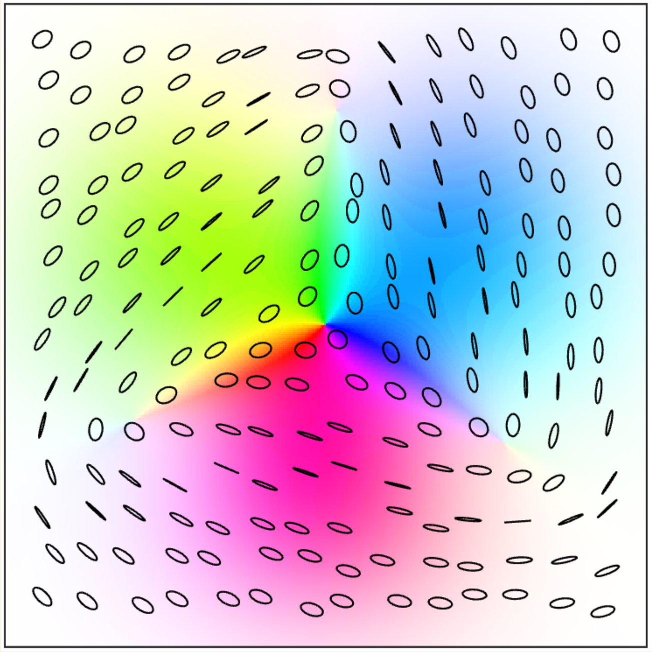
Optical Beams with Spatially Variable Polarization



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OSA Webinar
2017

Contributions

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- Joshua Jones
- Anthony D'Addario '18
- Ben Cvarch '17
- Ishir Dutta '17
- Jon Zeosky '16
- Kidane Kebede '16
- Brian Regan '15
- Kory Beach '15
- Flora Cheng '14
- Brett Rojec '14
- Kevin McCullough '14
- Shreeya Khadka '14
- Carrie Burgess '14
- William Schubert '12
- Matt Novenstern '11

Colgate U., Hamilton New York

- Liberal-arts college
- 2900 students
- ~20 P&A majors/yr

Collaborators: G. Millione, R. Alfano, N. Viswanathan, B. Piccirillo, L. Marrucci, M. Dennis

- Funding:

NSF

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Summary

- Polarization and Spatial Modes
- Poincaré Beams
- Polarization disclinations
- Asymmetric disclinations in polarization: Monstars
- 3-D Patterns: Möbius strips and twisted ribbons
- Conclusions

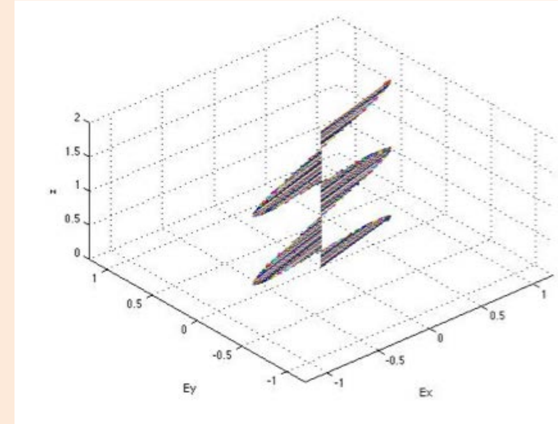
States of Polarization

- **Linear**

$$\vec{E}=e^{i(kz-\omega t)}(E_{0x}\hat{e}_x \pm E_{0y}\hat{e}_y)$$

$k=\frac{2\pi}{\lambda}$ ω= angular frequency

$\hat{e}_x,\hat{e}_y$= unit vectors

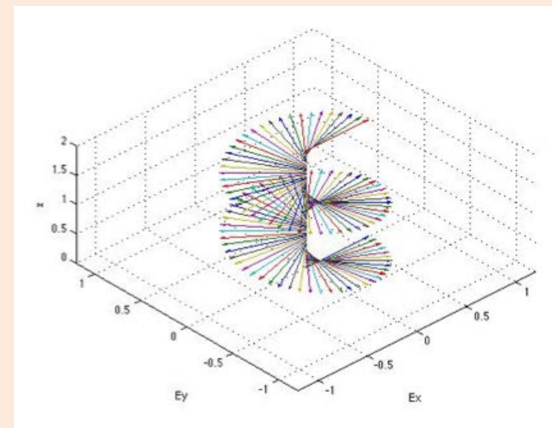


Fixed t

- **Circular**

$$\vec{E}=e^{i(kz-\omega t)}E_0(\hat{e}_x \pm i\hat{e}_y)$$

We can define: $\hat{e}_R=\frac{1}{\sqrt{2}}\left(\hat{e}_x-\mathrm{i}\hat{e}_y\right),\ \hat{e}_L=\frac{1}{\sqrt{2}}\left(\hat{e}_x+\mathrm{i}\hat{e}_y\right)$

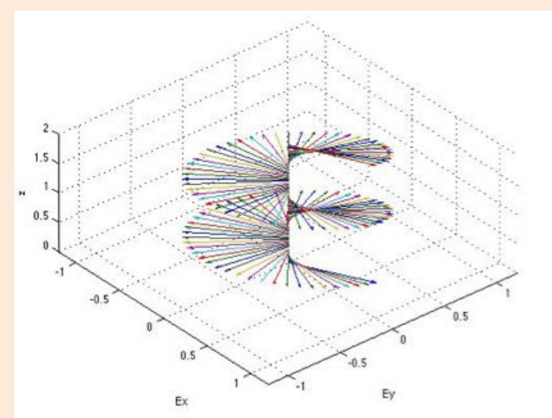


- **Elliptic**

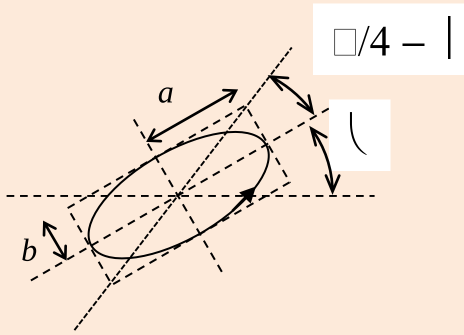
$$\vec{E}=e^{i(kz-\omega t)}(E_x\hat{e}_x+E_y e^{-2i\delta}\hat{e}_y)$$

2δ = relative phase

$E_y/E_x=\tan\alpha$ = relative amplitude



Polarization ellipse



Elipticity $\epsilon = \pm \frac{b}{a} = \tan\left(\pi/4 - \chi\right)$

Orientation: θ

How do we relate the amplitude and phases to the ellipse parameters? Not Trivially!

$$\vec{E} = E_0 e^{i(kz-\omega t)}(\cos\alpha\,\hat{e}_x + e^{\pm i2\delta}\sin\alpha\,\hat{e}_y)$$

$$\cos 2\alpha = \cos 2\theta \sin 2\chi$$

$$\cos 2\chi = \sin 2\alpha \sin 2\delta$$

The Alternative: to use the circular basis

$$\hat{e} = (\cos\chi e^{+i\theta}\hat{e}_R + e^{-i\theta}\sin\chi\,\hat{e}_L)$$

The Poincaré Sphere

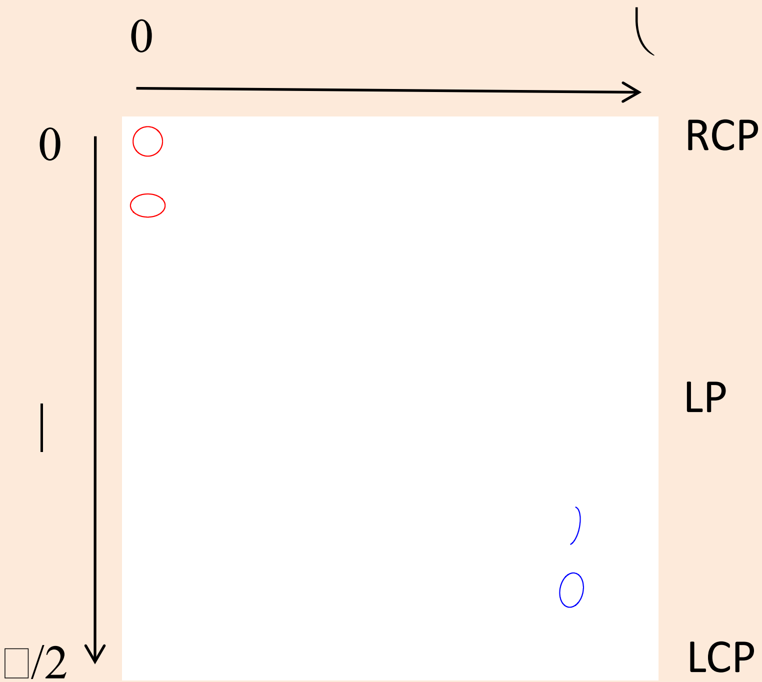
Henri Poincare’
1854-1912

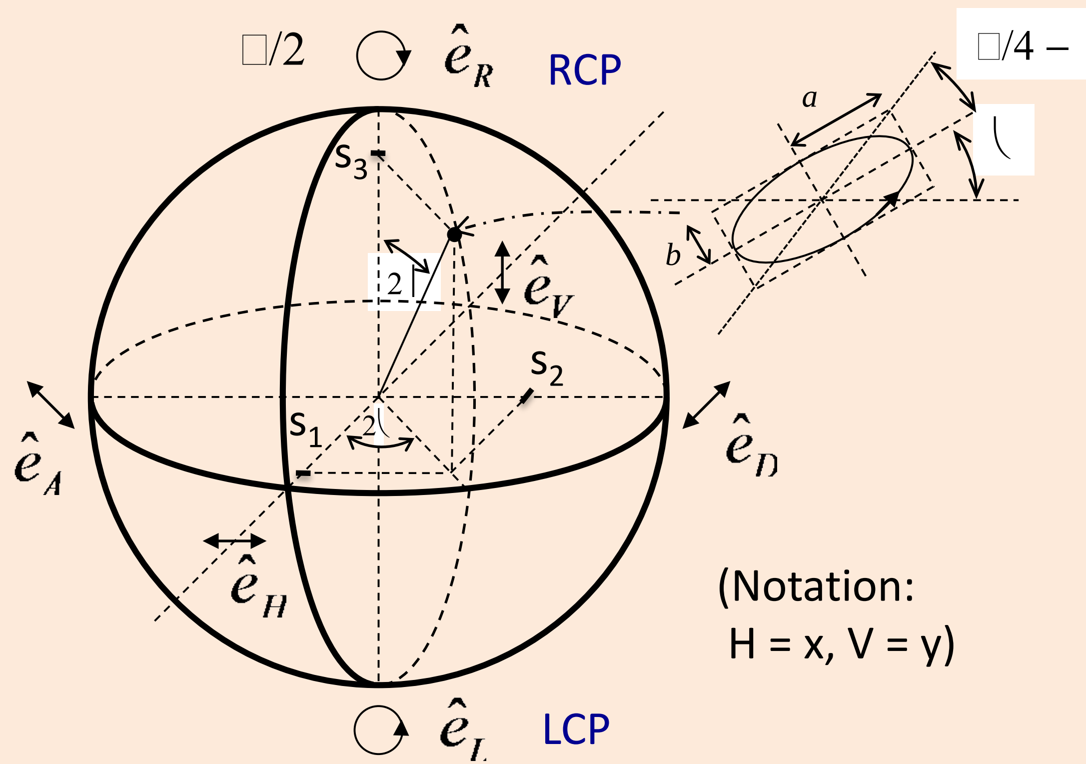
The state of polarization in the circular basis:

$$\hat{e} = (\cos\chi e^{+i\theta}\hat{e}_R + e^{-i\theta}\sin\chi\hat{e}_L)$$

Poincaré sphere (1892):

