Simplified Design of Freeform Optics for Beam Shaping and Illumination Applications

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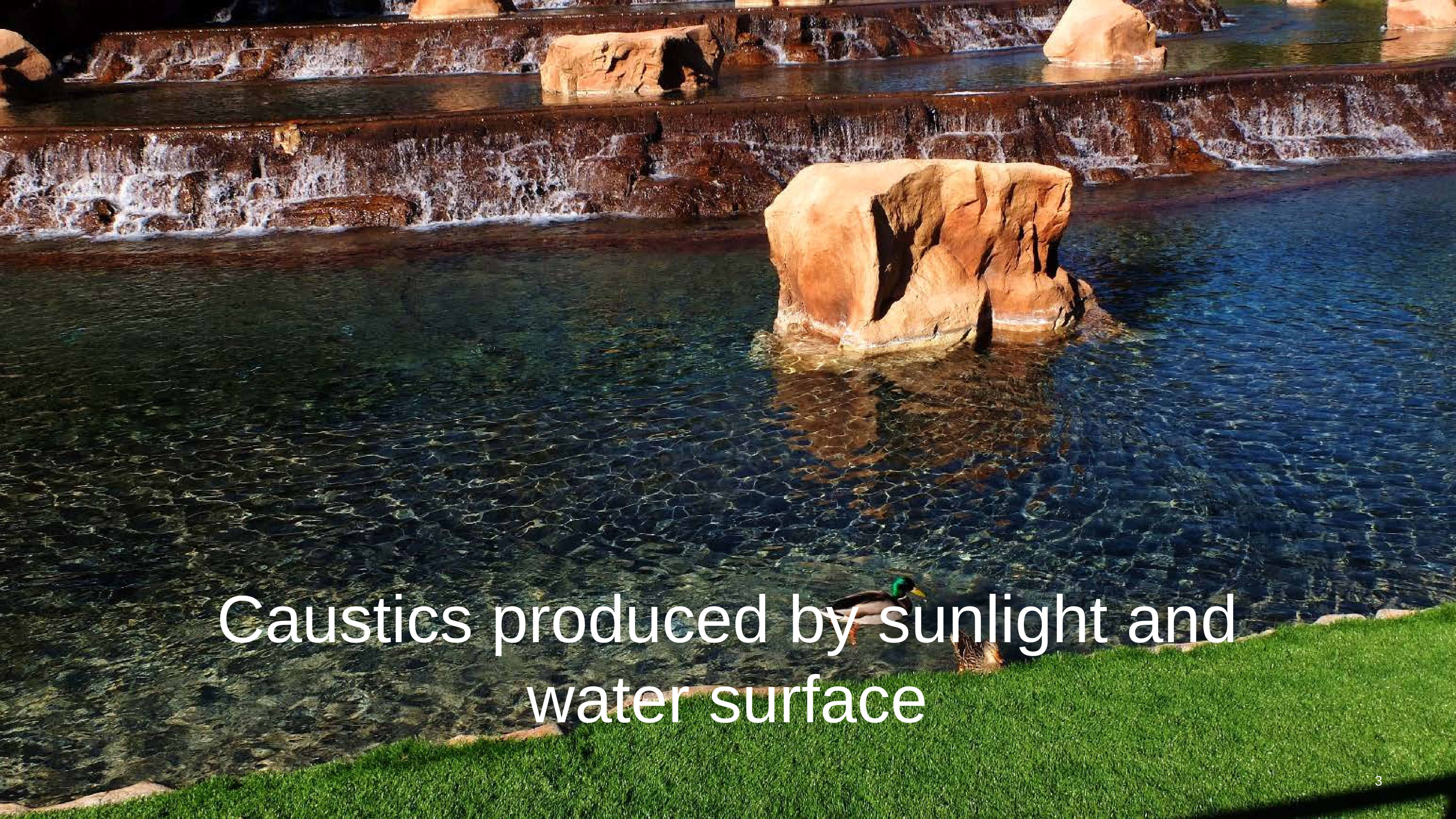
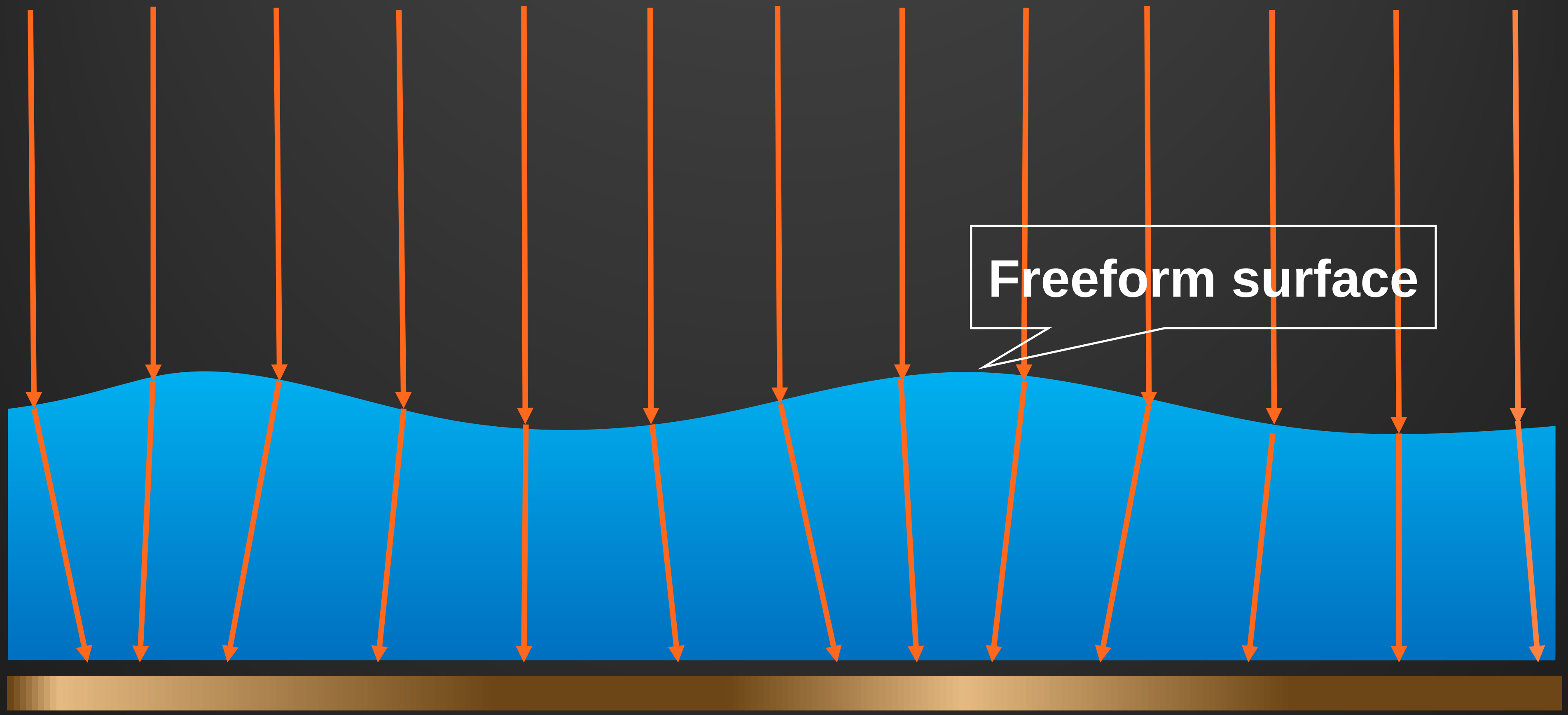
I. Introduction



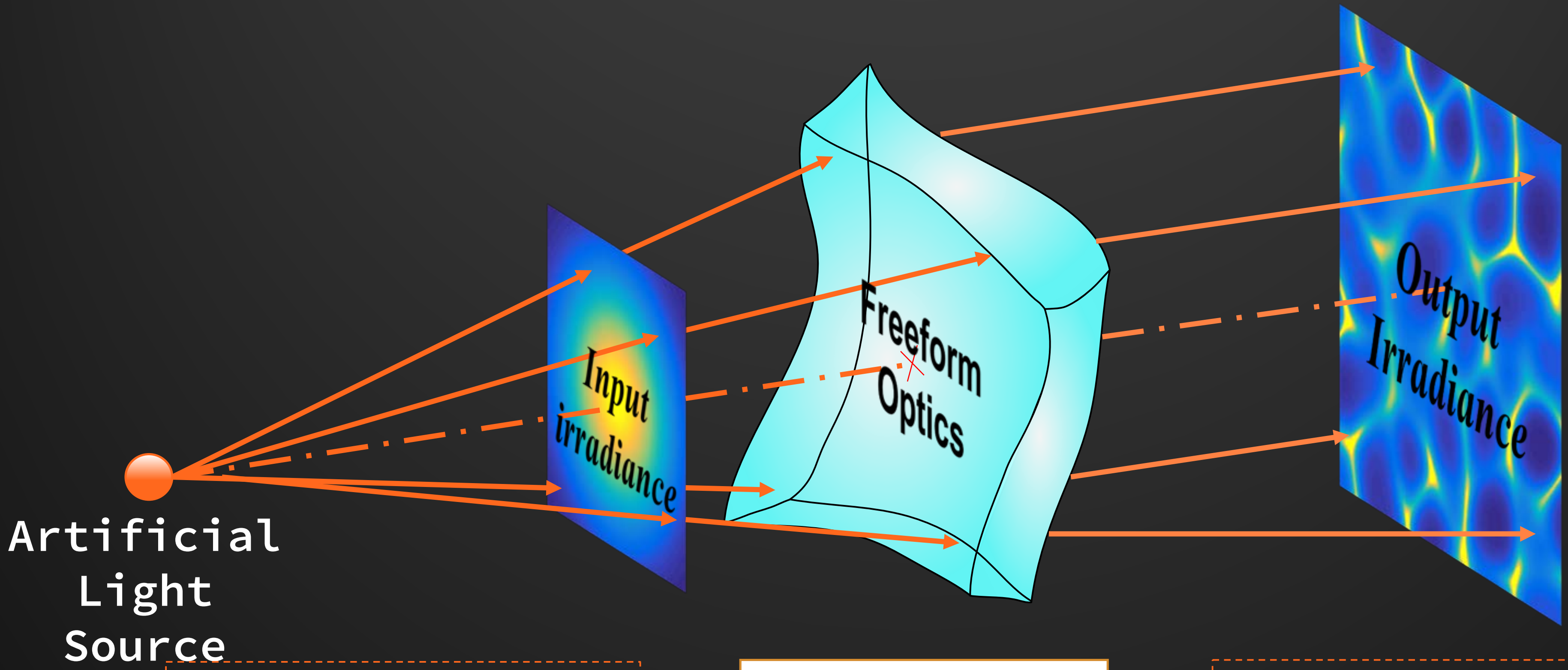
Caustics produced by sunlight and water surface



“Water’s wavy surface can be thought as a series of **positive** and **negative** lenses. The positive lenses focus sun light onto the bottom creating the bright network. The negative lenses refract light out of the beam and increase contrast of the network”

