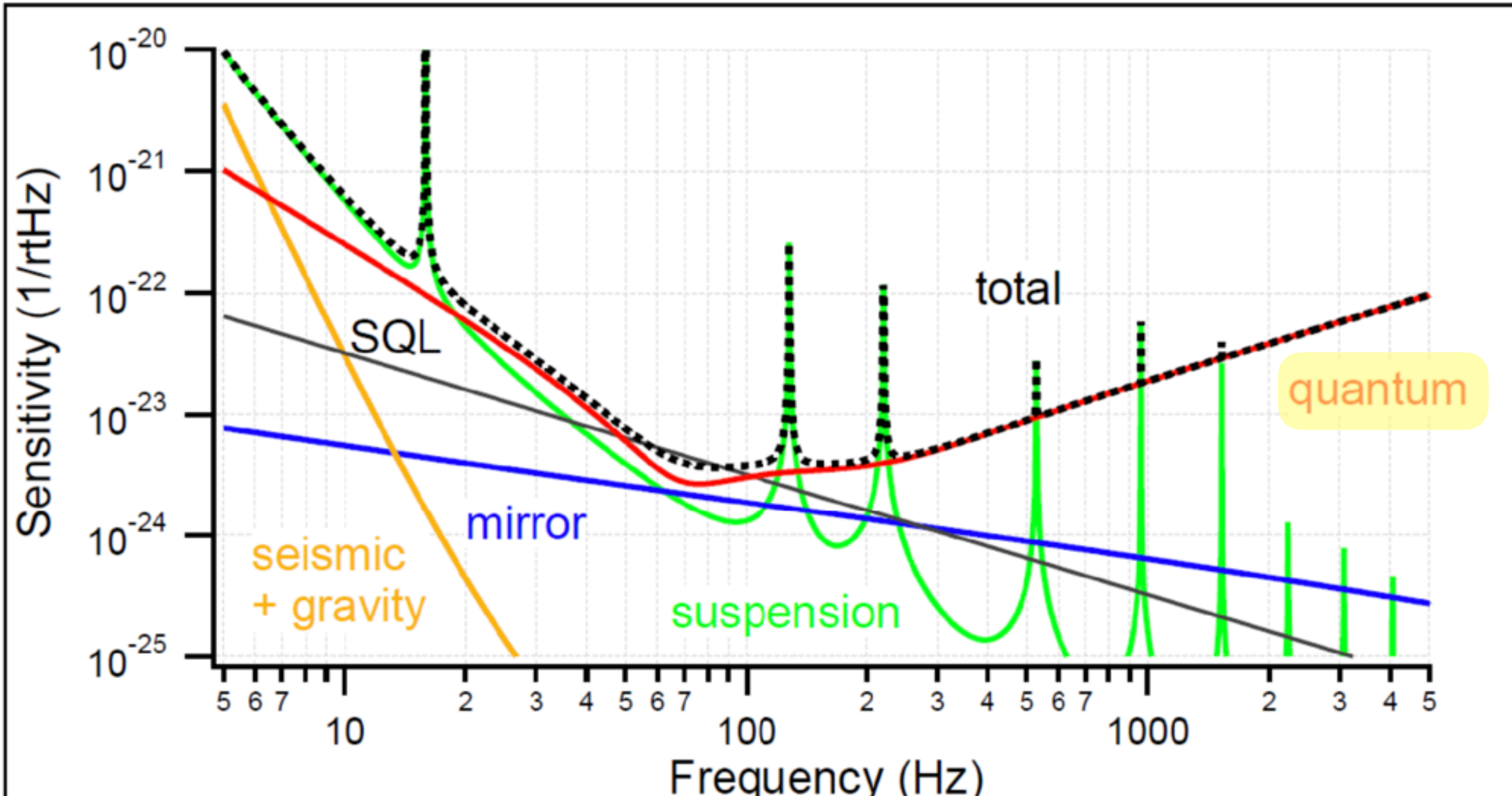
- Quantum Optics in Phase Space
- Quantum Noise Squeezing (SQZ)
- Machine-Learning enhanced Quantum State Tomography