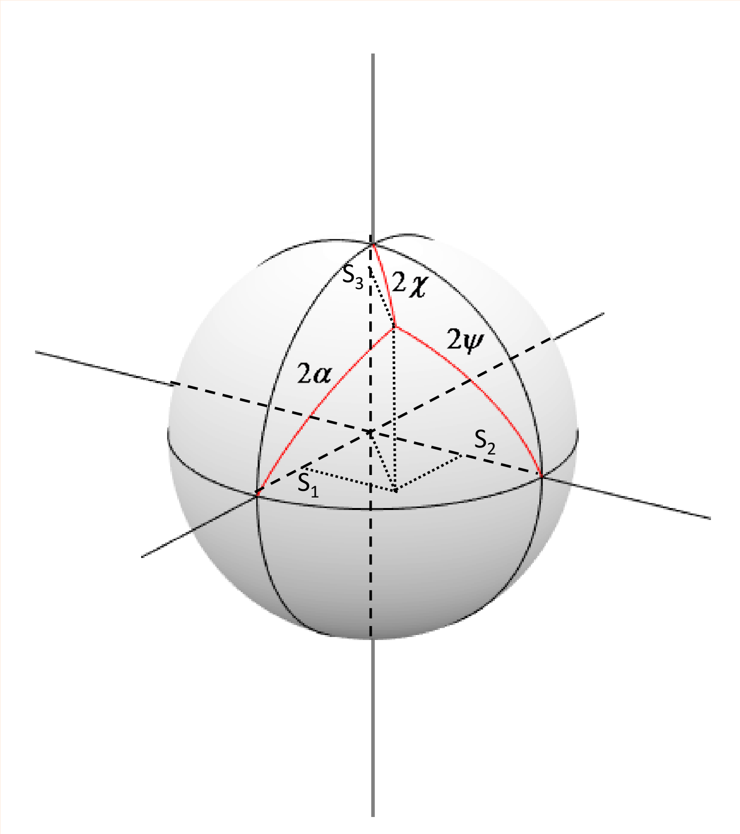
- Same lattitud = same elipticity (specified by χ)
- Same longitude = same orientation (specified by θ)





(Notation:
H = x, V = y)

Stokes Parameters



Components of a point on the sphere:

$$s_3 = \cos 2\chi = \frac{I_R - I_L}{I_0}$$

$$s_1 = \cos 2\alpha = \frac{I_H - I_V}{I_0}$$

$$s_2 = \cos 2\psi = \frac{I_D - I_A}{I_0}$$

Ellipse parameters:

Elipticity: $$\chi = \frac{1}{2}\cos^{-1}\left(\frac{s_3}{\sqrt{s_1^2 + s_2^2 + s_3^2}}\right)$$

Orientation: $$\theta = \frac{1}{2}\tan^{-1}\left(\frac{s_2}{s_1}\right)$$

Laguerre-Gauss modes: spatial modes that carry phase singularities or optical vortices

$$LG^{\ell}(r,\phi) = A_{\ell}\; r^{|\ell|}\; e^{-r^2/w^2}\; e^{i\ell\phi}$$

amplitude phase

- For *l* ≠ 0, wavefront is made of intertwined spirals of pitch *l*
- Carry *orbital* angular momentum.
- Forked diffraction gratings generate modes in the diffraction orders.

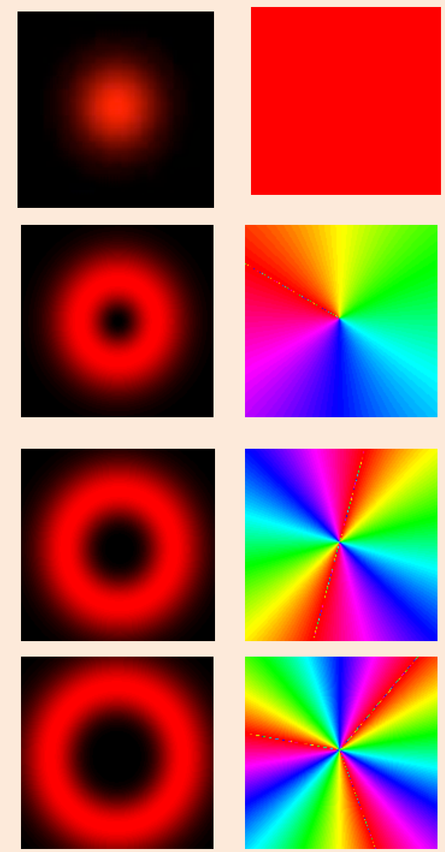
Intensity Phase Wavefront

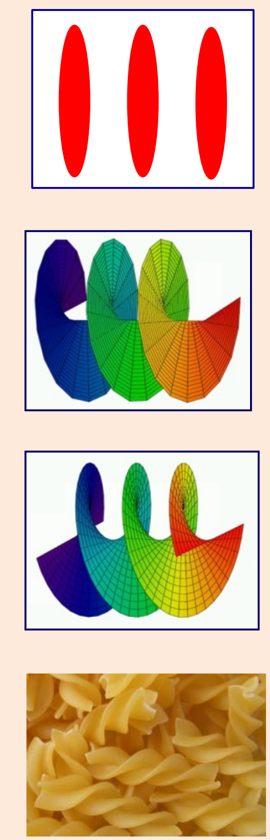
l = 0

l = 1

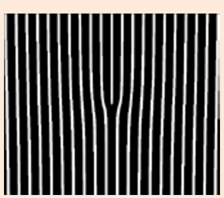
l = 2

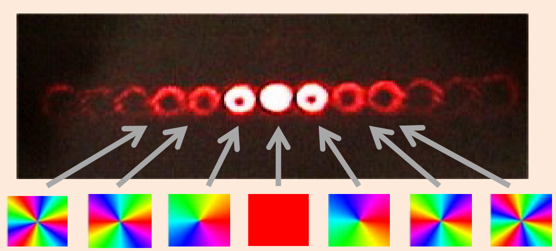
l = 3



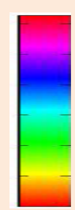


(Fusilli pasta)





ℓ = -3 -2 -1 0 +1 +2 +3



2□

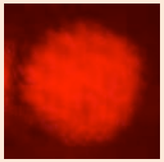
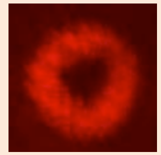
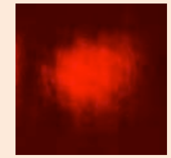
□

0

Phase encoded with color.

Poincaré mode: has spatially-variable polarization

$$V(r,\ \phi) = \frac{1}{\sqrt{2}}\left(e^{i\alpha} LG^{\ell_1}(r,\phi)\ \hat{e}_R + e^{-i\alpha} LG^{\ell_2}(r,\phi)\ \hat{e}_L \right)$$

We can rewrite it as:

$$V(r,\phi) = e^{i\theta} \cos\chi\ \hat{e}_R + e^{-i\theta} \sin\chi\ \hat{e}_L$$

where:

$$\chi = \tan^{-1}\left(\frac{A_{LG_2}}{A_{LG_1}} \right) = \tan^{-1}\left(\frac{A_{\ell_2} r^{|\ell_2|}}{A_{\ell_1} r^{|\ell_1|}} \right)$$

Ellipticity depends only on r

and:

$$\theta = (\ell_1 - \ell_2)\phi/2 + \alpha$$

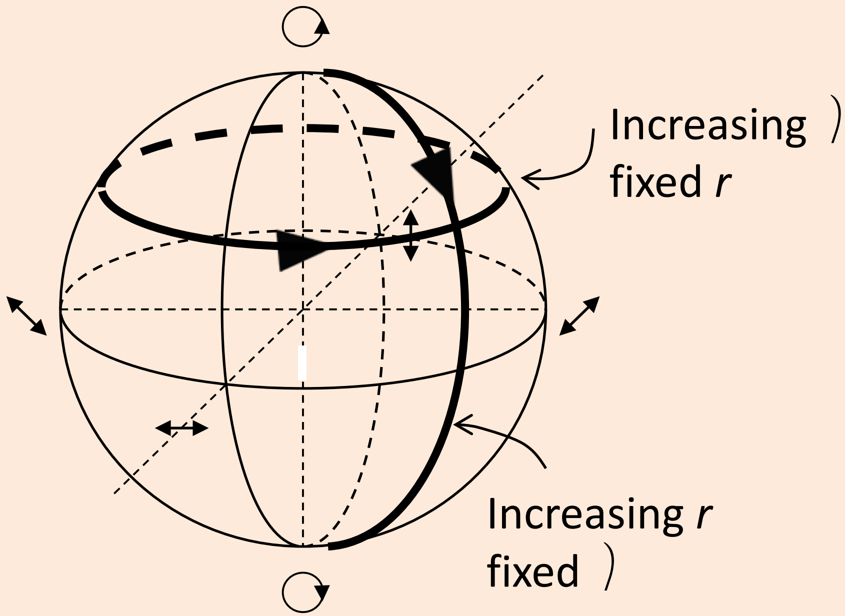
Orientation depends only on)

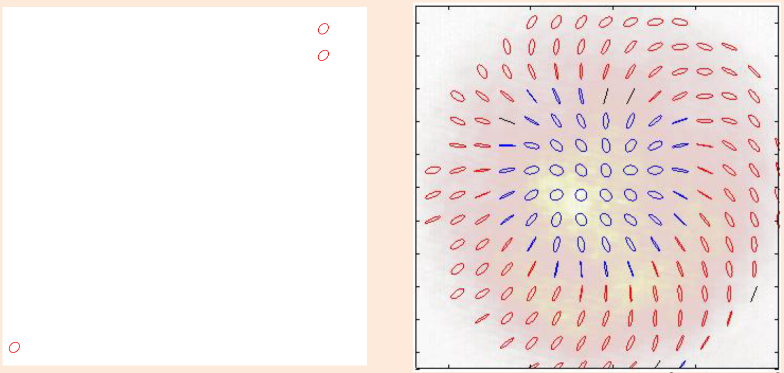
Spatial/Polarization modes:

$$V = \frac{1}{\sqrt{2}}\left(e^{i\alpha} LG_0^{\ell_1}\ \hat{e}_R + e^{-i\alpha} LG_0^{\ell_2}\ \hat{e}_L\right)$$

Non-separable superposition

Case: l_1=2, l_2=0 ellipticity: $\chi = \tan^{-1}\left(\sqrt{2}\, r^2\right)$ orientation: $\theta = \phi$ radial





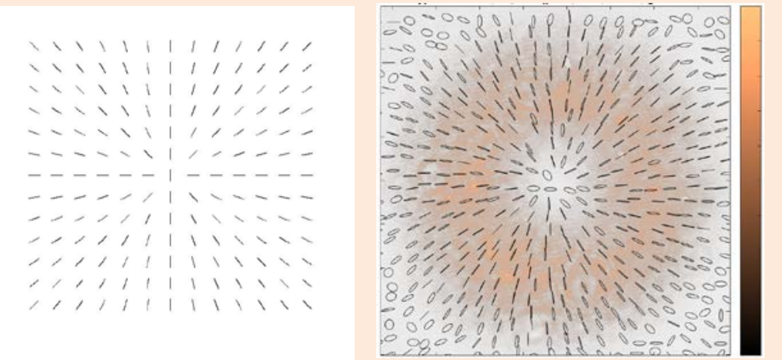
Beckley et al Opt. Exp. 18, 10777 (2010)
Galvez et al Appl. Opt. **51**, 2125 (2012)
Cardano et al Opt. Express 21, 8815 (2013)

Case: l_1=+1, l_2=-1 ellipticity: $\chi = \pi / 4$ linear

orientation: $\theta = \phi$ radial

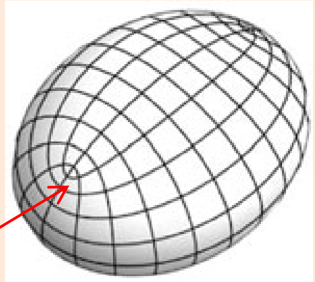
Also known as radial vector beam

Tidwell et al App. Opt 29 (1996)



The patterns of orientation are topological disclinations: dislocations in the rotational order. A few examples:

Surface topology



singularity

Bauer et al 2010

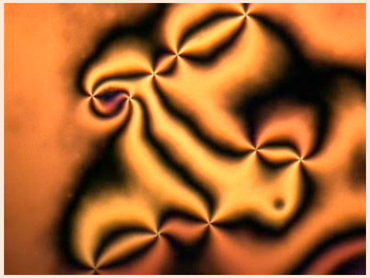
Loop

Delta

Fingerprints: natural disclinations in skin formation (R. Penrose 1979)

Outstanding mathematical problem: Carathéodory conjecture. Any closed convex surface must have at least two umbilical points

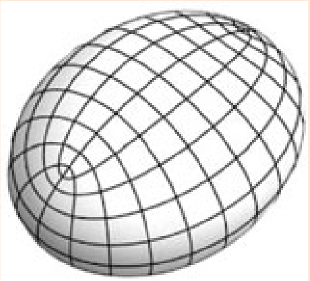
Bicep2 data on the polarization of the cosmic microwave background. Expected to reveal information about gravitational waves in the early big bang. Data corrupted by scattering from galactic dust.
Bicep3 is on the way…



In liquid crystal disclinations appear in the molecular directors (Kent State)

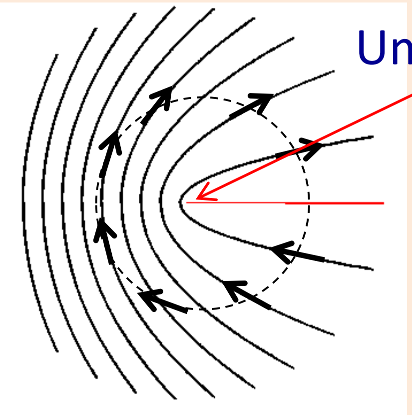
Studying disclinations:

Berry & Hannay J. Phys. A **10**, 1809 (1977)
Used line patterns to model surface topology.