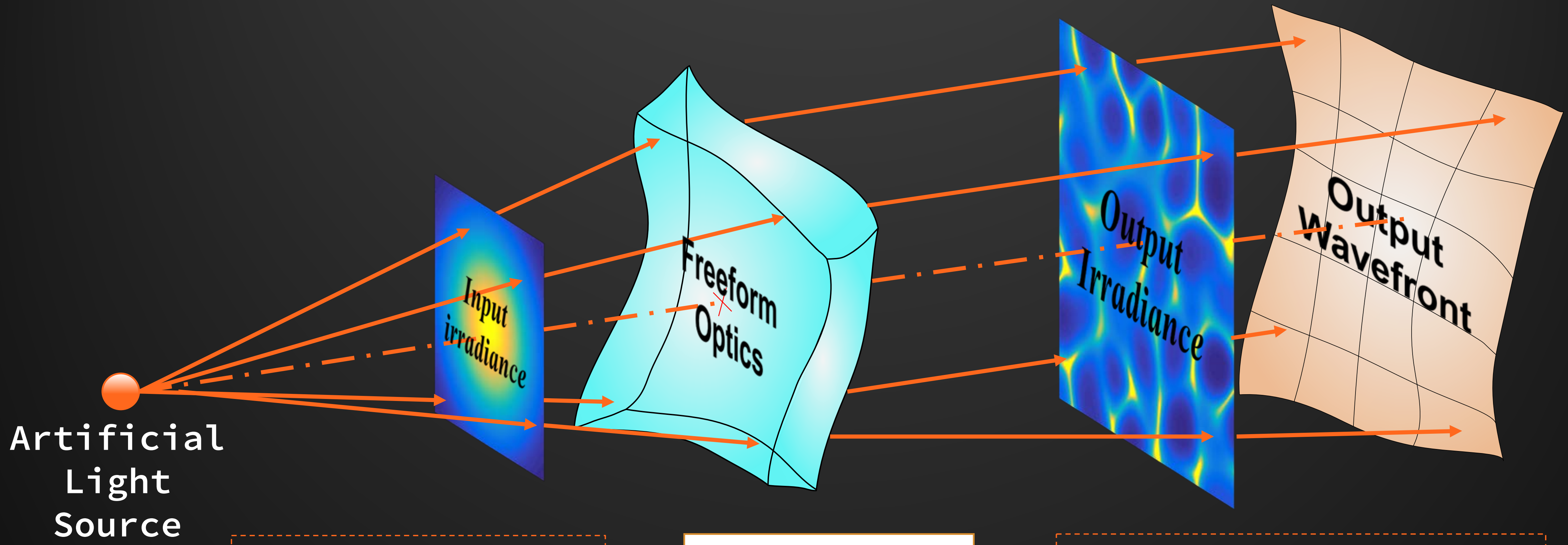
- D. K. Lynch and W. Livingston, Color and Light in Nature, (Cambridge university press, 1995), pp. 93-94.
- http://philippebompas.com/2014/07/architectural-caustics/

INVERSE PROBLEM FOR CONTROLLING **IRRADIANCE**



Input Beam Information → **Freeform Optics?** → Output Irradiance

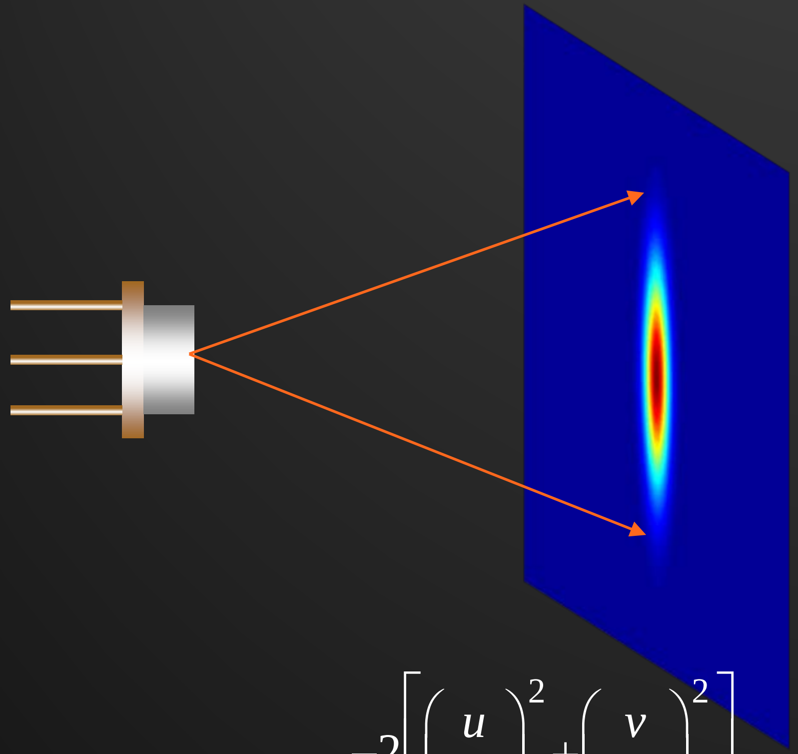
INVERSE PROBLEM FOR CONTROLLING BOTH **IRRADIANCE** & **WAVEFRONT**



Input Beam Information → **Freeform Optics?** → Output Irradiance & Wavefront

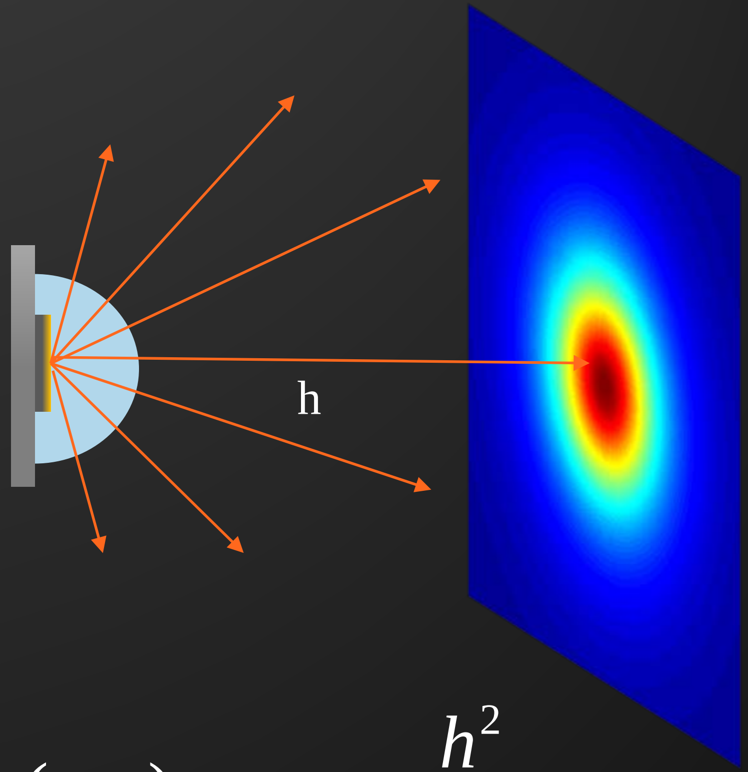
EVOLUTIONARY ARTIFICIAL LIGHT SOURCES

LASERs



$$I_{in}(u,v) \propto e^{-2\left[\left(\frac{u}{w_1}\right)^2 + \left(\frac{v}{w_2}\right)^2\right]}$$

LEDs



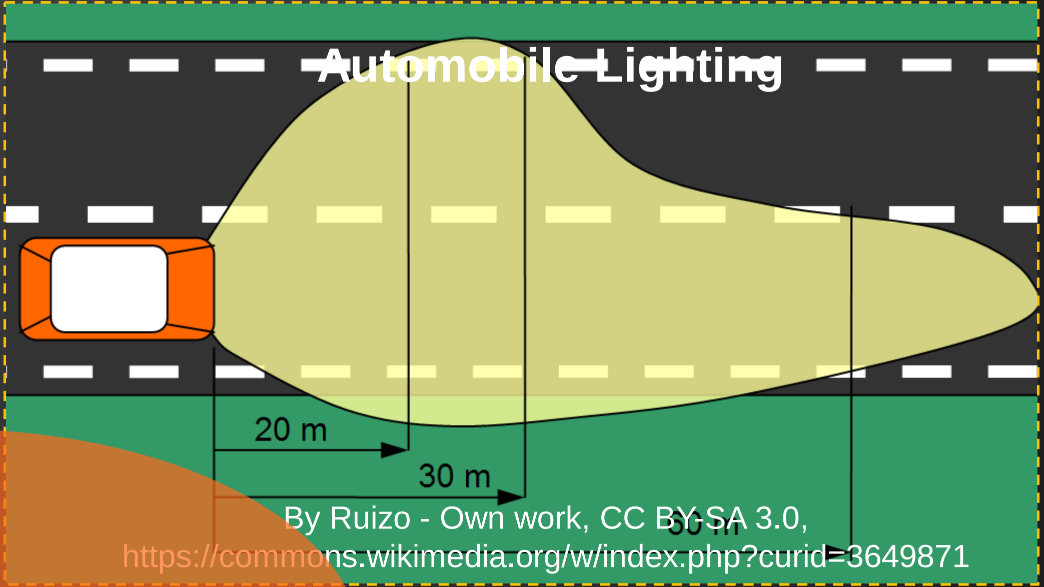
$$I_{in}(u,v) \propto \frac{h^2}{(u^2+v^2+h^2)^2}$$

REQUIREMENTS

Road Lighting

Y. Luo, “Energy-saving LED light sources,” SPIE Newsroom

Automobile Lighting



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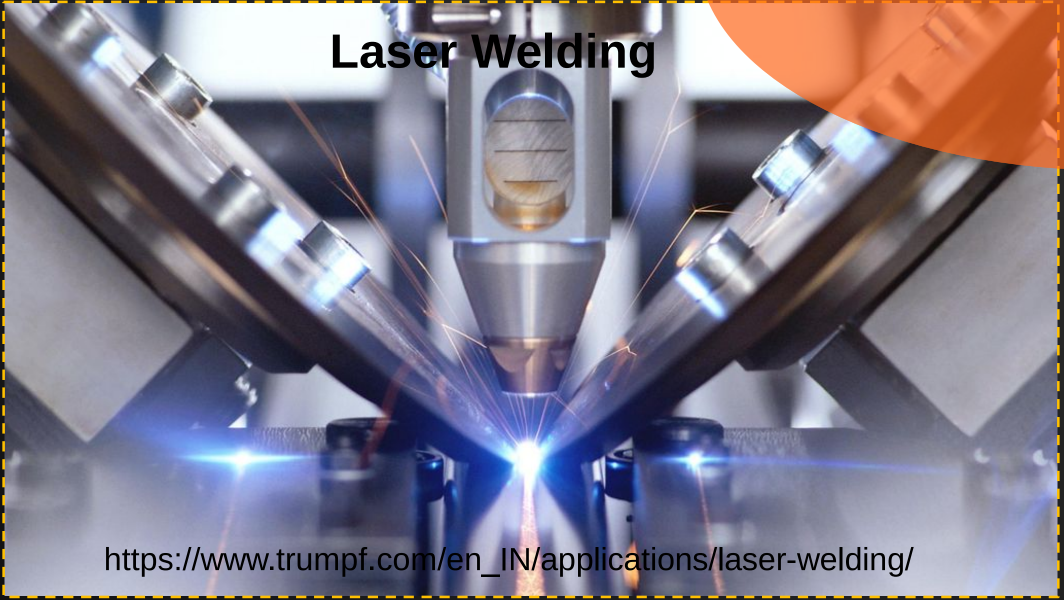
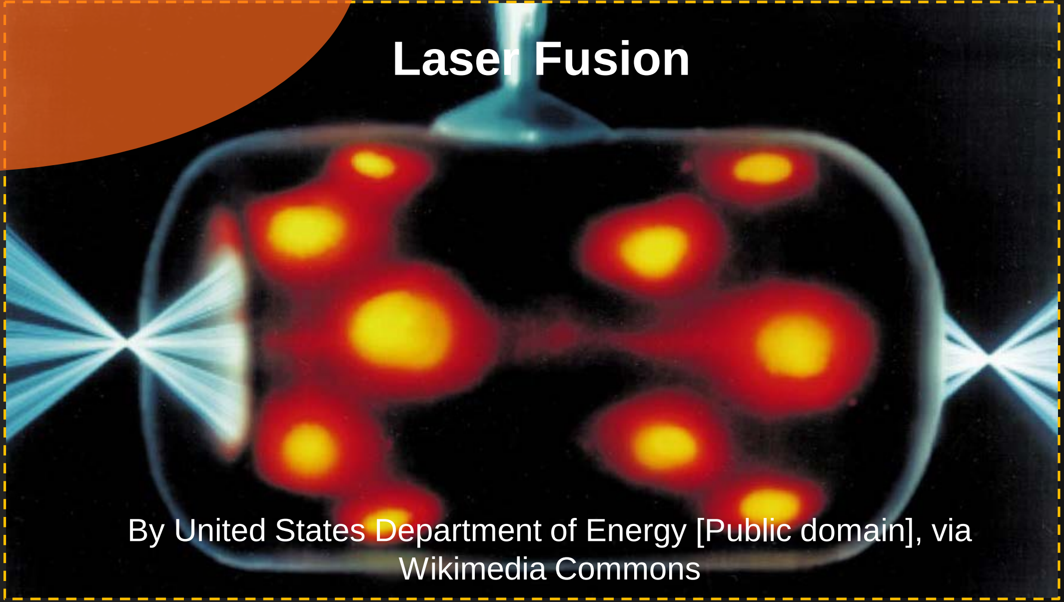
Laser Welding



https://www.trumpf.com/en_IN/applications/laser-welding/

Laser Fusion



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II. Formulation of the Inverse Problems