- Applications with SQZ:
  - Optical Cat states
  - Quantum Photonic Chips
  - Error-Correction Code: GKP states
  - Quantum Random Number Generator
  - Gravitational Wave Detectors



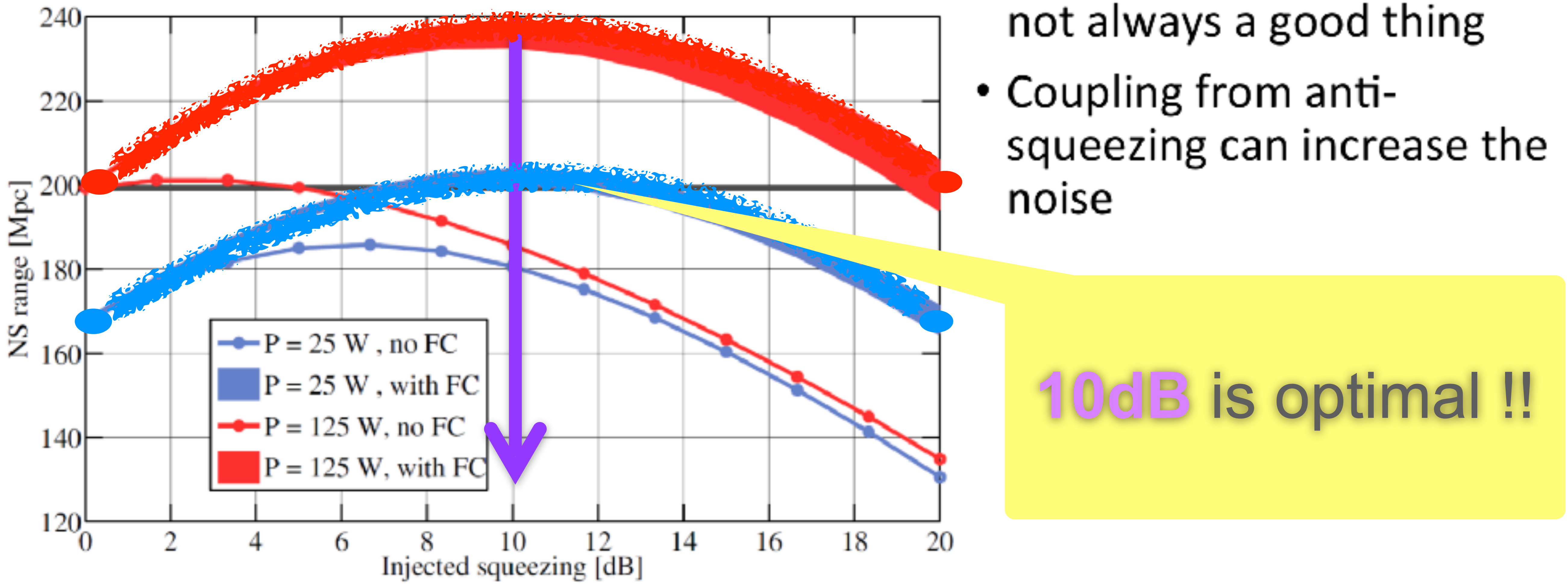
(a)
vacuum state
Vacuum is NOT empty !
0
π
2π
3π
4π
5π
6π
(b)
Circle
Δp^2
Δx^2
7
0
−7
−7
0
7
wave-nature
Fluctuation of photons reflecting from a suspended mirror causes mirror motion
• Quantum Fluctuation
wave-nature
particle-nature
Quantum Noise Squeezing
Ellipse
LIGO-G140147-v1

Sensitivity Curves:



For aLIGO parameters, about 10dB injection is optimal.