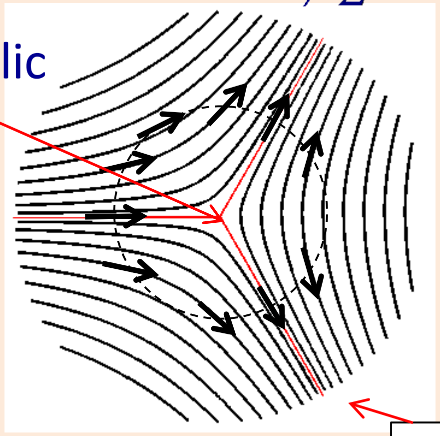


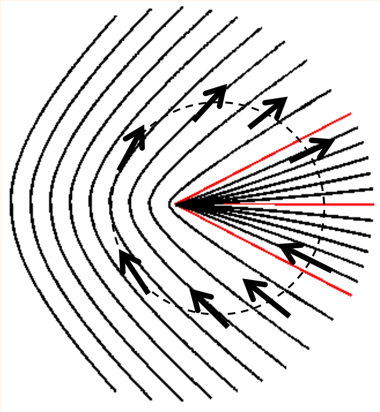
Bauer et al 2010

lemon $I_C = +\frac{1}{2}$ star $I_C = -\frac{1}{2}$ monstar $I_C = +\frac{1}{2}$



Umbilic



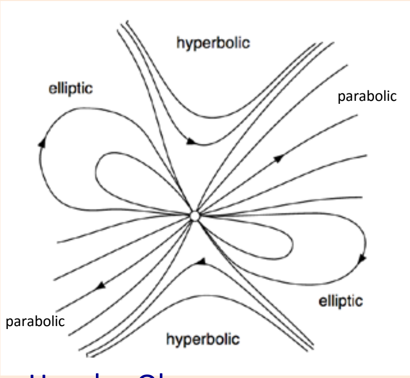


Radial line

We categorize them by their index:

$$I_C = \frac{\Delta\theta_{\text{per turn}}}{2\pi}$$

Ivar Bendixson
1861-1935



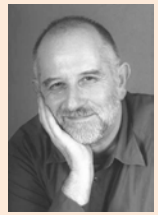
Hanche-Olsen

Bendixson 1901 formula:

$$I_C = \frac{\Delta\theta_{\text{per turn}}}{2\pi} = 1 + \frac{e}{2} - \frac{h}{2}$$

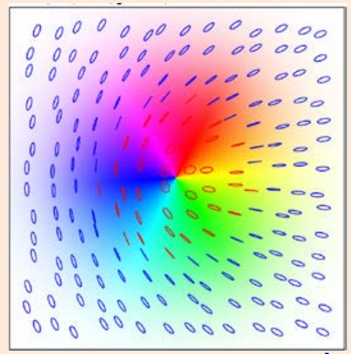
Measuring Disclinations in Polarization

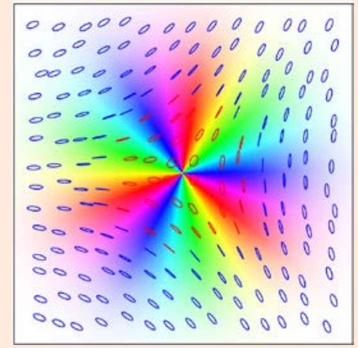
For polarization, ellipse orientations follow lines.
Nye, R. Proc. Soc (1983);
Berry SPIE 4403, 1, (2001)



$$|\psi\rangle = e^{i\ell_1}|R\rangle + e^{i\ell_2}|L\rangle$$

$(\ell_1, \ell_2) = (+1, 0)$ $(-1, 0)$



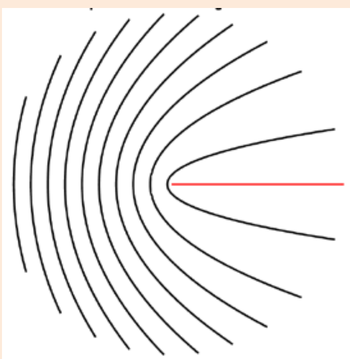


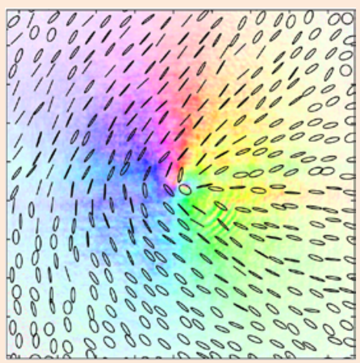
Color: relative to radial (radial=yellow; blue=tangential).

$$I_C = \frac{\ell_1 - \ell_2}{2}$$

(Centers are singular points of orientation)

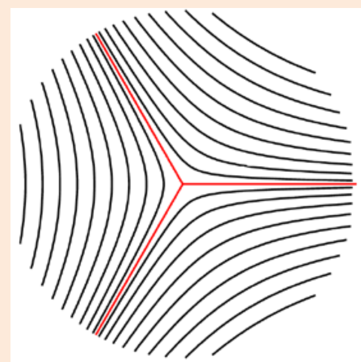
$(\ell_1, \ell_2) = (+1, 0)$ $I_C = +\frac{1}{2}$ (lemon)

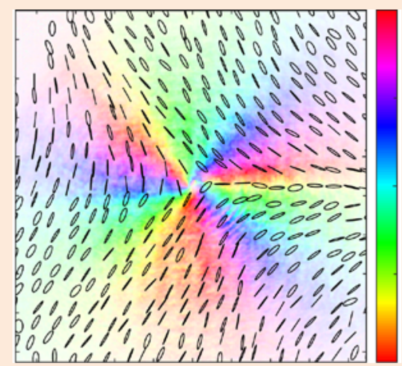


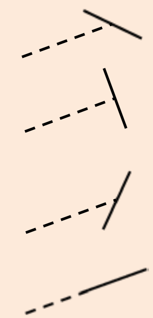


$N = 1$

$(-1, 0)$ $I_C = -\frac{1}{2}$ (star)







$N = 3$

Other mode combinations

$(+2, +1)$
$(+3, +2)$

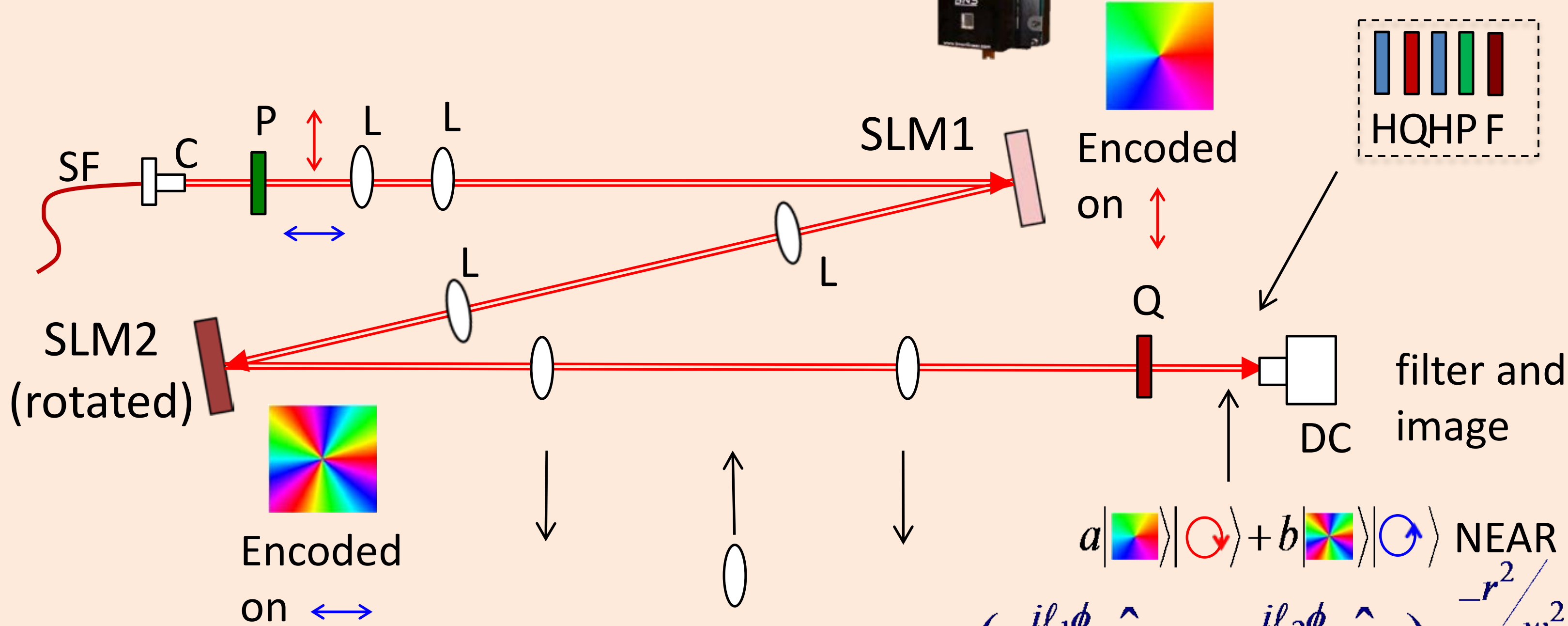
$(-2, -1)$
$(-3, -2)$

Radial lines (Freund OC 2010):

$$N = |2I_C - 2| = |(\ell_1 - \ell_2) - 2|$$

Near-Field Measurements of polarization disclinations

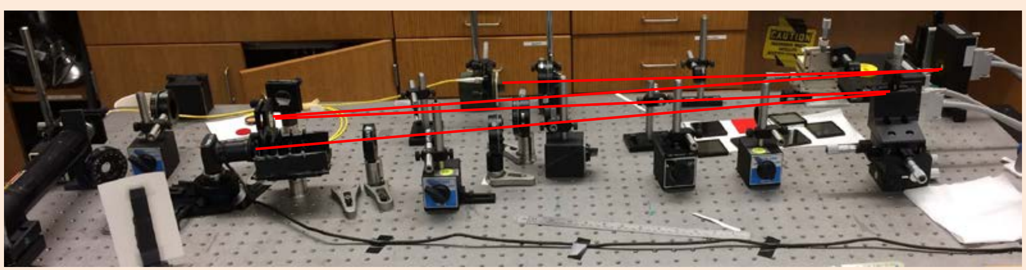
Polarimetry



$a|\cdot\rangle|\circlearrowright\rangle + b|\cdot\rangle|\circlearrowleft\rangle$ NEAR field

$$(e^{i\ell_1\phi}\ \hat{e}_R + e^{i\ell_2\phi}\ \hat{e}_L)e^{-r^2/w^2}$$

Khajavi & Galvez J. Opt. (2016)