APPROXIMATIONS

Geometrical Optics

Point Light Source

Monotonic Ray Bending

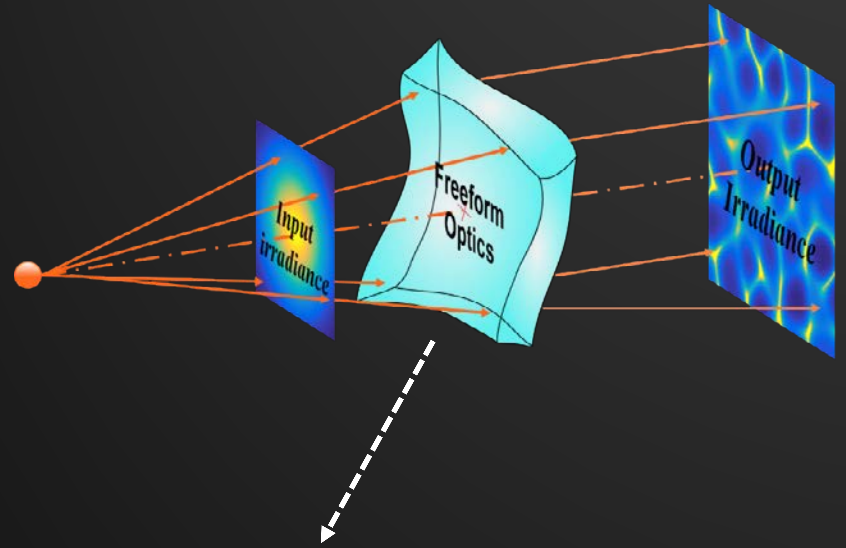
Lossless System

SOME FORMULATION REFERENCES

- J. S. Schruben, “Formulation of a reflector-design problem for a lighting fixture,”J. Opt. Soc. Am. 62, 1498-1501 (1972).
- R. Winston, J. C. Miñano, and P. Benítez, eds., Nonimaging Optics (Elsevier, 2005), PP. 174-178.
- H. Ries and J. Muschaweck, “Tailored freeform optical surfaces,” JOSA A 19(3), 590-595, (2002)
- H. Ries, “Laser beam shaping by double tailoring,” Proc. SPIE 5876, 587607, (2005)
- Wu, L. Xu, P. Liu, Y. Zhang, Z. Zheng, H. Li, and X. Liu, “Freeform illumination design: a nonlinear boundary problem for the elliptic Monge-Ampère equation,” Opt. Lett. **38**, 229-231 (2013).
- Y. Zhang, R. Wu, P. Liu, Z. Zheng, H. Li, and X. Liu, “Double freeform surfaces design for laser beam shaping with Monge-Ampère equation method,” Opt. Commun. 331, 297-305 (2014).
- S. Chang, R. Wu, A. Li and Z. Zheng, “Design beam shapers with double freeform surfaces to form a desired wavefront with prescribed illumination pattern by solving a Monge-Ampère type equation”J. Opt. 18, 125602 (2016).

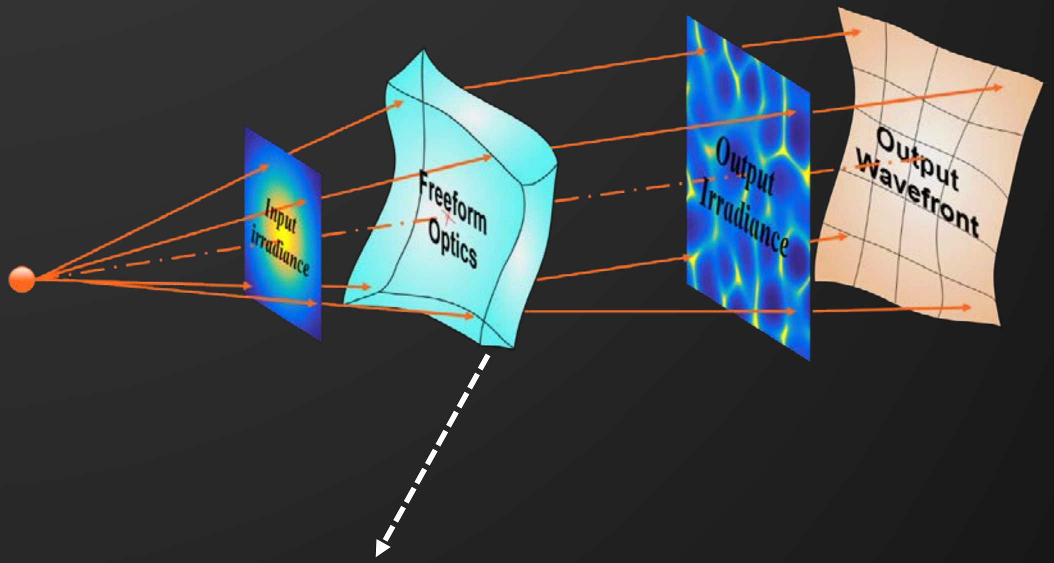
NUMBER OF FREEFORM SURFACES

Irradiance Control



At least **one** freeform surface

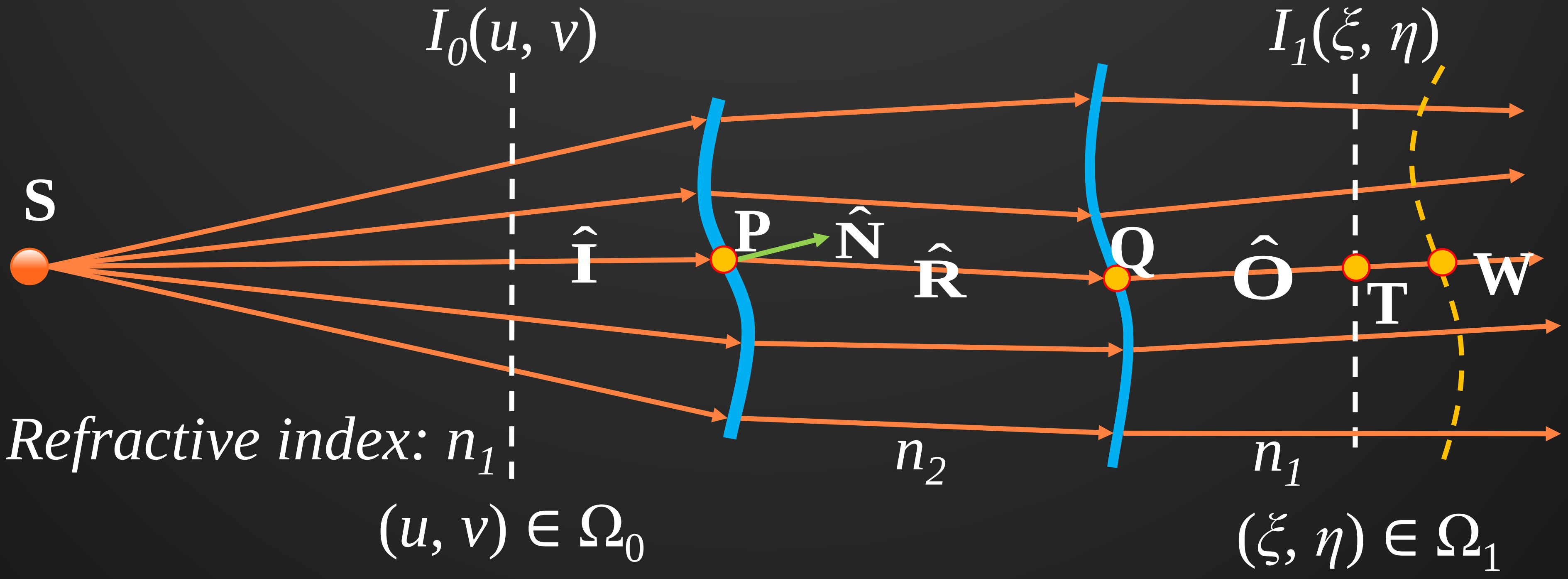
Irradiance & Wavefront Control



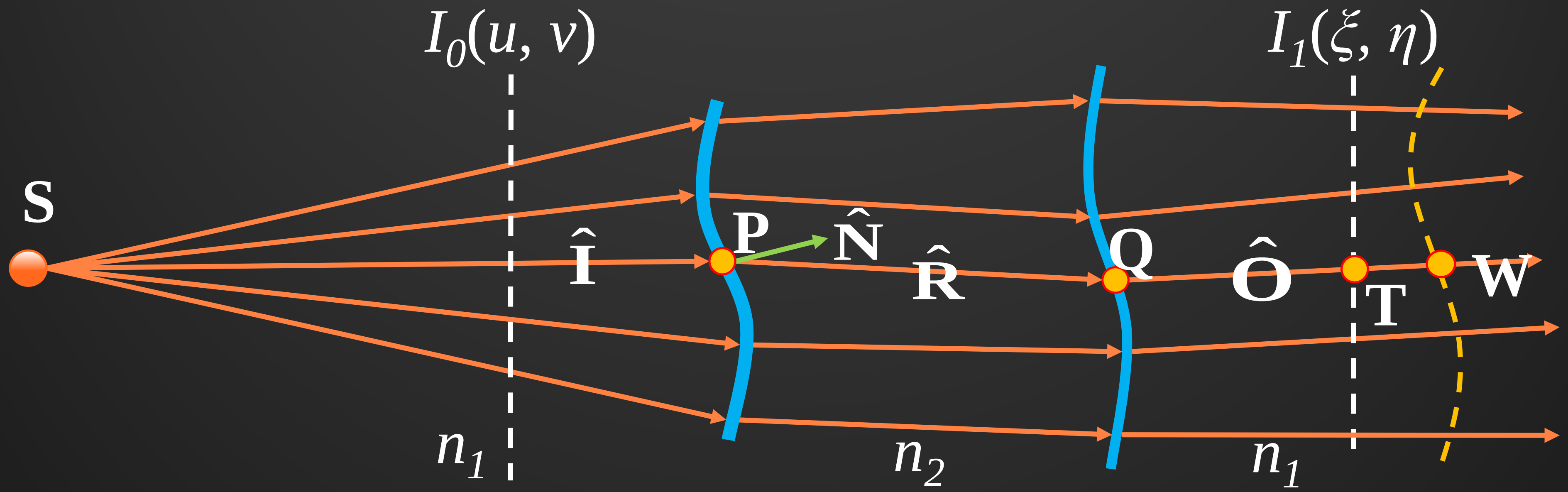
At least **two** freeform surfaces

If we use two freeform optical surfaces to control the irradiance only, we must provide an additional constraint.