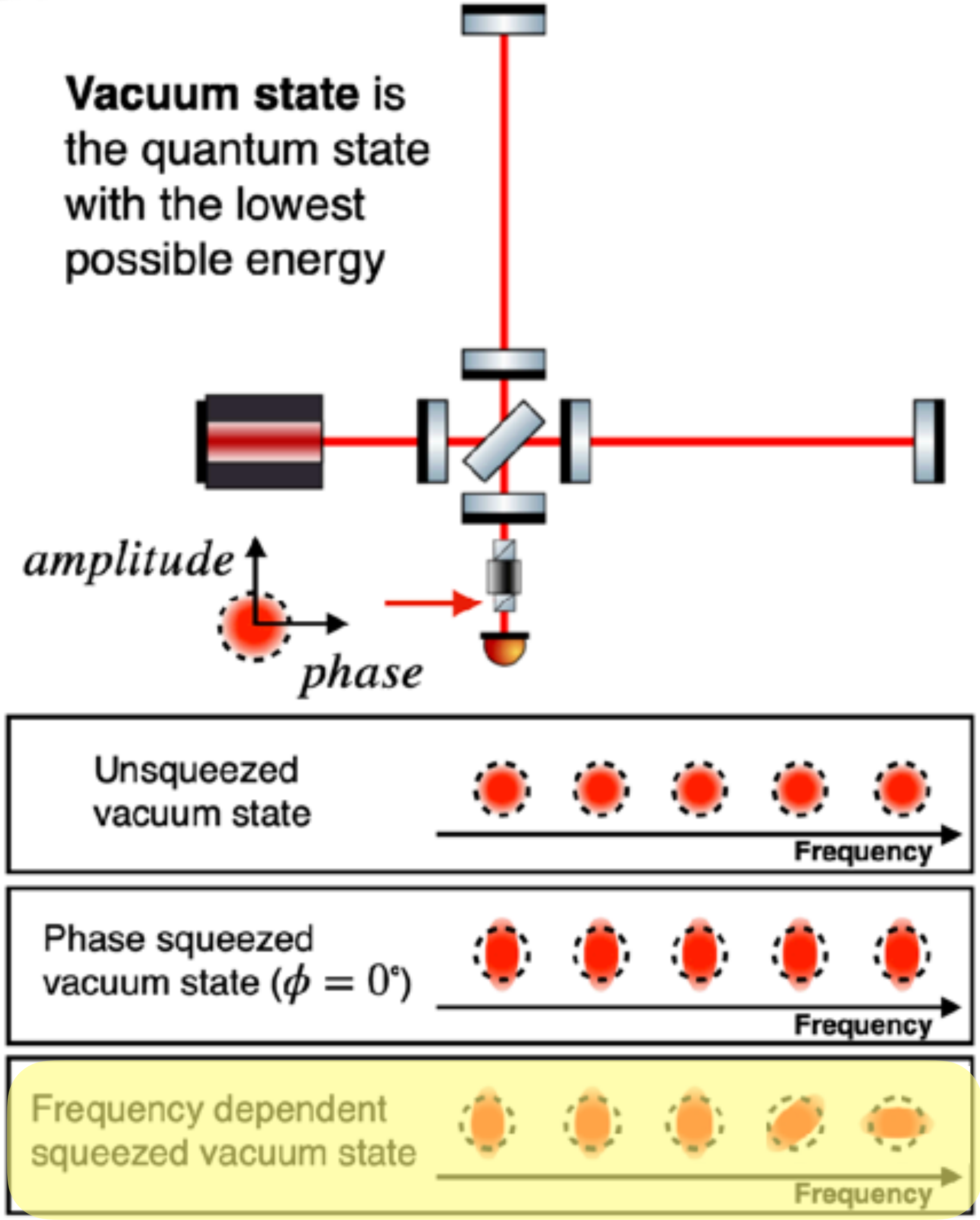
Range v squeezing

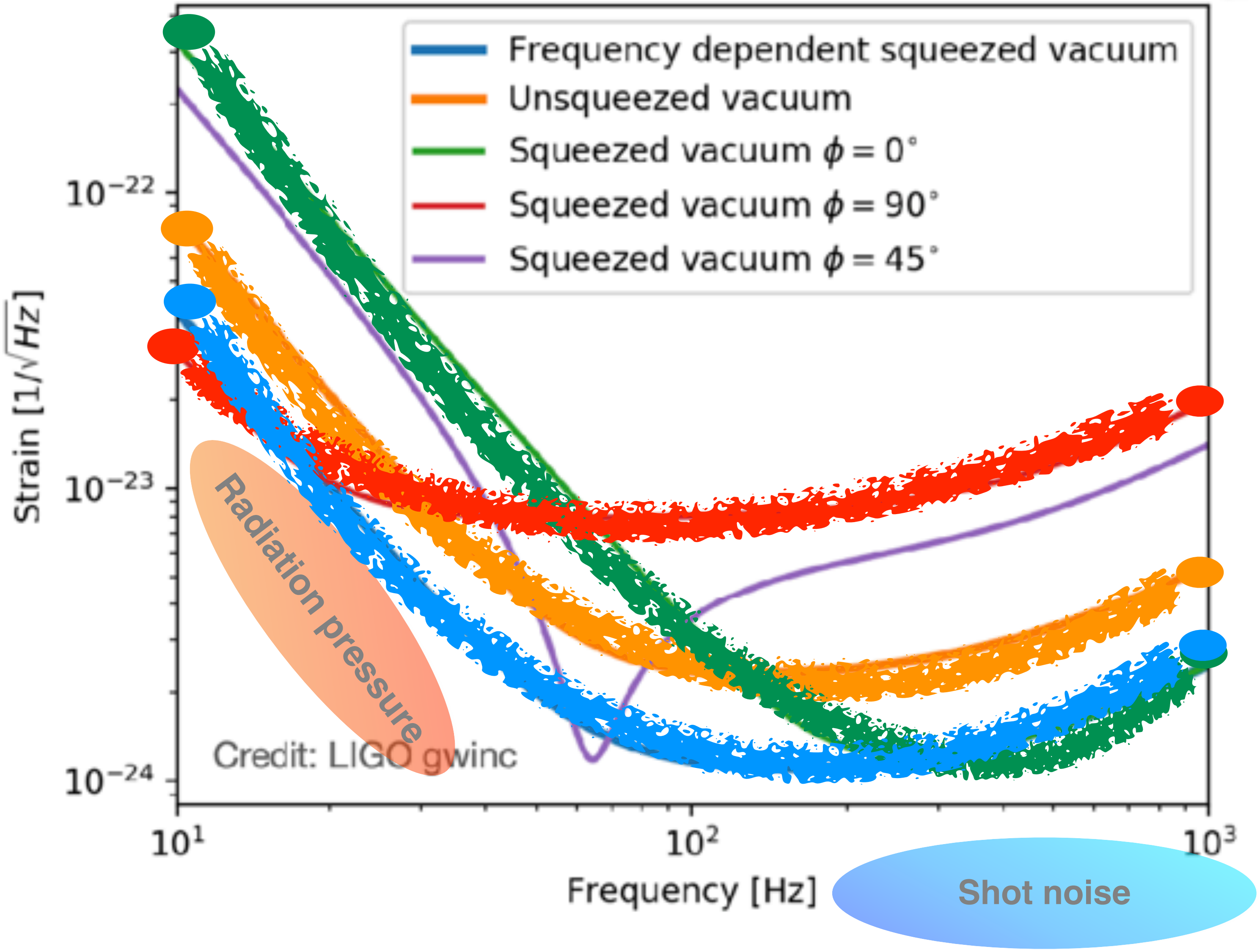


- Injecting more squeezing is not always a good thing
- Coupling from anti-squeezing can increase the noise