$a|\cdot\rangle|\circlearrowright\rangle + b|\cdot\rangle|\circlearrowleft\rangle$ FAR field

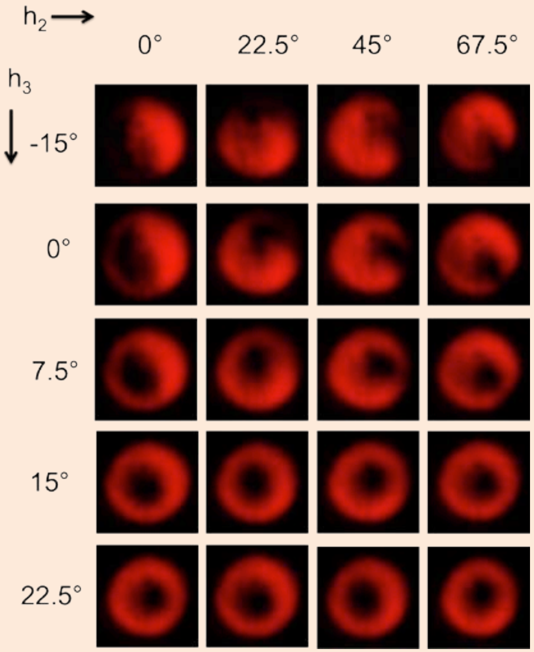
$$LG_0^{\ell_1}\ \hat{e}_R + LG_0^{\ell_2}\ \hat{e}_L$$

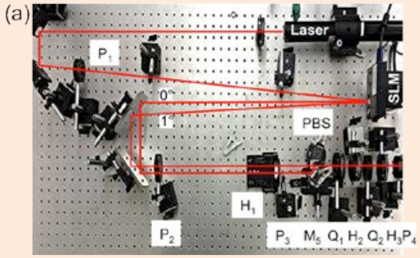
Precursors:
Morgensen & Gluckstad (2000)
Davis et al (2005)

Khajavi & Galvez Opt. Eng. (2015)

Create an optics lab to learn teach states of polarization:

- We create a beam with a polarization that varies from point to point.
- Then use a polarization filter to block a given polarization state.
- Map out the polarization of the mode at each point.





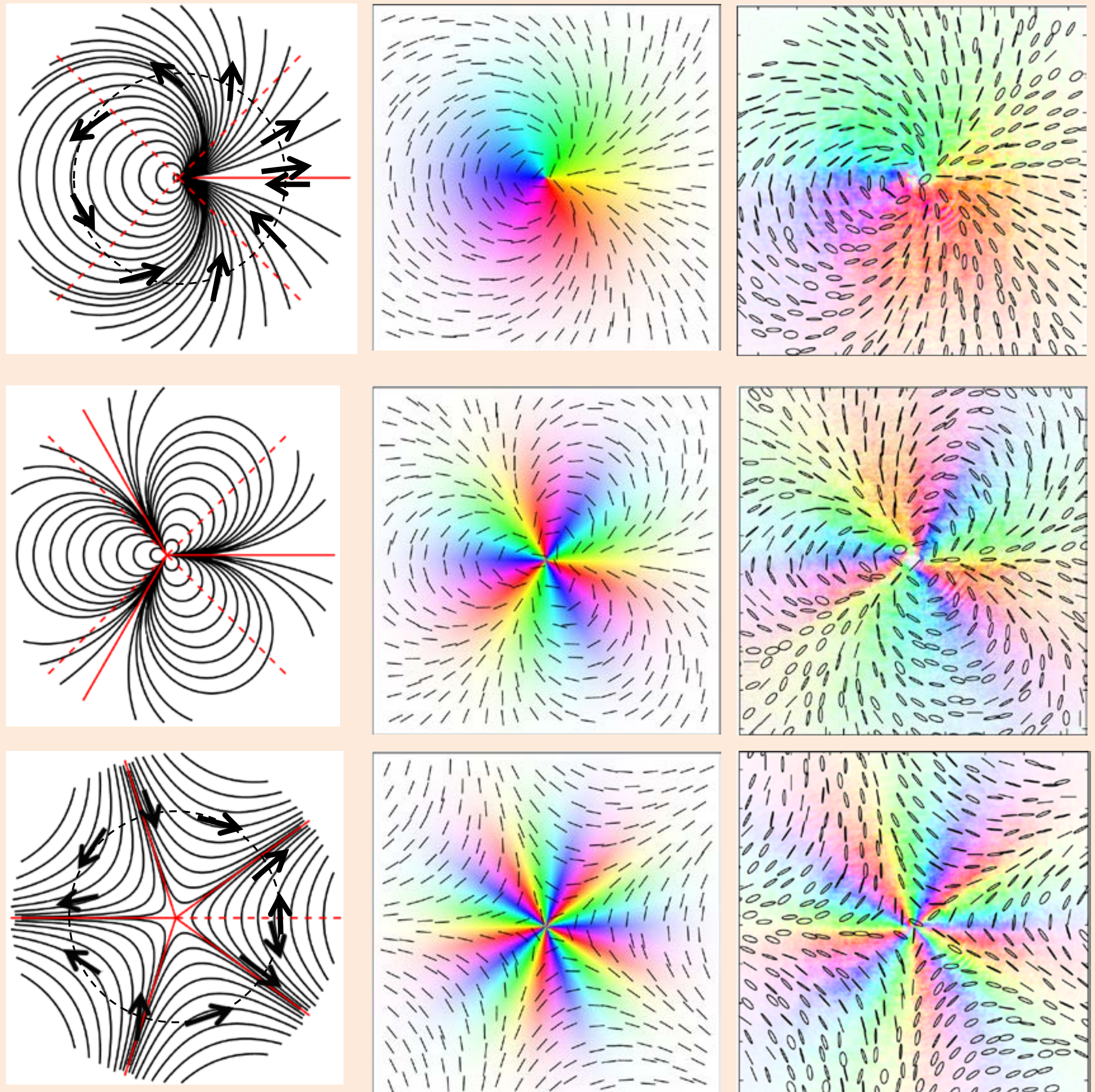
Jones et al Am. J. Phys 84, 822 (2016).

High-order disclinations

$$I_C = 1 + \frac{e}{2} - \frac{h}{2}$$

Sectors:
e= #elliptic
h= #hyperbolic
Bendixon (1901)

Line pattern | Modeling | Measurements



(hyper lemon) "spider"

$I_C = +3/2$
$(+2,-1)$
$N = 1$

(hyper lemon) "flower"

$I_C = +5/2$
$(+2,-3)$
$N = 3$

(hyper star) "spider web"

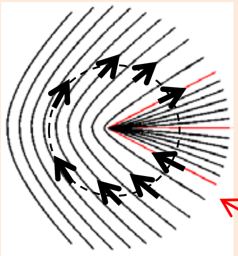
$I_C = -3/2$
$(-1,2)$
$N = 5$

Theory: Freund (2001)

Vector-beam measurements: Denz et al, Marrucci et al (2012), Khajavi & Galvez J. Opt (2016)

Asymmetric orientation dislocations: monstars

Predicted by Berry & Hannay 1977, Dennis 2002, Freund 2002



$$\left|\psi\right\rangle = \left(\cos\beta e^{+i\ell_1} + \sin\beta e^{-i\ell_1} e^{i\gamma}\right)\left|R\right\rangle + e^{i\ell_2}\left|L\right\rangle$$

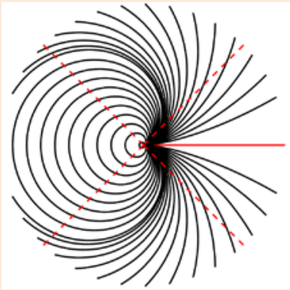
Asymmetric vortex: - charge $+\ell_1$ if $\beta < 45^\circ$

- charge $-\ell_1$ if $\beta > 45^\circ$

Phase shears and no vortex if $\beta = 45^\circ$

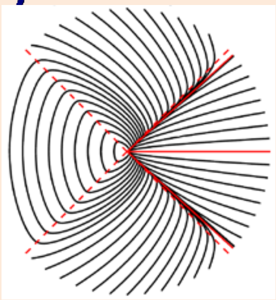
$(\ell_1, \ell_2) = (+2, -1)$

$\beta = 0$



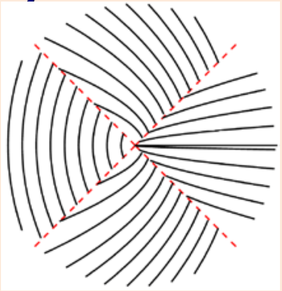
Lemon I_C=3/2

$\beta = 40^\circ$



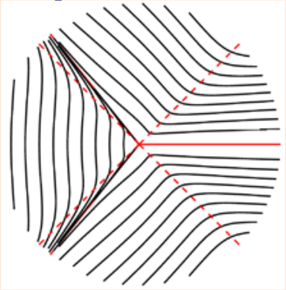
Monstar

$\beta = 45^\circ$



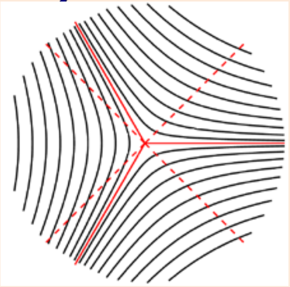
Shear

$\beta = 60^\circ$



Star

$\beta = 90^\circ$



Star I_C=-1/2

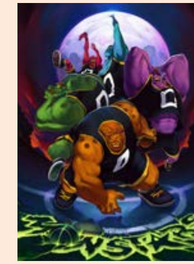
Depending on ℓ_1 and ℓ_2 we may have: lemon -> star

lemon -> lemon

star -> star

$$I_C = 1 + \frac{e}{2} - \frac{h}{2}$$

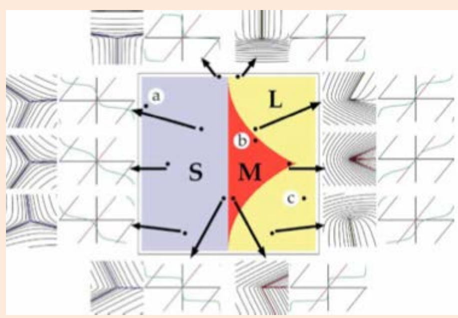
Monstardom: The space of monstars

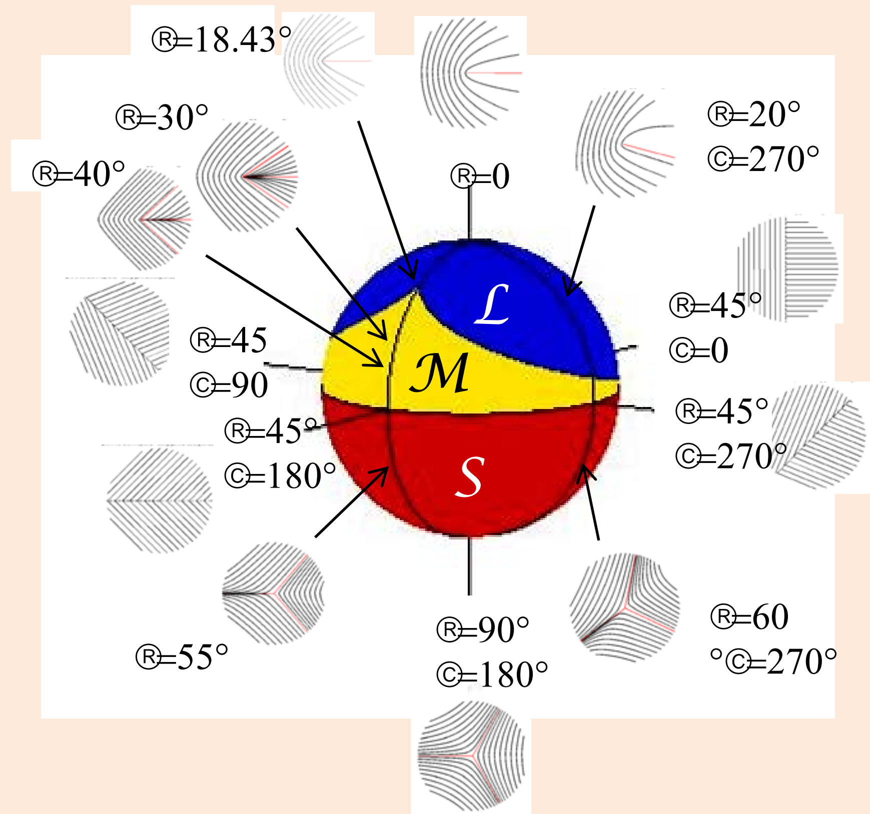


$$|\psi\rangle = \left(\cos\beta\, e^{+i\ell_1} + \sin\beta\, e^{-i\ell_1} e^{i\gamma}\right)|R\rangle + e^{i\ell_2}|L\rangle$$