EXAMPLE: DOUBLE FREEFORM SURFACE DESIGN FOR IRRADIANCE & WAVEFRONT CONTROL



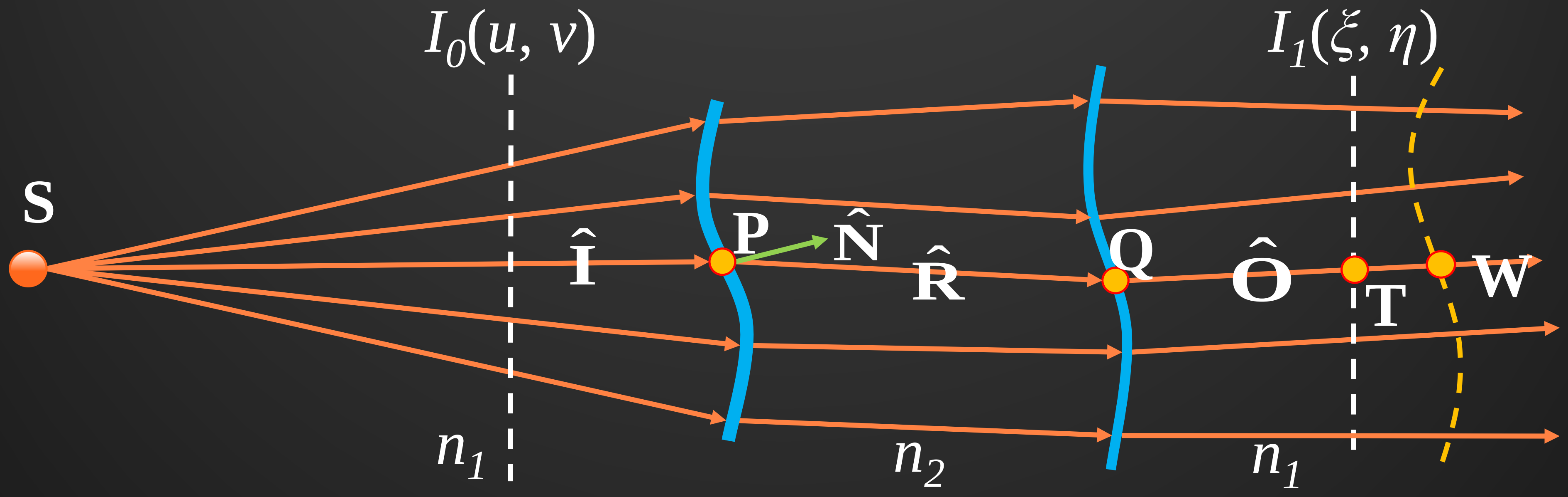
Energy Conservation



$$I_0(u,v)dudv = I_1(\xi,\eta)d\xi d\eta$$

$$\Rightarrow \quad I_0(u,v) = I_1(\xi,\eta)\left|\frac{\partial\xi}{\partial u}\frac{\partial\eta}{\partial v} - \frac{\partial\eta}{\partial u}\frac{\partial\xi}{\partial v}\right|$$

Ray Tracing Equations

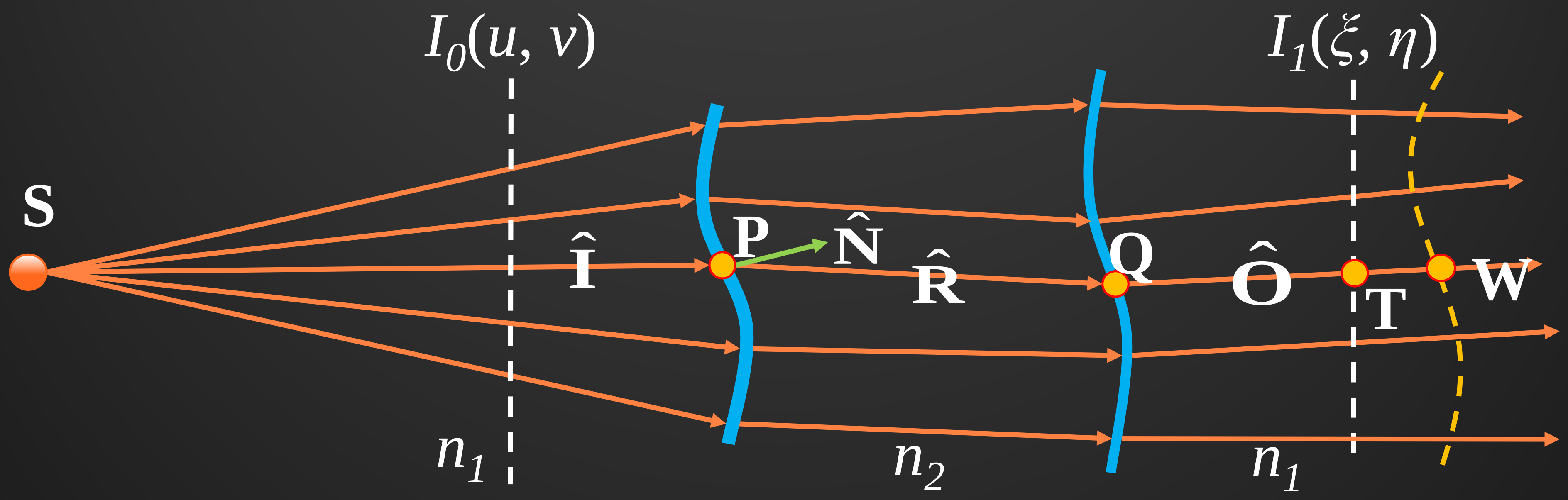


$$\begin{cases} \mathbf{P} = [\mathbf{O}, \mathbf{P}]\hat{\mathbf{I}} = \rho\hat{\mathbf{I}} \\ \mathbf{Q} = \mathbf{P} + [\mathbf{P}, \mathbf{Q}]\hat{\mathbf{R}} = \mathbf{P} + r\hat{\mathbf{R}} \\ \mathbf{W} = \mathbf{Q} + [\mathbf{Q}, \mathbf{W}]\hat{\mathbf{O}} = \mathbf{Q} + t\hat{\mathbf{O}} \end{cases}$$

$$\hat{\mathbf{R}} = \frac{n_1}{n_2}\hat{\mathbf{I}} + \gamma\hat{\mathbf{N}}$$ Snell's Law

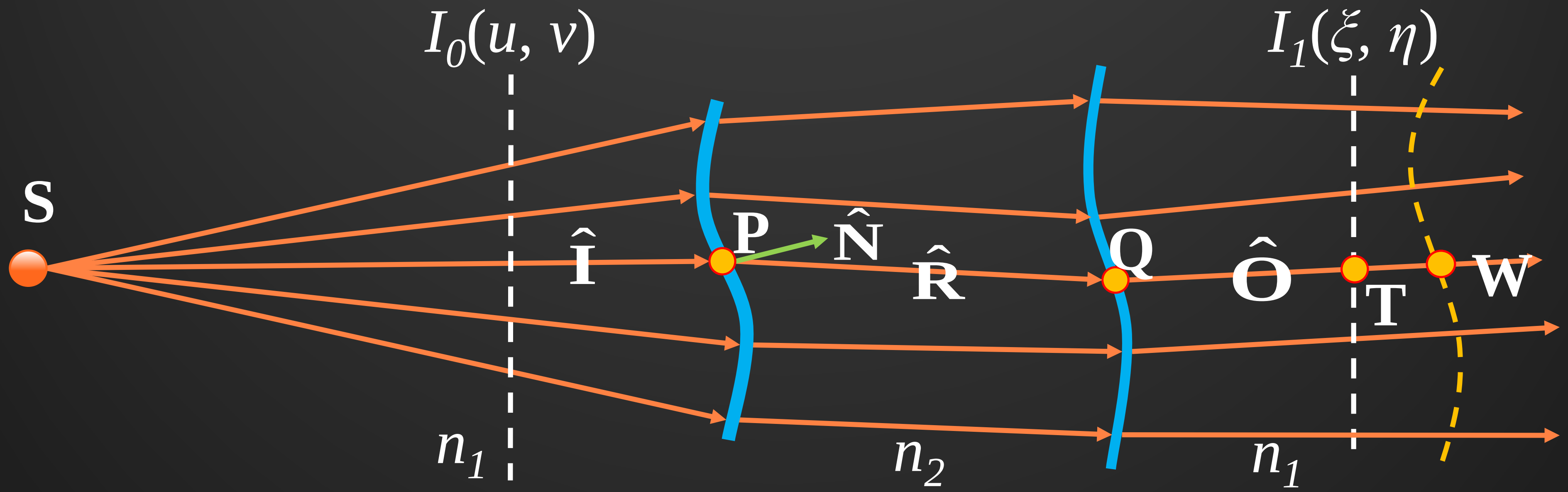
$$\gamma = -\frac{n_1}{n_2}(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}}) + \sqrt{1 - (\frac{n_1}{n_2})^2[1 - (\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})^2]}$$

OPL Constancy



$$n_1[\mathbf{S},\mathbf{P}] + n_2[\mathbf{P},\mathbf{Q}] + n_1[\mathbf{Q},\mathbf{W}] = Const$$

Surface Normals and Continuity



$$\hat{\mathbf{N}} = \left(\mathbf{P}_u \times \mathbf{P}_v\right) / \left|\mathbf{P}_u \times \mathbf{P}_v\right|$$

$$\hat{\mathbf{O}} = \left(\mathbf{W}_\xi \times \mathbf{W}_\eta\right) / \left|\mathbf{W}_\xi \times \mathbf{W}_\eta\right|$$

Parametric surface normals

$$\frac{\partial^2 \rho}{\partial u \partial v} = \frac{\partial^2 \rho}{\partial v \partial u}$$

Surface continuity

VERY COMPLICATED DERIVATION PROCESS

Ray Tracing Equations

+

Parametric Surface Normals

+

OPL Constancy

$$\begin{cases} \xi = \xi(u, v, \rho, \rho_u, \rho_v) \\ \eta = \eta(u, v, \rho, \rho_u, \rho_v) \end{cases}$$

Energy Conservation

$$I_0(u,v) = I_1(\xi,\eta)\left|\frac{\partial\xi}{\partial u}\frac{\partial\eta}{\partial v} - \frac{\partial\eta}{\partial u}\frac{\partial\xi}{\partial v}\right|$$

+

Surface Continuity

$$\frac{\partial^2\rho}{\partial u\partial v} = \frac{\partial^2\rho}{\partial v\partial u}$$