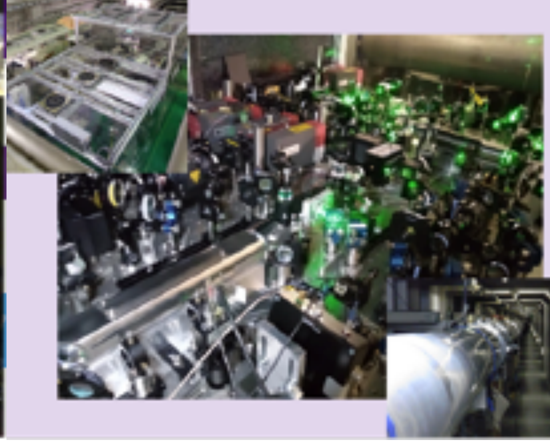
Figure Credit: John Miller

Freq. (In-)Dependent SQZ: FIS/FDS





Frequency dependent squeezed vacuum needs proper angle at specific frequencies, which is realized by filter cavity in this plot



Synopsis: Feeling the Squeeze at All Frequencies

April 28, 2020 • *Physics* 13, s55

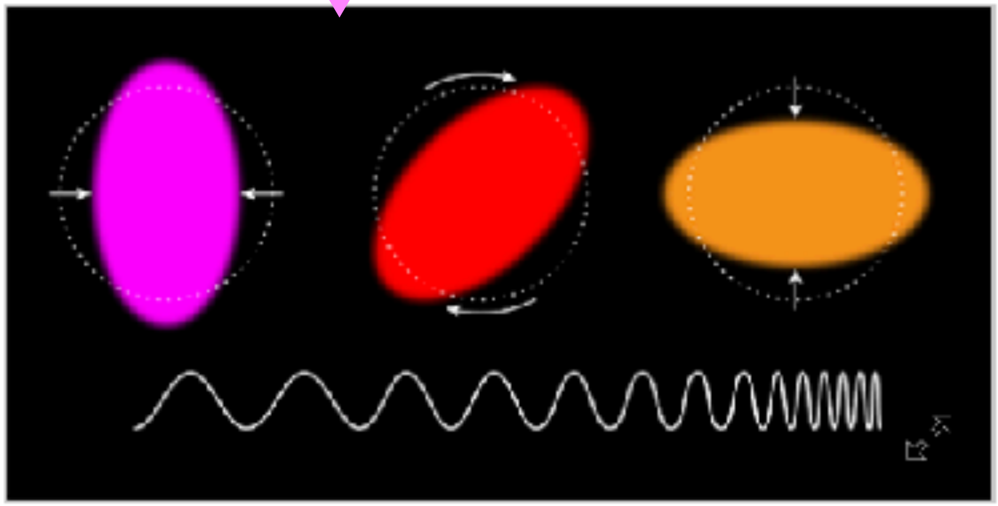
Two teams demonstrate frequency-dependent quantum squeezing, which could double the sensitivity of gravitational-wave detectors.

Frequency-Dependent Squeezed Vacuum Source for Broadband Quantum Noise Reduction in Advanced Gravitational-Wave Detectors

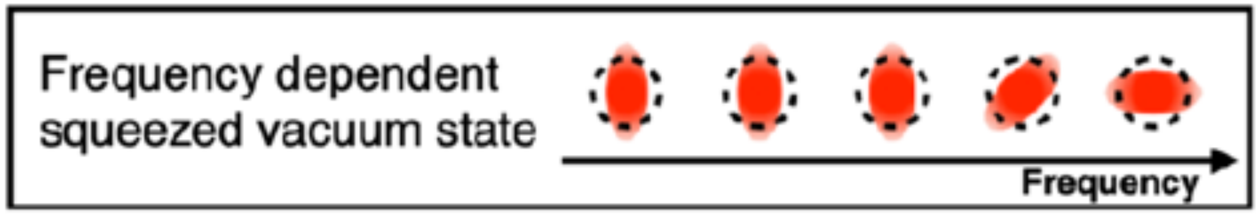
Yuhang Zhao, Naoki Aritomi, Eleonora Capocasa, Matteo Leonardi, Marc Eisenmann, Yuefan Guo, Eleonora Polini, Akihiro Tomura, Koji Arai, Yoichi Aso, Yao-Chin Huang, Ray-Kuang Lee, Harald Lück, Osamu Miyakawa, Pierre Prat, Ayaka Shoda, Matteo Tacca, Ryutaro Takahashi, Henning Vahlbruch, Marco Vardaro, Chien-Ming Wu, Matteo Barsuglia, and Raffaele Flaminio