For: $\beta = 30^\circ$

Predicted by M. Dennis OL **33**, 2572 (2008)



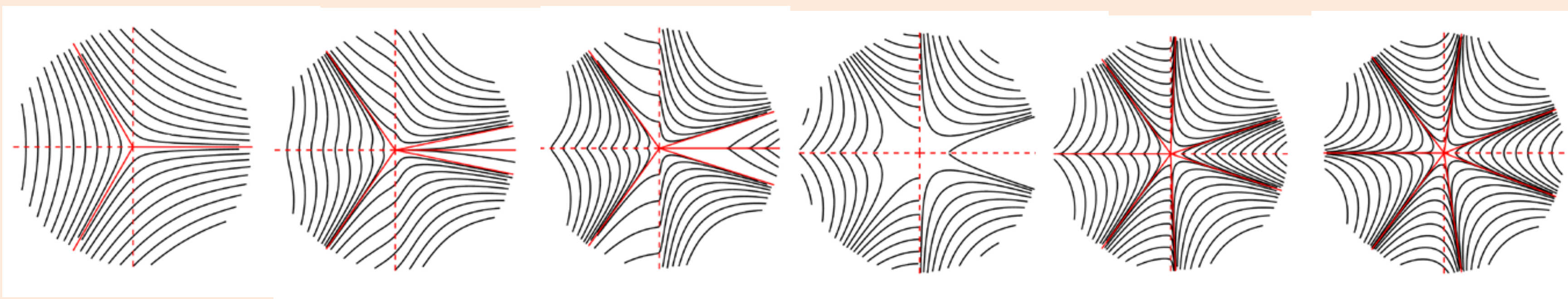


Polar angle is β
azimuthal angle is γ

Negative-index monstars

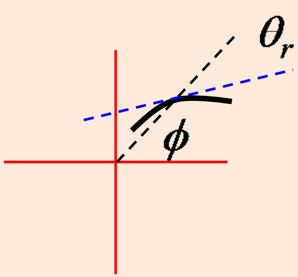
$$(\cos\beta\, e^{+i\ell_1\phi} + \sin\beta\, e^{i\gamma}\ e^{-i\ell_1\phi})\,\hat{e}_R + e^{+i\ell_2\phi}\hat{e}_L \qquad (+2,+3),\ \gamma = \pi$$

0° | 30° | 44° | 45° | 46° | 60°

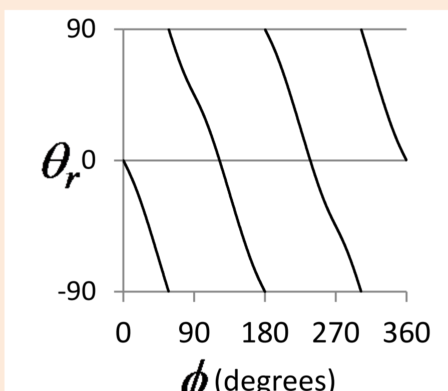


star | monstar | monstar | shear | star | star

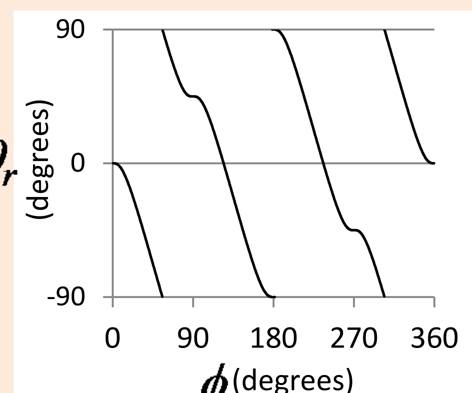
Radial Orientation:



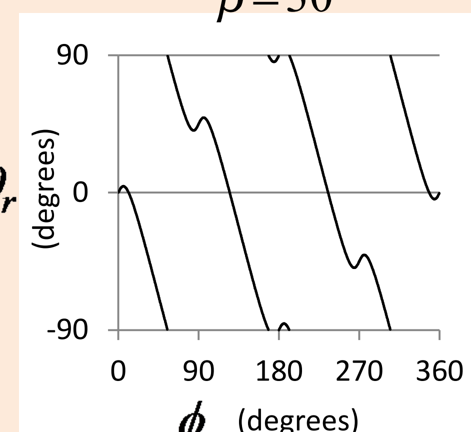
$\beta = 10^\circ$



$\beta = 23.33^\circ$

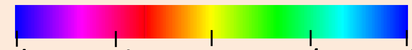


$\beta = 30^\circ$



New monstars…

Angle relative to radial:



$(+2,-3)$

$\beta = 50^\circ$

$I_C = +\frac{1}{2}$

$(+2,3)$

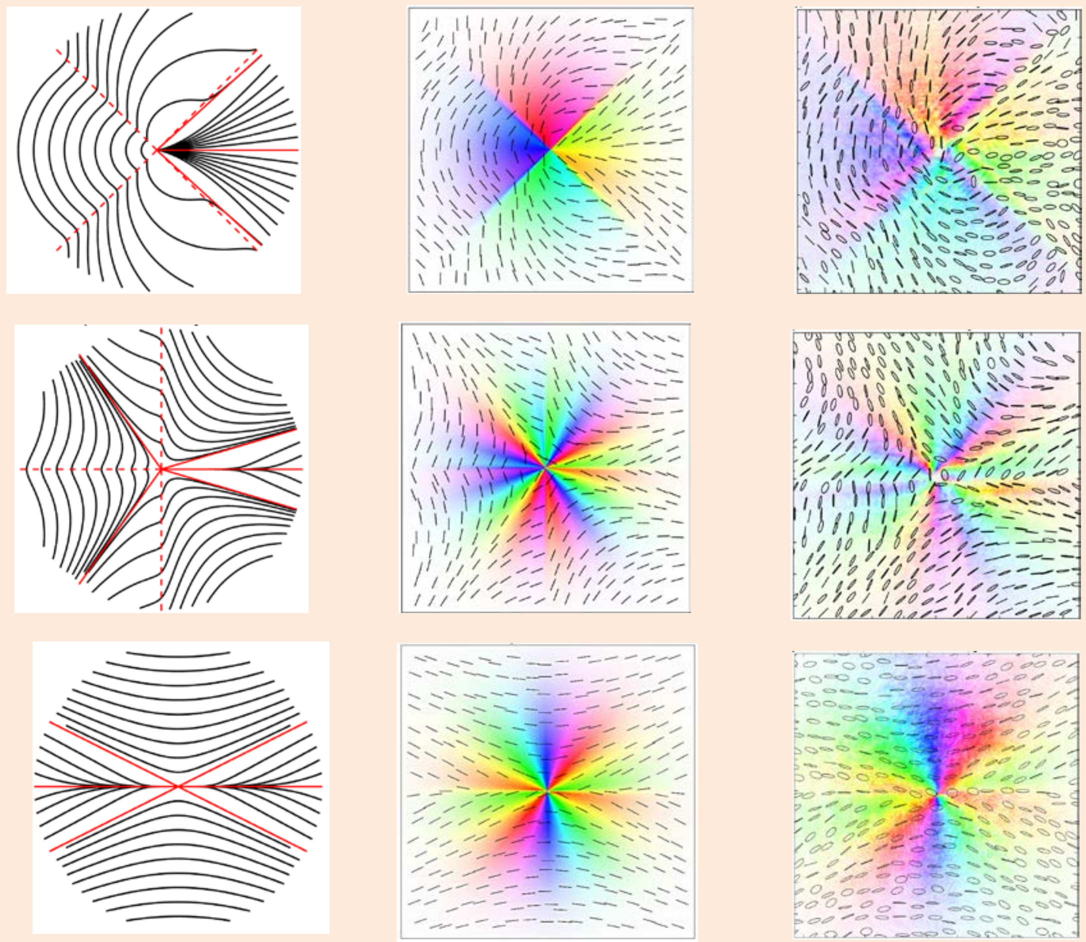
$\beta = 40^\circ$

$I_C = -\frac{1}{2}$

$(1,1)$

$\beta = 40^\circ$

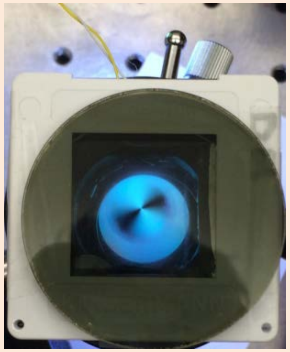
$I_C = 0$



Khajavi & Galvez J. Opt (2016);

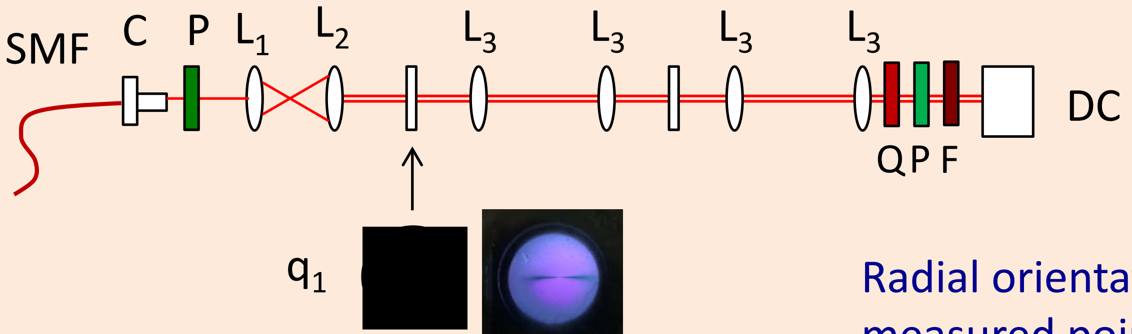
Galvez & Khajavi JOSAA (2017)

Superposition is not the only way: q-plates



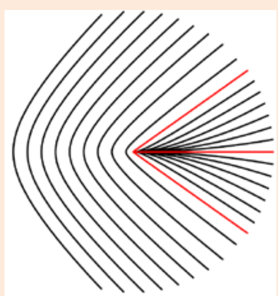
- Liquid crystal cells with directors forming a disclination pattern, forming retardation plates with spatially-dependent fast axis.
- Light passing through *acquires the encoded disclination*.
  Cardano et al Appl Opt 51, C1 (2012); Cardano et al Opt. Express 21, 8815 (2013)

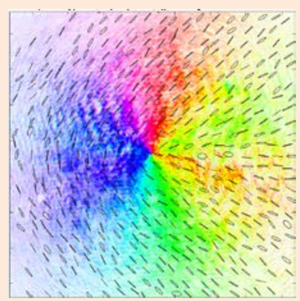
Apparatus:

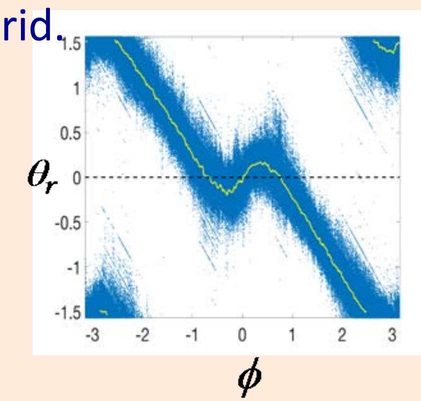


Radial orientation for each measured point in 200x200 grid.