SECOND-ORDER NONLINEAR PDE

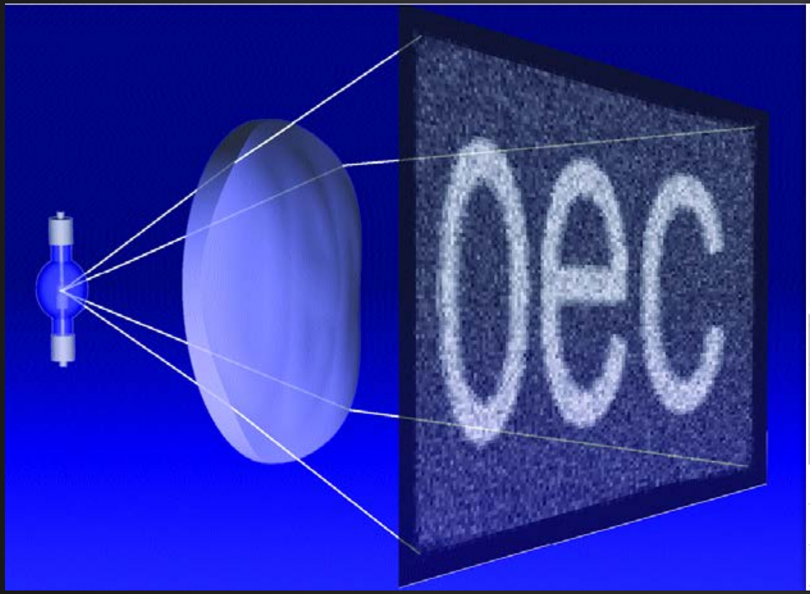
$$A\left[\frac{\partial^2 \rho}{\partial u^2}\frac{\partial^2 \rho}{\partial v^2} - \left(\frac{\partial^2 \rho}{\partial u \partial v}\right)^2\right] + B\frac{\partial^2 \rho}{\partial u^2} + 2C\frac{\partial^2 \rho}{\partial u \partial v} + D\frac{\partial^2 \rho}{\partial v^2} + E = 0$$

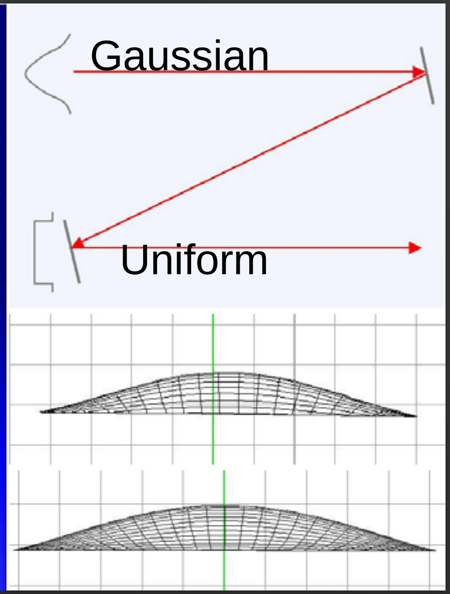
Boundary condition: $\begin{cases} \xi = \xi(u, v, \rho, \rho_u, \rho_v) \\ \eta = \eta(u, v, \rho, \rho_u, \rho_v) \end{cases} : \ \Omega_0 \to \Omega_1$

$$\begin{cases} A = A(u, v, \rho, \rho_u, \rho_v) \\ B = B(u, v, \rho, \rho_u, \rho_v) \\ C = C(u, v, \rho, \rho_u, \rho_v) \\ D = D(u, v, \rho, \rho_u, \rho_v) \\ E = E(u, v, \rho, \rho_u, \rho_v) \end{cases}$$

Monge–Ampère equation with tedious coefficients

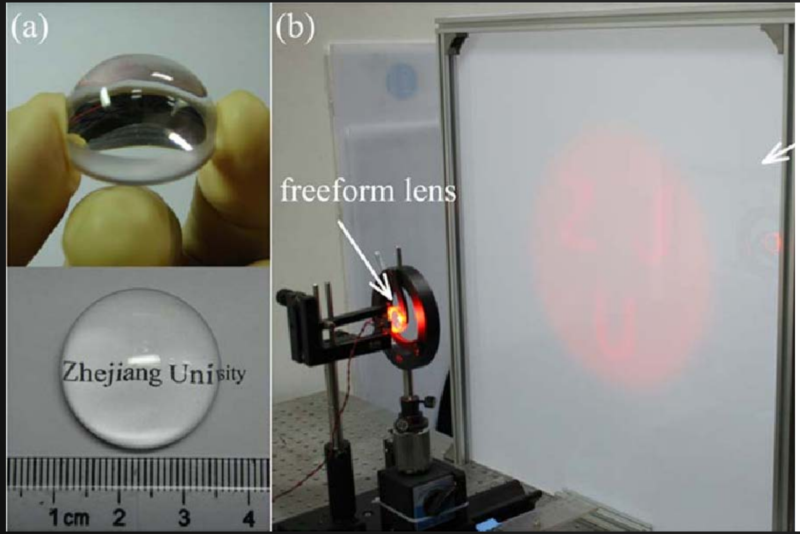
DIRECT DETERMINATION METHODS

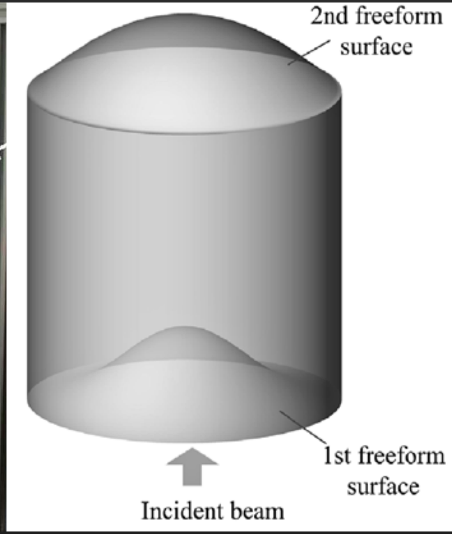




- H. Ries & J. Muschaweck, JOSA A 19, 590-595, (2002)
- H. Ries, Proc. SPIE 5876, 587607 (2005)

Numerical technique: Multi-grid algorithm

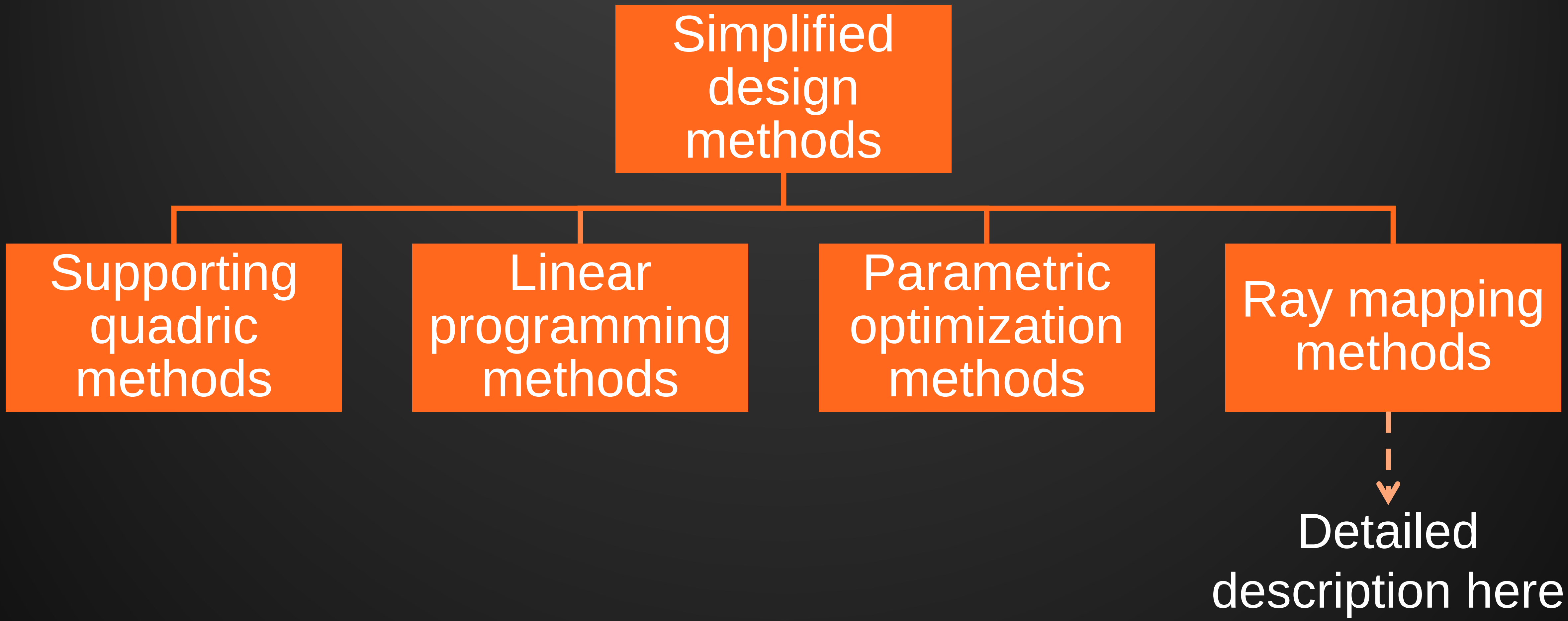




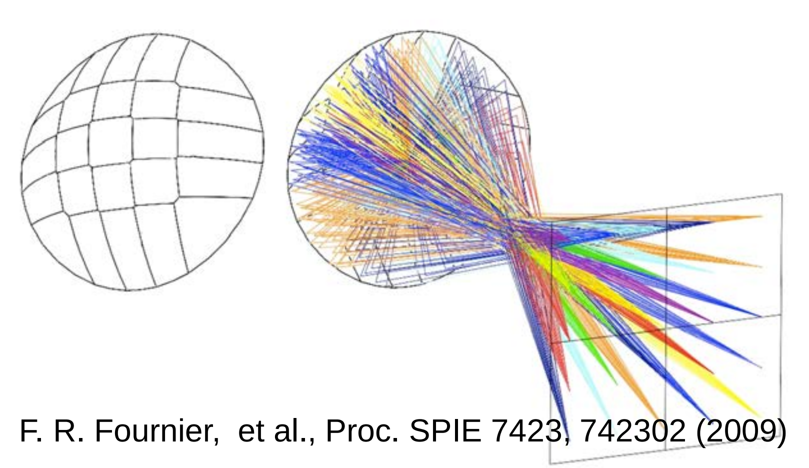
- R. Wu, et al., Opt. Lett. 38, 229-231 (2013).
- Y. Zhang, et al., Opt. Commun. 331, 297–305 (2014).