Phys. Rev. Lett. 124, 171101 (2020)
Published April 28, 2020

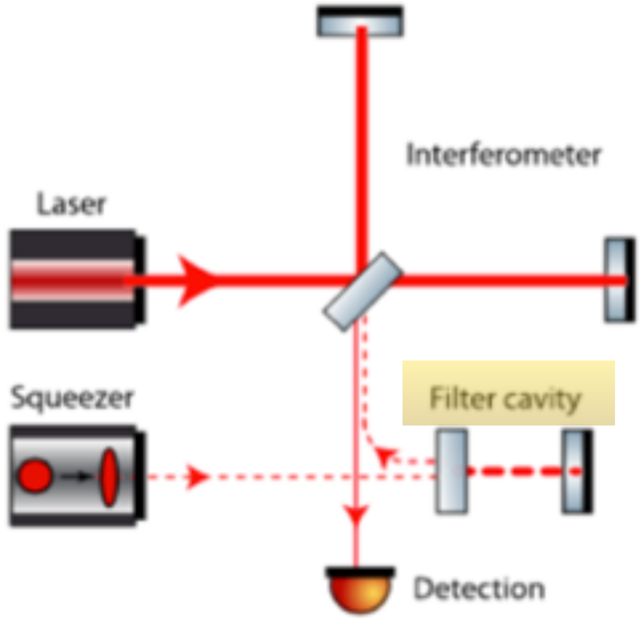
FDS



APS/Alan Stonebraker

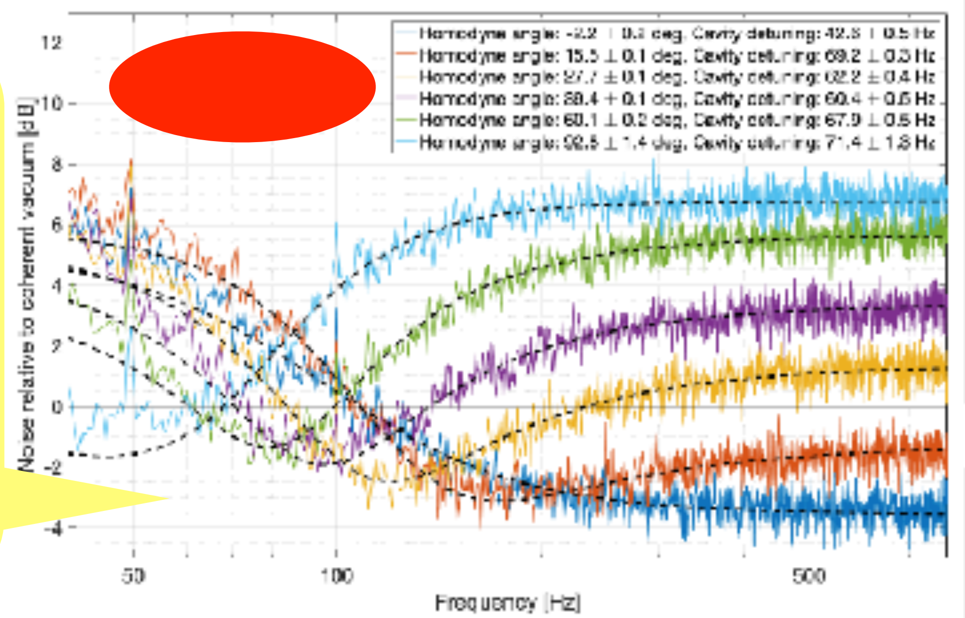


KAGRA Filter Cavity (KFC) Team



First Exp. on FDS,

Freq.-Dep. Squeezing, at 100 Hz



Thanks for your attentions ^.^

http://mx.nthu.edu.tw/~rklee