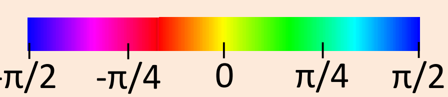
Monstars with elliptically symmetric q-plates





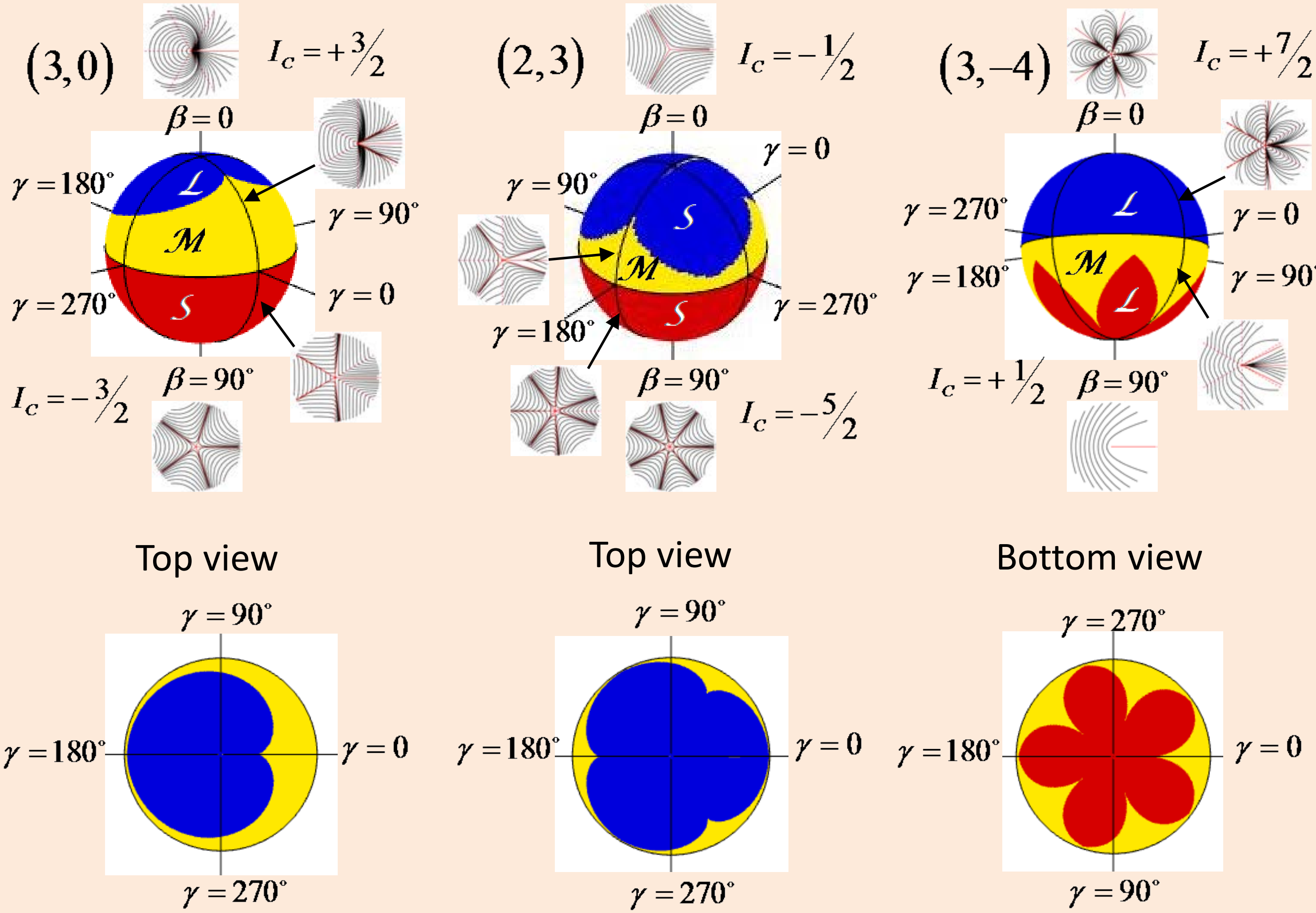


Angle relative to radial:



Cvarch et a submitted

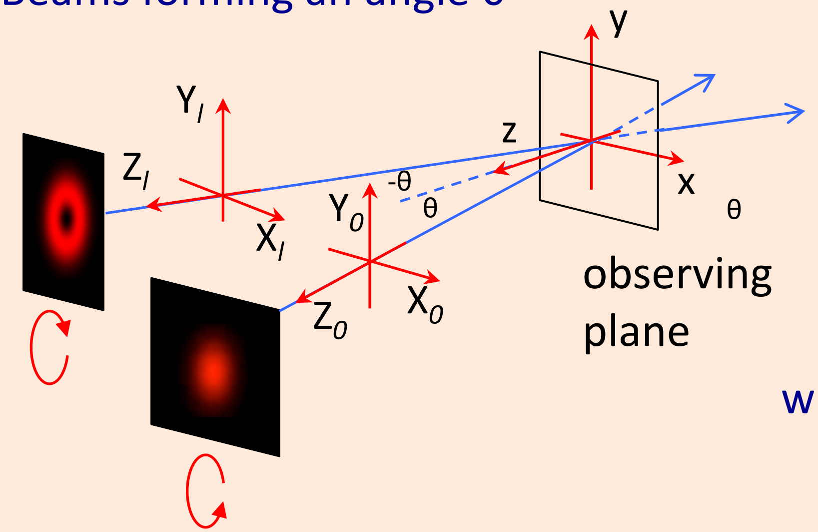
Each modal combination has its own space:



Galvez et al PRA 2014
Khajavi & Galvez J. Opt (2016);

3-dimensional spatially-variable polarization

Beams forming an angle θ



Each beam in their local frame:

$$\vec{E}_\ell = A\, e^{i\ell \tan^{-1}(Y_\ell/X_\ell)}\; e^{ikZ_\ell}\, G\,\hat{e}_R$$

$$\vec{E}_0 = A\, e^{ikZ_0}\, G\,\hat{e}_L$$

where $G_\ell = e^{-(X_\ell^2+Y_\ell^2)/w^2}$ $\quad G_0 = e^{-(X_0^2+Y_0^2)/w^2}$

Transform local frames to observing frame: $\begin{cases}(X_\ell, Y_\ell, Z_\ell) \to (x,y,z)\\ (X_0, Y_0, Z_0) \to (x,y,z)\end{cases}$

Rewriting the fields in terms of the observing plane coordinates:

$$\vec{E}_\ell = A\, e^{i\ell \tan^{-1}(y/x\cos\theta)}\, e^{-ikx\sin\theta}\, G'\left(\cos\theta\,\hat{e}_x - i\,\hat{e}_y + \sin\theta\,\hat{e}_z\right)$$

$$\vec{E}_0 = A\, e^{ikx\sin\theta}\, G'\left(\cos\theta\,\hat{e}_x + i\,\hat{e}_y - \sin\theta\,\hat{e}_z\right)$$

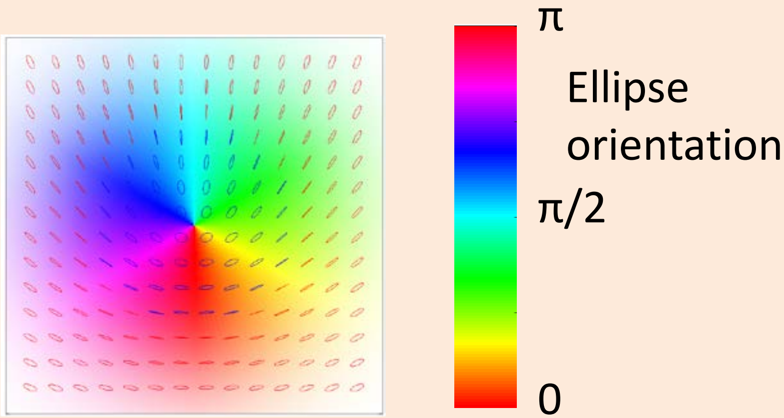
$$(G' \sim G)$$

Notice:

- Z-component of the field
- Relative phase depends on x, θ
- Azimuthal phase depends on θ

As we increase θ…

θ = 0

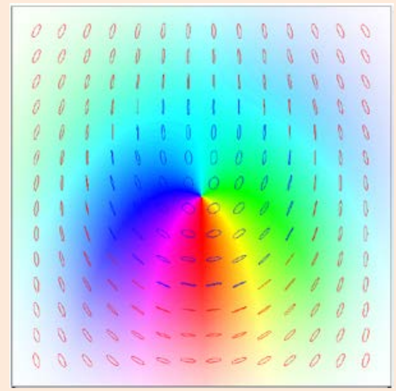


Lemon C-point:
net – ½ turn (ccw)

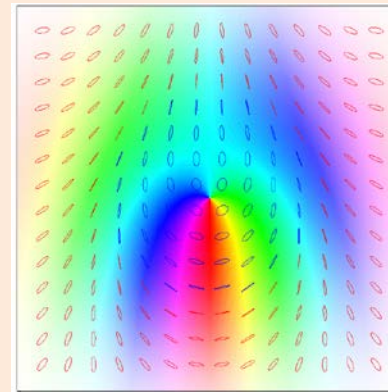
$$I_C = +\frac{1}{2}$$

As we increase θ orientation varies more rapidly with x cordinate:

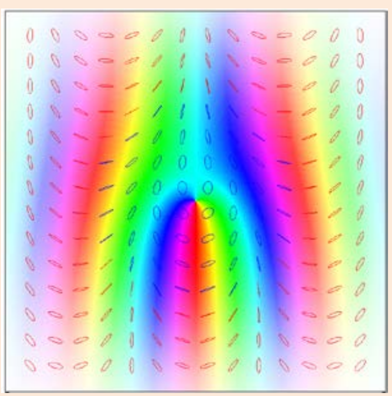
θ = 0.3 arcmin



θ = 0.6 arcmin

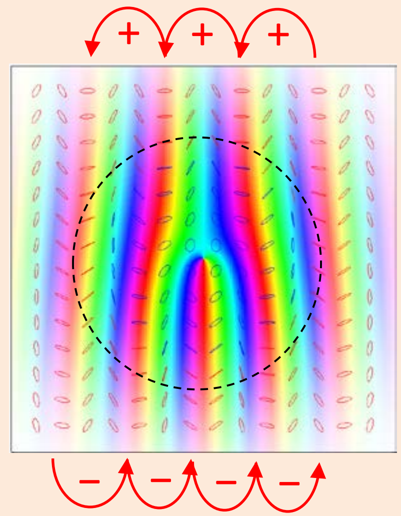


θ = 1.2 arcmin



As θ keeps increasing , orientation fringes appear

θ = 2.4 arcmin



In the XY plane the orientation rotates, but the total polarization rotation is still a net ½ turn.