Numerical technique: Newton's method

III. Simplified Design Methods


Simplified design methods
Supporting quadric methods
Linear programming methods
Parametric optimization methods
Ray mapping methods
Detailed description here

SUPPORTING QUADRIC METHODS



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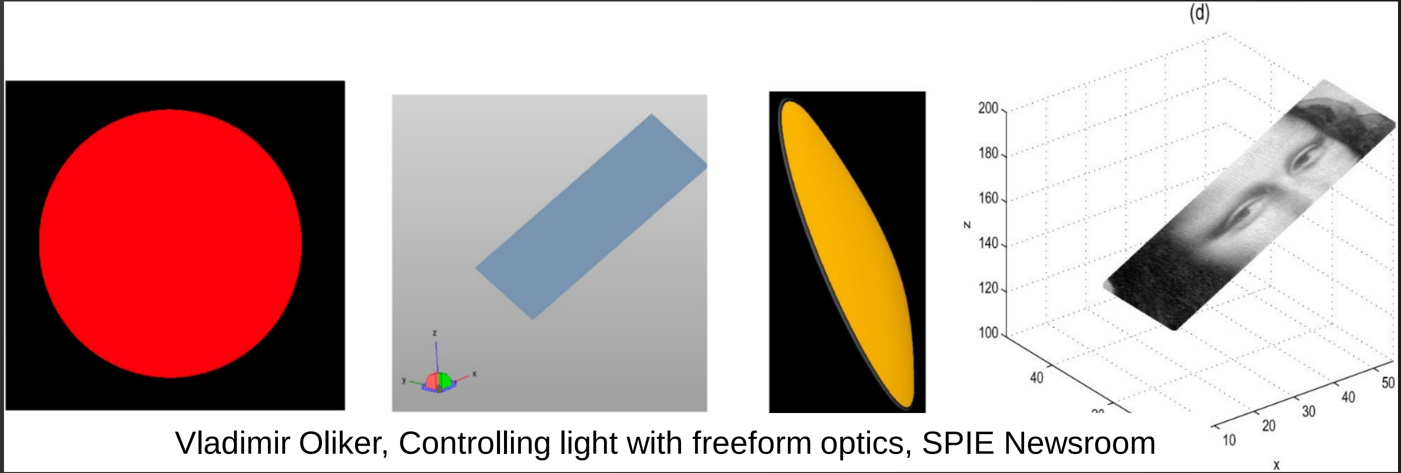
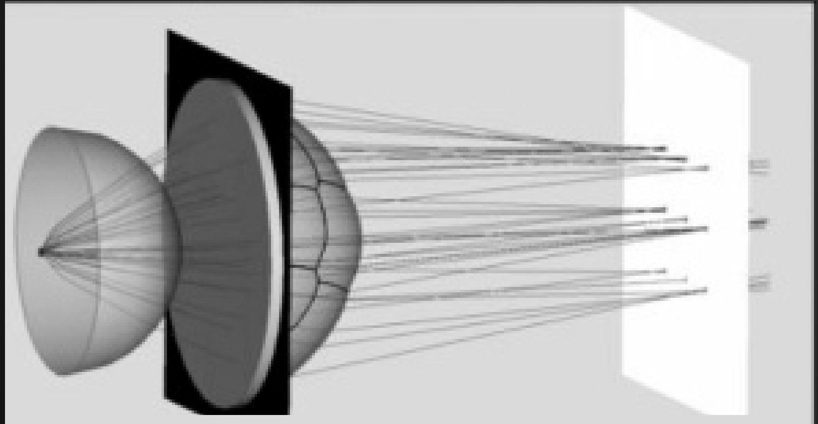


Vladimir Oliker, Controlling light with freeform optics, SPIE Newsroom

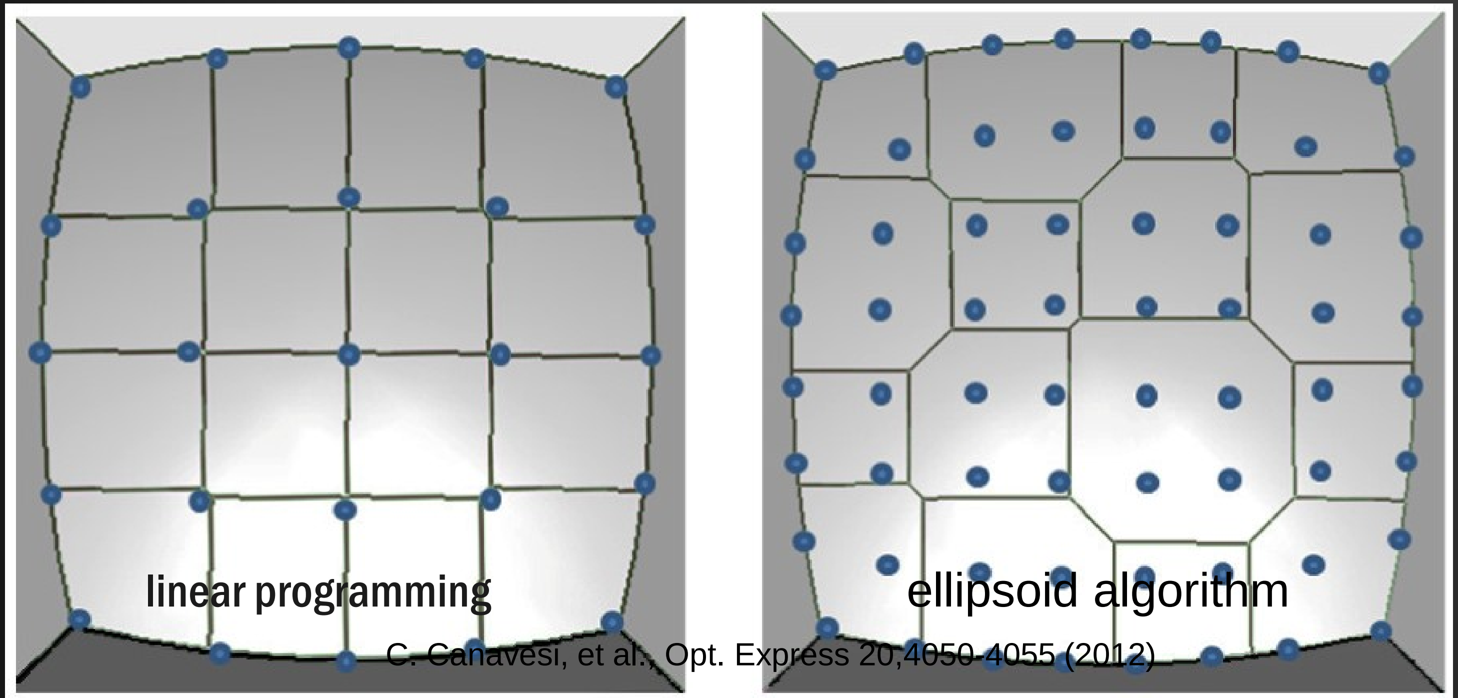


D. Michaelis, et al., Opt. Lett. 36, 918-920 (2011)

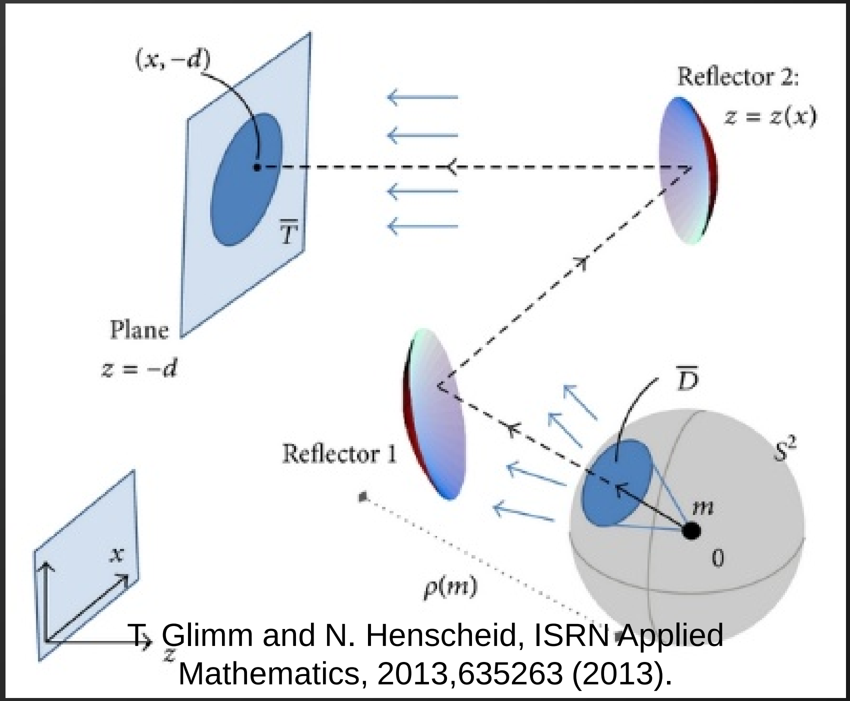
- V. I. Oliker, "Mathematical aspects of design of beam shaping surfaces in geometrical optics," Trends in Nonlinear Analysis, pp. 191–222 (2002).
- F. R. Fournier, et al., "Fast freeform reflector generation using source-target maps," Opt. Express 18, 5295-5304 (2010).
- D. Michaelis, et al., “Cartesian oval representation of freeform optics in illumination systems,” Opt. Lett. 36, 918-920 (2011)
- S. Magarill, “Skew-faceted elliptical reflector,” Opt. Lett. 36, 532-533 (2011).
- L. L. Doskolovich, et al., "Design of mirrors for generating prescribed continuous illuminance distributions on the basis of the supporting quadric method," Appl. Opt. 55, 687-695 (2016)
- V. Oliker, “Controlling light with freeform multifocal lens designed with supporting quadric method(SQM),” Opt. Express 25, A58-A72 (2017).

Quadrics: Cartesian ovals, ellipsoid, paraboloid, and hyperboloid

LINEAR PROGRAMMING METHODS



C. Canavesi, et al., Opt. Express 20,4050-4055 (2012)

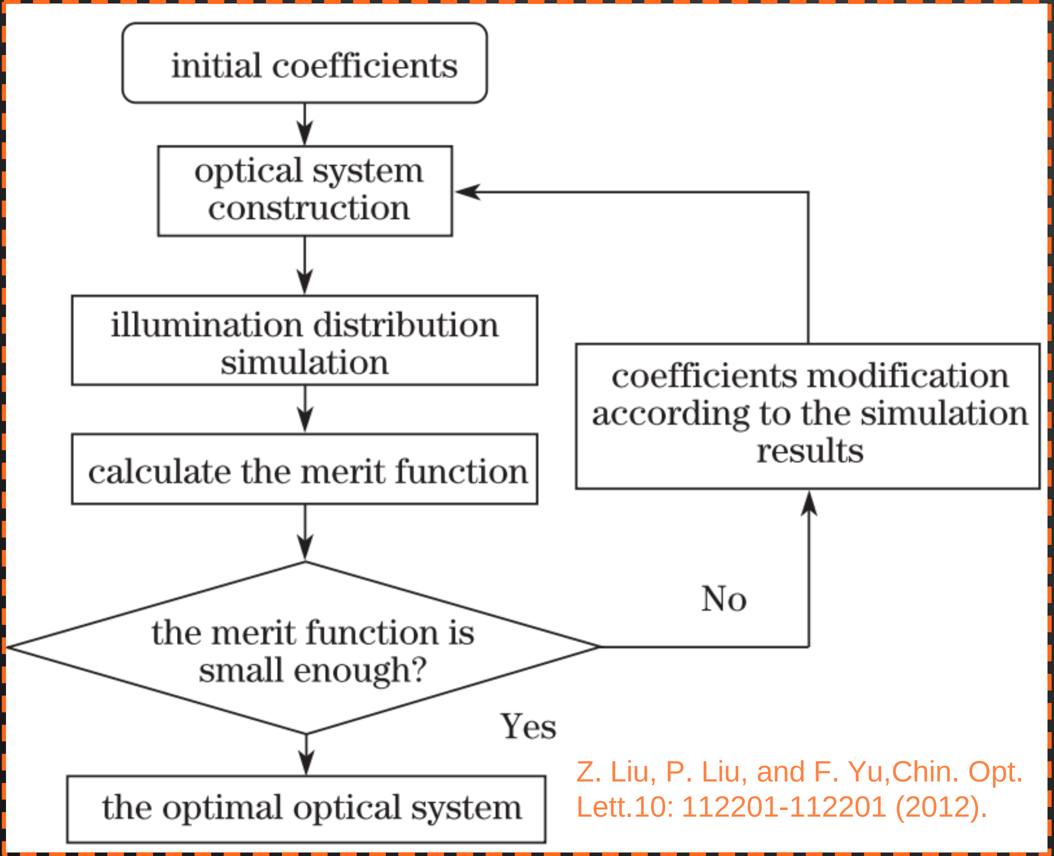


T. Glimm and N. Henscheid, ISRN Applied Mathematics, 2013,635263 (2013).

$$\max\ c^T x, \text{ s.t. } \mathrm{A}x \le b$$

- T. Glimm and V. Oliker, “Optical design of single reflector systems and the Monge-Kantorovich mass transfer problem”, J. of Mathematical Sciences, 117(3), 4096-4108 (2003).
- Xu-Jia Wang, “On the design of a reflector antenna II,” Calc. Var. 20, 329–341 (2004).
- V. Oliker, “Geometric and variational methods in optical design of reflecting surfaces with prescribed irradiance properties”, Proc. SPIE 5942, 594207 (2005).
- T. Glimm and N. Henscheid. Iterative Scheme for Solving Optimal Transportation Problems Arising in Reflector Design. ISRN Applied Mathematics, 2013,635263 (2013).
- C. Canavesi, W. J. Cassarly, and J. P. Rolland. Observations on the linear programming formulation of the single reflector design problem. Opt. Express 20,4050-4055 (2012)

PARAMATRIC OPTIMIZATION METHODS



Z. Liu, P. Liu, and F. Yu,Chin. Opt. Lett.10: 112201-112201 (2012).

$$\min f(v), \text{ s.t. } v \in K$$

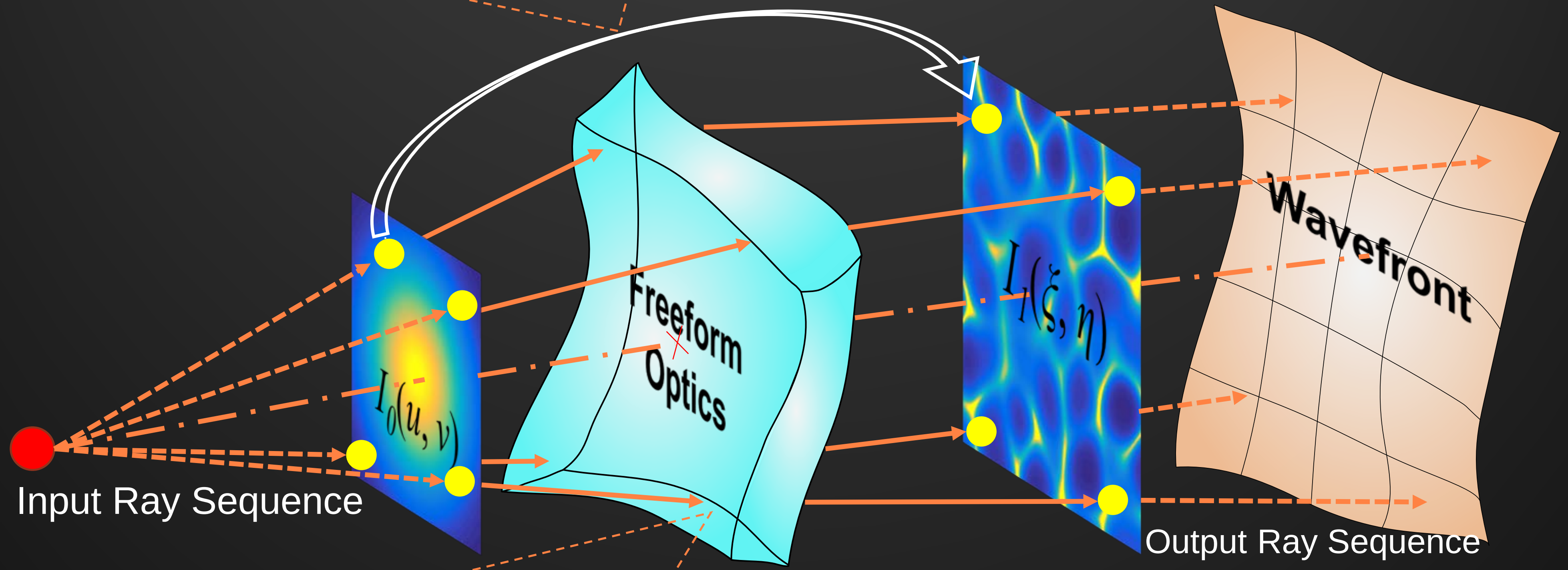
K is the feasible region

$$z = \sum_{i+j<N} v_{i,j} x^i y^j$$

- Pablo Benítez and Juan C. Miñano, "The Future of Illumination Design," Optics & Photonics News 18(5), 20-25 (2007).
- Z. Liu, P. Liu, and F. Yu, Parametric optimization method for the design of high-efficiency freeform illumination system with a LED source. Chin. Opt. Lett.10: 112201-112201 (2012).

RAY MAPPING METHODS

Step 1: Find an appropriate ray Map : $\xi = \xi(u, v), \quad \eta = \eta(u, v)$



Step 2: Construct the freeform optics following the ray map

VARIABLE SEPARABLE RAY MAP

$$I_0(u,v)dudv = I_1(\xi,\eta)d\xi d\eta$$

$$\Downarrow$$

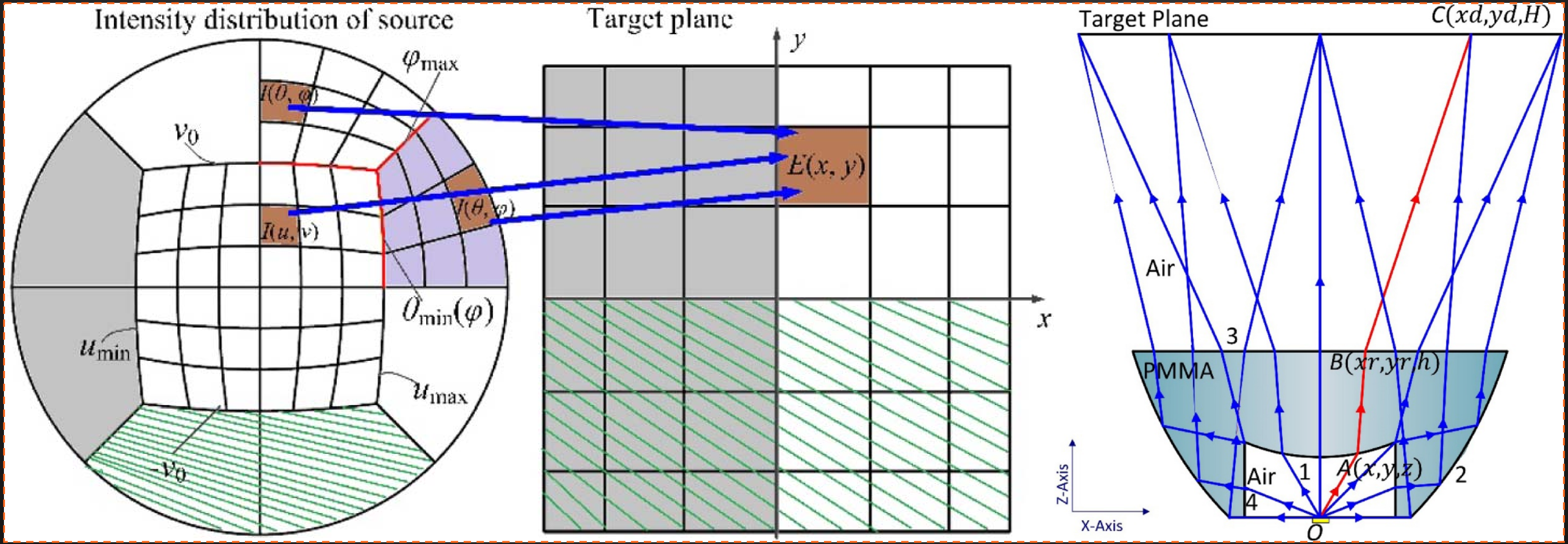
$$I_{0,u}(u)I_{0,v}(v)dudv = I_{1,\xi}(\xi)I_{1,\eta}(\eta)d\xi d\eta$$

$$\Downarrow$$

$$\boxed{\xi = \xi(u),\ \ \eta = \eta(v)}$$

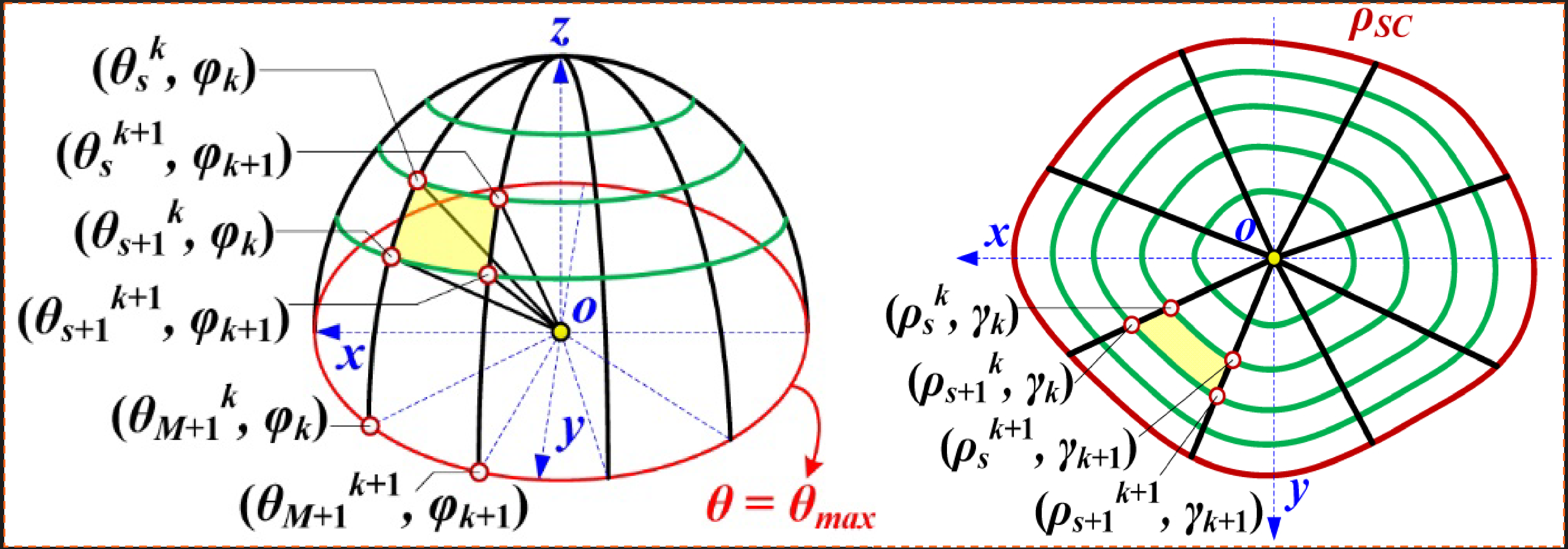
- W. A. Parkyn, “Illumination lenses designed by extrinsic differential geometry”, SPIE 3482, 389-396 (1998).
- D. L. Shealy and S. Chao, “Geometric optics-based design of laser beam shapers,” Opt. Eng. 42, 3123–3138 (2003).
- L. Wang, K. Qian and Y. Luo, “Discontinuous free-form lens design for prescribed irradiance”, Appl. Opt. 46, 3716-3723 (2007).
- Y. Ding, X. Liu, Z. R. Zheng, and P. F. Gu, “Freeform LED lens for uniform illumination,” Opt. Express 16, 12958-12966 (2008).