But something happens in the z-coordinate

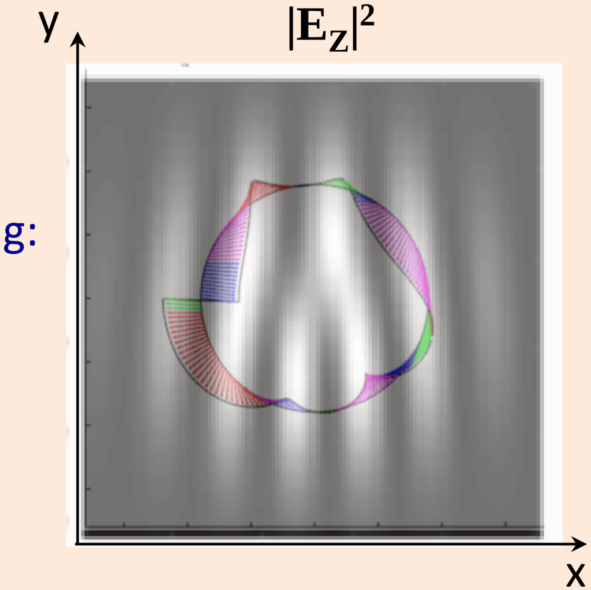
I. Freund Opt. Lett. 35, 148 (2008)

We can extract the semi-axes of the ellipse using:

Semi-major: $$\vec{a} = \frac{1}{\left|\sqrt{\vec{E}\cdot\vec{E}}\right|}\mathrm{Re}\left(\vec{E}\sqrt{\vec{E}^{*}\cdot\vec{E}^{*}}\right)$$

Semi-minor: $$\vec{b} = \frac{1}{\left|\sqrt{\vec{E}\cdot\vec{E}}\right|}\mathrm{Im}\left(\vec{E}\sqrt{\vec{E}^{*}\cdot\vec{E}^{*}}\right)$$

Berry J. Opt. A 6, 675 (2004)

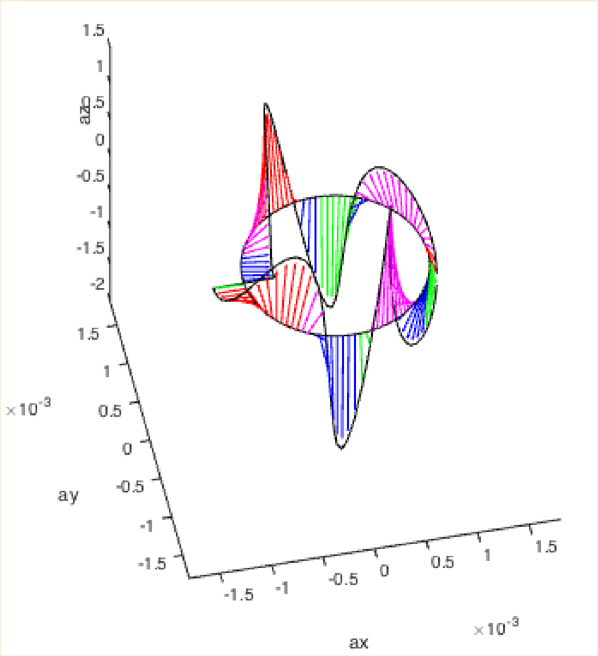


7 zero crossings

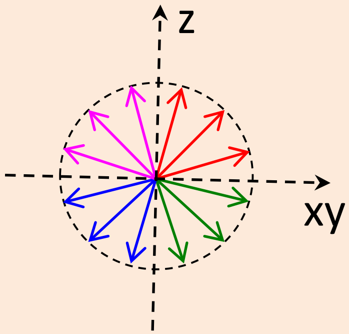
for $r = w/\sqrt{2}$

From crossing to crossing orientation makes half twist

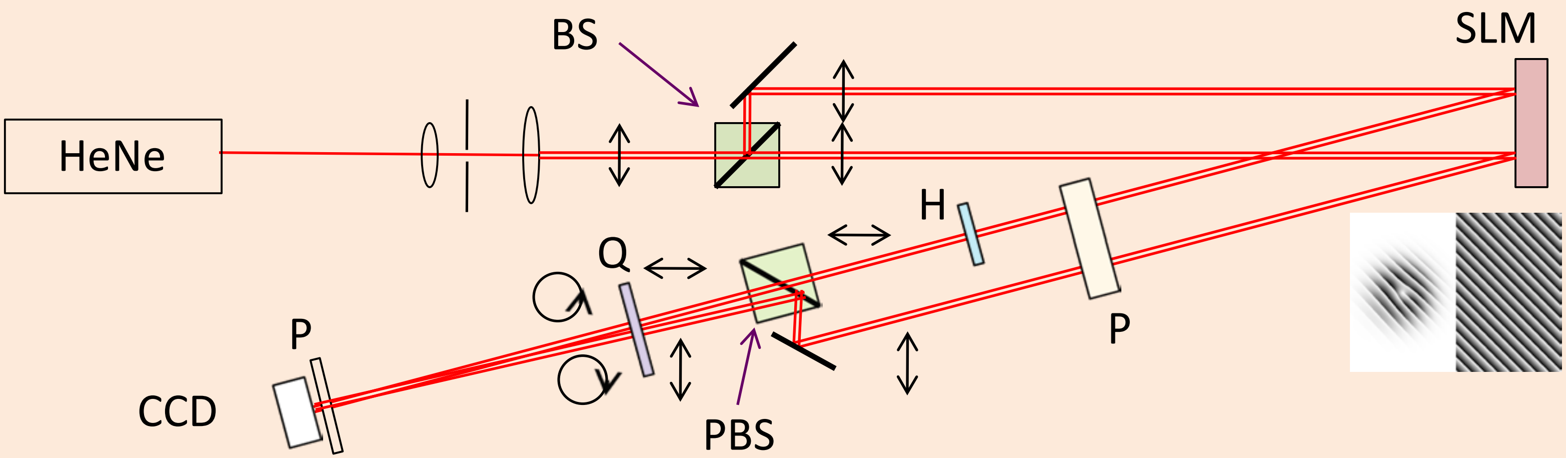
The combination of 2-D rotations with z- ocillations makes the polarization ellipse twist in 3 dimensions, describing Möbius strips or twisted ribbons.



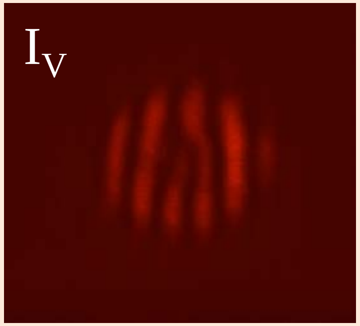
Color coding: red/magenta above plane; blue/green below plane.

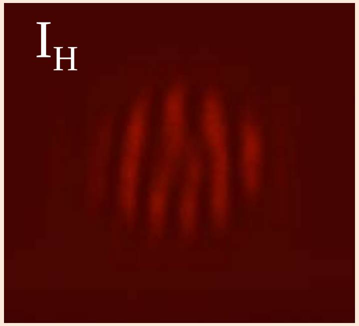


Extracting the pattern with polarization projections



Measure H, V, D A projections with a polarizer to get the field to an overall phase.





For our small angles (few arcmin):

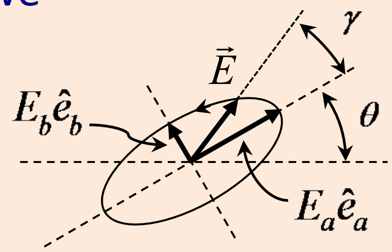
$$E_z \sim 10^{-2} E_y$$

Galvez et al Proc. SPIE 2015

Getting the semi-major axis $\vec{a}$: we can express the field as:

$$\vec{E} = e^{-i\gamma}\left(E_a\hat{e}_a - iE_b\hat{e}_b\right)$$

so… $\vec{a} = E_a\hat{e}_a = \mathrm{Re}(\overline{E^* e^{i\gamma}})$



The rectification phase γ is the instantaneous angle that the field makes with the semi-major axis. We get it doing:

$$e^{i\gamma} = \frac{\sqrt{\vec{E}\cdot\vec{E}}}{\left|\sqrt{\vec{E}\cdot\vec{E}}\right|}$$

Galvez & Dutta Proc. SPIE 2017

Case θ = 36 arcsec

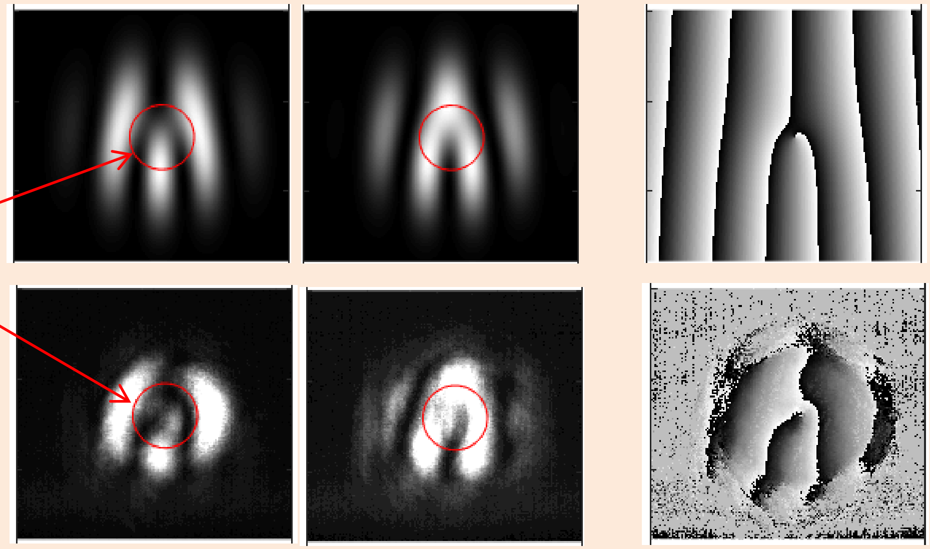
½ turn
Möbius strip

I_V I_H θ

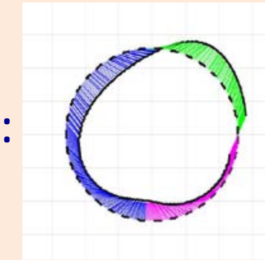
Prediction:

loop

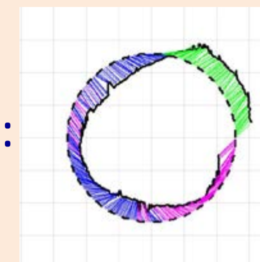
Measurements:



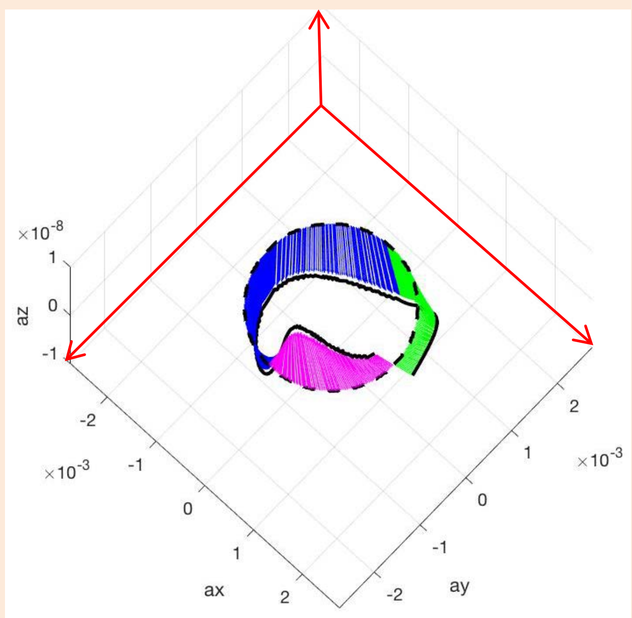
Calculated:

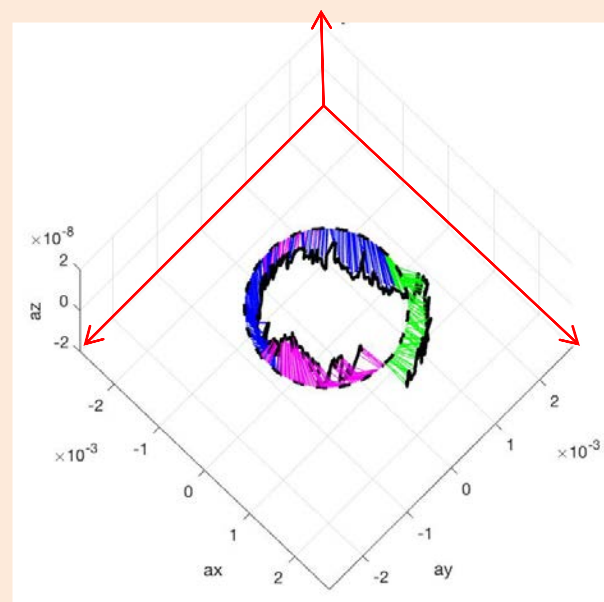


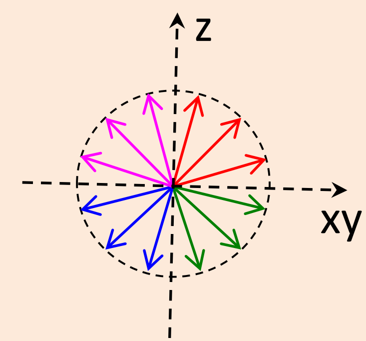
Measured:



3D views:







E. Galvez et al (in preparation)

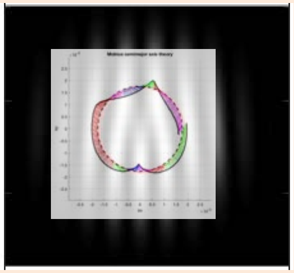
Case θ = 1.9 arcmin

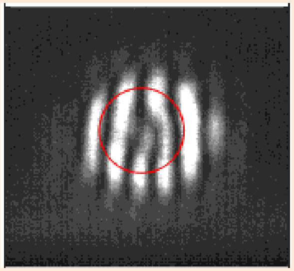
$\ell = 1$

I_V

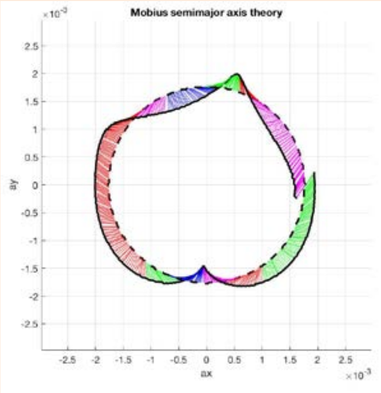
Calculated

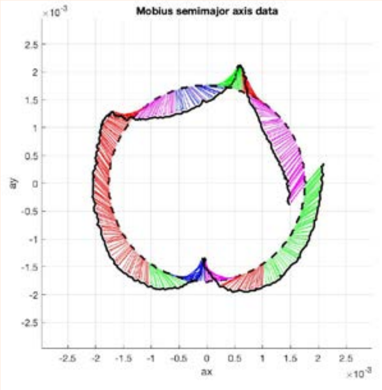
Measured

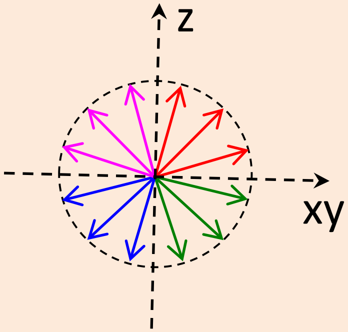




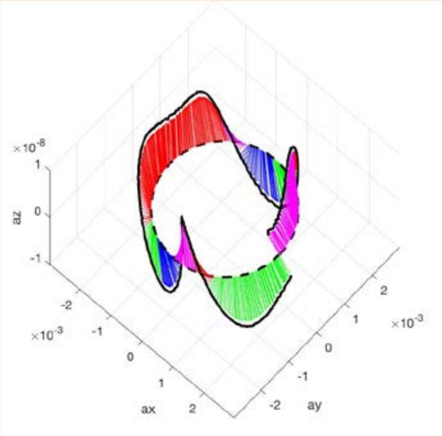
2 ½ turn Möbius strip

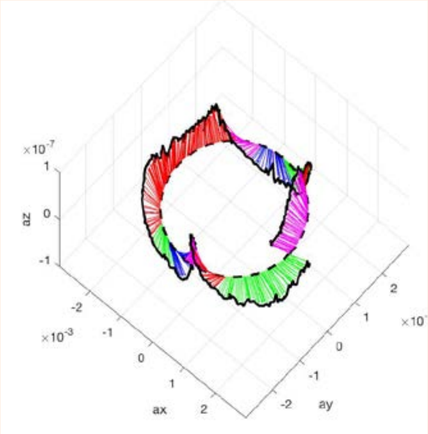






3D views:





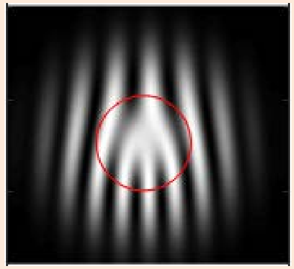
Case θ = 1.9 arcmin

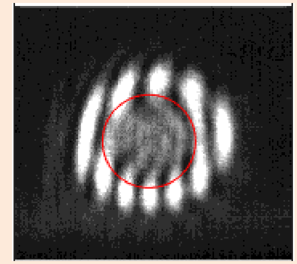
$\ell = 2$

Calculated

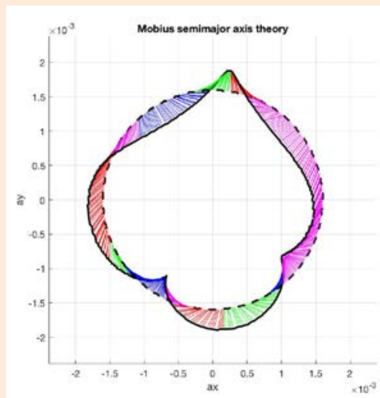
Measured

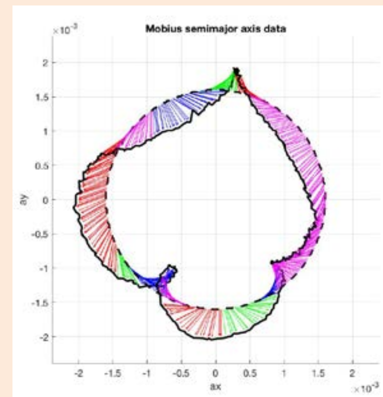
I_V

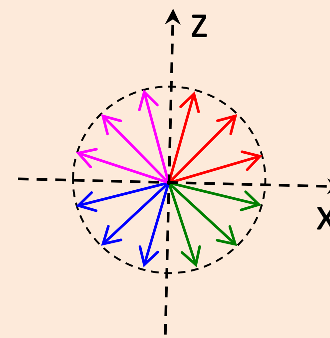




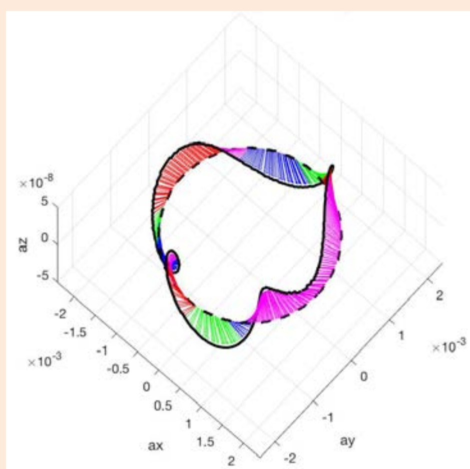
3 turn
twisted ribbon

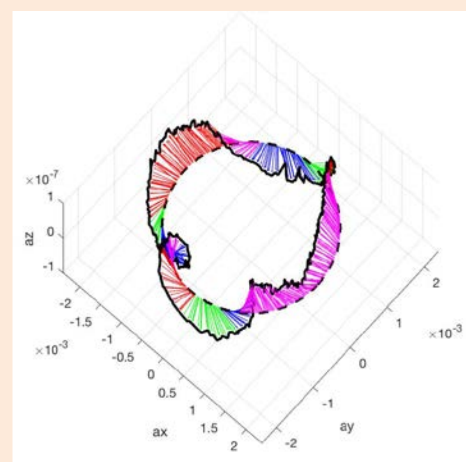






3D views:



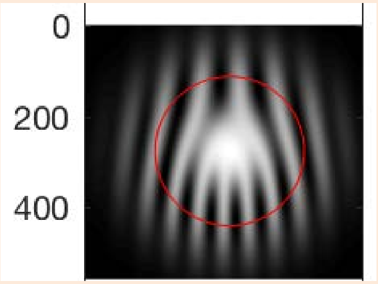


Case θ = 1.9 arcmin

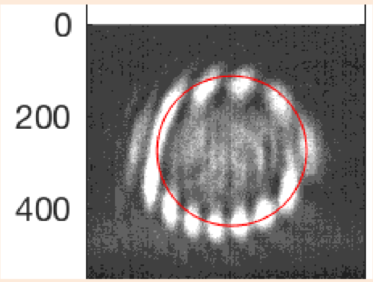
$\ell = 3$

I_V

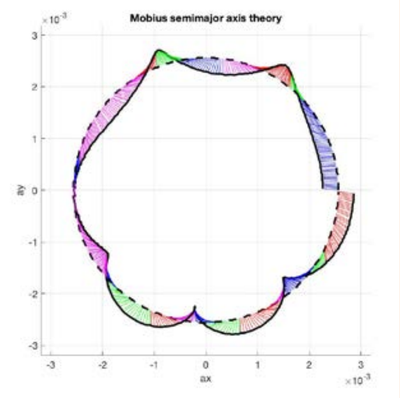
Calculated

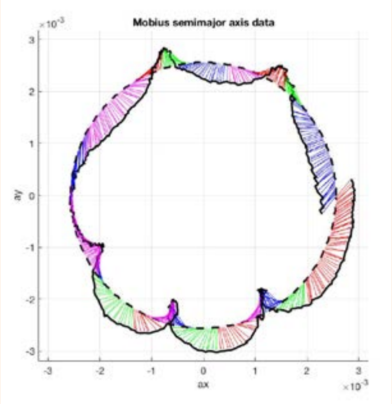


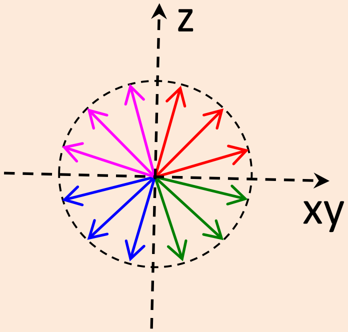
Measured



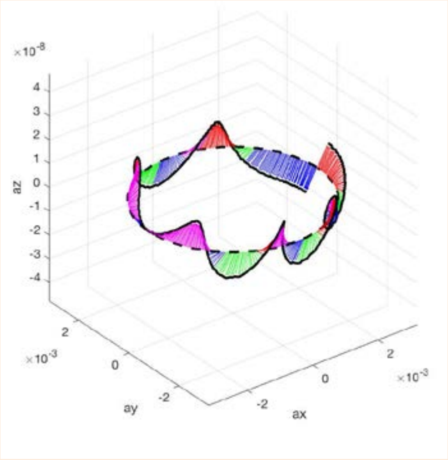
5 1/2 turn
Möbius strip

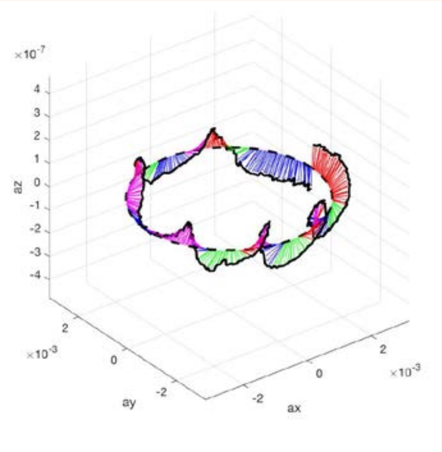






3D views:





Conclusions:

- Spatially-variable polarization patterns are produced by non-separable superpositions of polarization and spatial mode.
- They contain polarization singularities, and allow the exploration of patterns of disclinations not studied before.
- Encode information in the joint space of polarization and spatial mode.
- Polarization and liquid-crystal molecules interact strongly, allowing the patterning of light by matter and *vice versa*.

Thank You for Your Attention!