COMPOSITE RAY MAP



D. Ma, Z. Feng, and R. Liang, "Freeform illumination lens design using composite ray mapping," Appl. Opt. 54, 498-503 (2015)

POLAR-GRIDS MAP

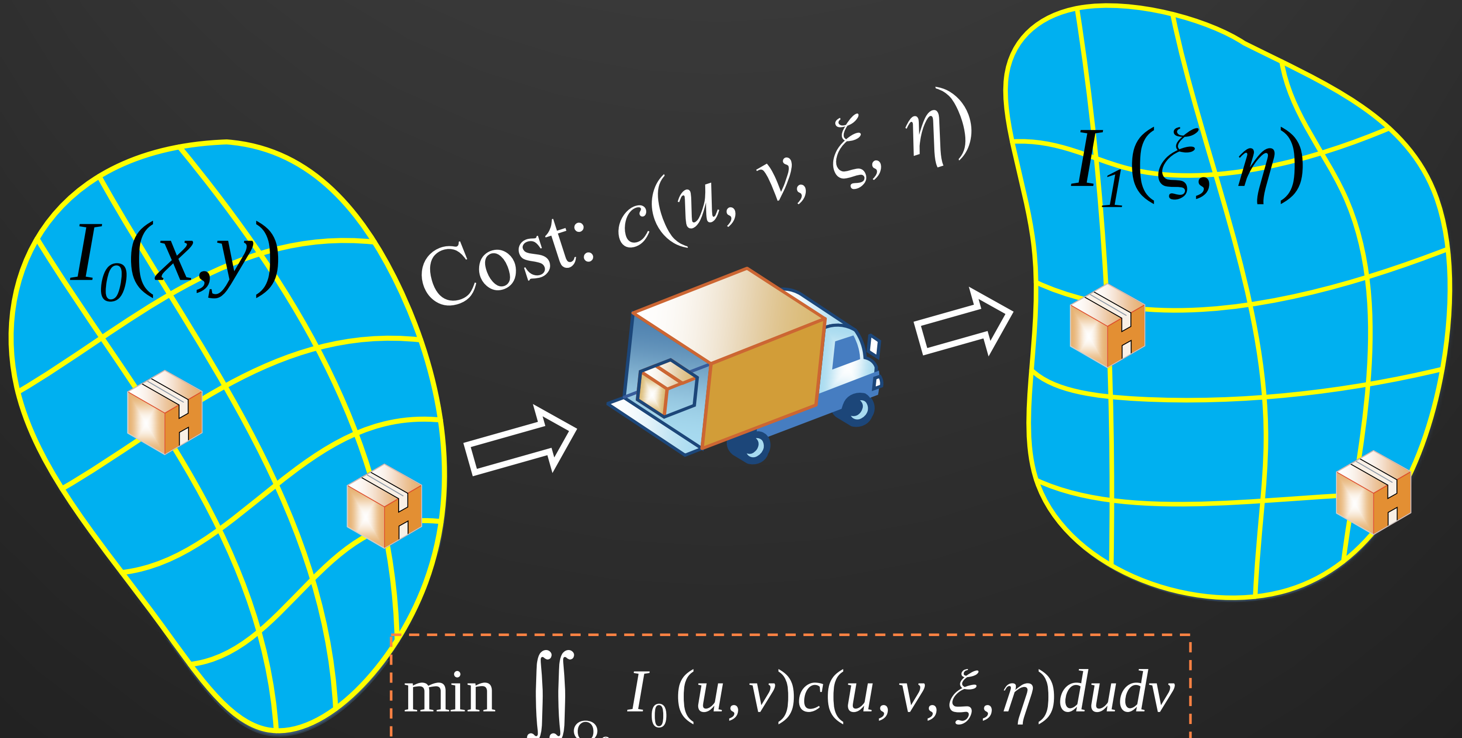


Grid division of the source based on spherical coordinates

Target division of the source based on polar coordinates using non-uniform sampling

X. Mao, H. Li, Y. Han, and Y. Luo, "Polar-grids based source-target mapping construction method for designing freeform illumination system for a lighting target with arbitrary shape," Opt. Express 23, 4313-4328 (2015)

OPTIMAL TRANSPORT MAP



$$\min \iint_{\Omega_0} I_0(u,v)c(u,v,\xi,\eta)dudv$$

$$\text{s.t. } I_0(u,v)dudv = I_1(\xi,\eta)d\xi d\eta$$

- J. Rubinstein and G. Wolansky, "Intensity control with a free-form lens," J. Opt. Soc. Am. A 24, 463-469 (2007).
- A. Bruneton, A. Bäuerle, P. Loosen, and R. Wester, "Freeform lens for an efficient wall washer," Proc. SPIE 8167, 816707 (2011).
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- A. Bruneton, A. Bäuerle, R. Wester, J. Stollenwerk, and P. Loosen, "High resolution irradiance tailoring using multiple freeform surfaces,“ Opt. Express 21, 10563-10571 (2013).
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- Z. Feng, B. D. Froese, and R. Liang, "Freeform illumination optics construction following an optimal transport map," Appl. Opt. 55, 4301-4306 (2016).
- C. Bösel and H. Gross, "Ray mapping approach for the efficient design of continuous freeform surfaces," Opt. Express 24, 14271-14282 (2016).

L^2 OPTIMAL TRANSPORT

$$c(u,v,\xi,\eta)=\frac{1}{2}[(\xi-u)^2+(\eta-v)^2]$$

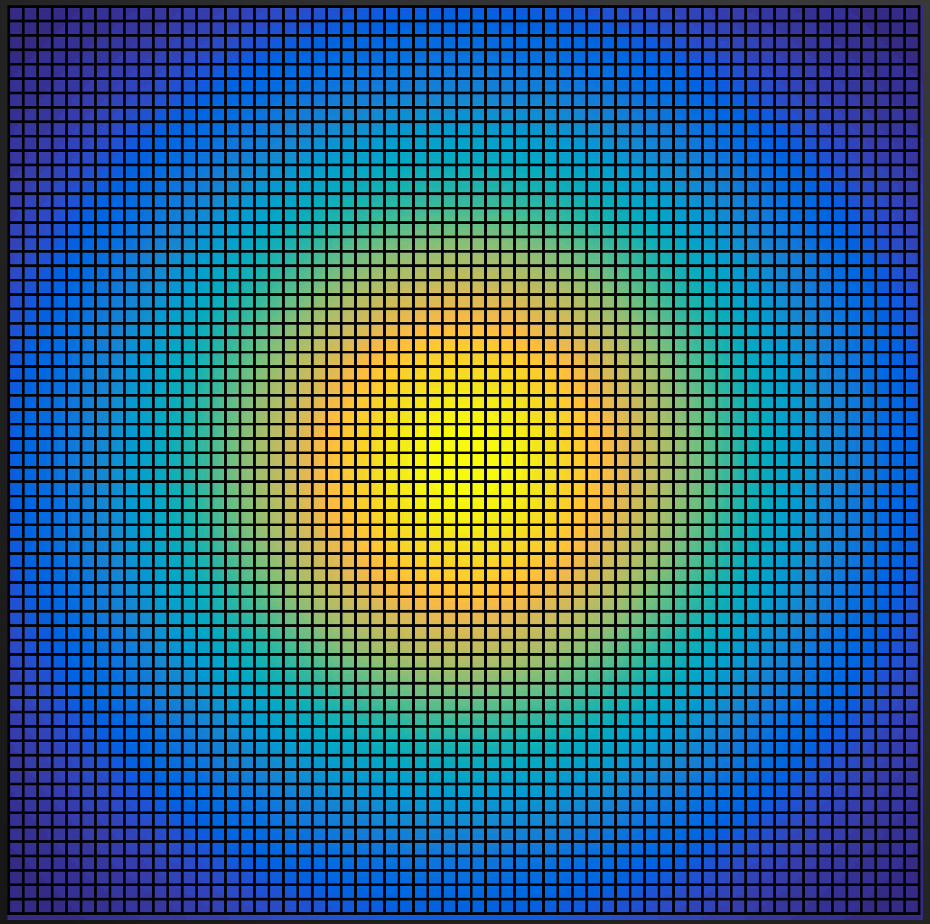
$$\frac{\partial^2\phi}{\partial u^2}\frac{\partial^2\phi}{\partial v^2}-\left(\frac{\partial^2\phi}{\partial u\partial v}\right)^2=\frac{I_0(u,v)}{I_1(\nabla\phi)}$$

$$\nabla\phi:\Omega_0\to\Omega_1$$

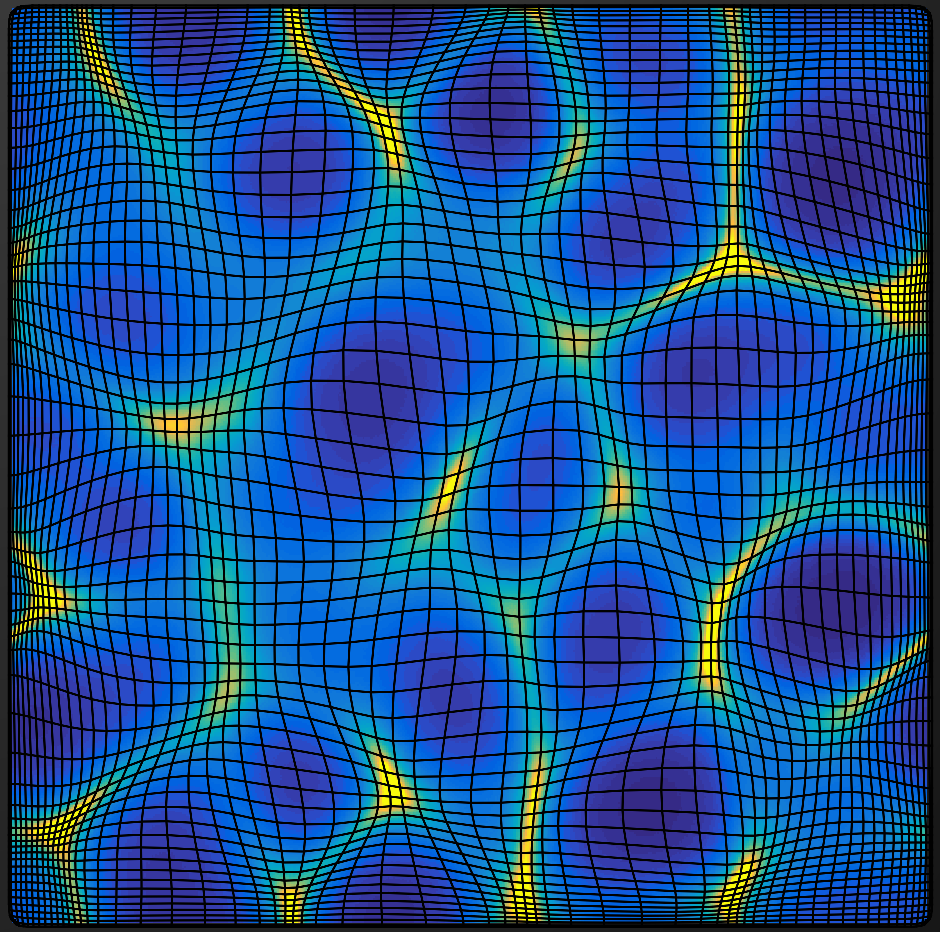
Standard Monge–Ampère Equation

- R. T. Rockafellar, “Characterization of the subdifferentials of convex functions,” Pacific J. Math. 17, 497-510 (1966).
- Robert J. McCann, “Existence and uniqueness of monotone measure-preserving maps,” Duke Math. J., 80, 309-323 (1995).

OPTIMAL TRANSPORT MAP



$I_0(u, v)$



$I_1(\xi, \eta)$

J. D. Benamou, B. D. Froese, and A. M. Oberman, “Numerical solution of the optimal ransportation problem using the Monge-Ampère equation,” J. Comput. Phys. 260, 107–126 (2014).

SURFACE CONSTRUCTION

Point-by-point

- W. A. Parkyn, “Illumination lenses designed by extrinsic differential geometry”, SPIE 3482, 389-396 (1998).
- L. Wang, K. Qian and Y. Luo, “Discontinuous free-form lens design for prescribed irradiance”, Appl. Opt. 46, 3716-3723 (2007).
- Z. Feng, L. Huang, G. Jin, and M. Gong, "Designing double freeform optical surfaces for controlling both irradiance and wavefront," Opt. Express 21, 28693-28701 (2013).

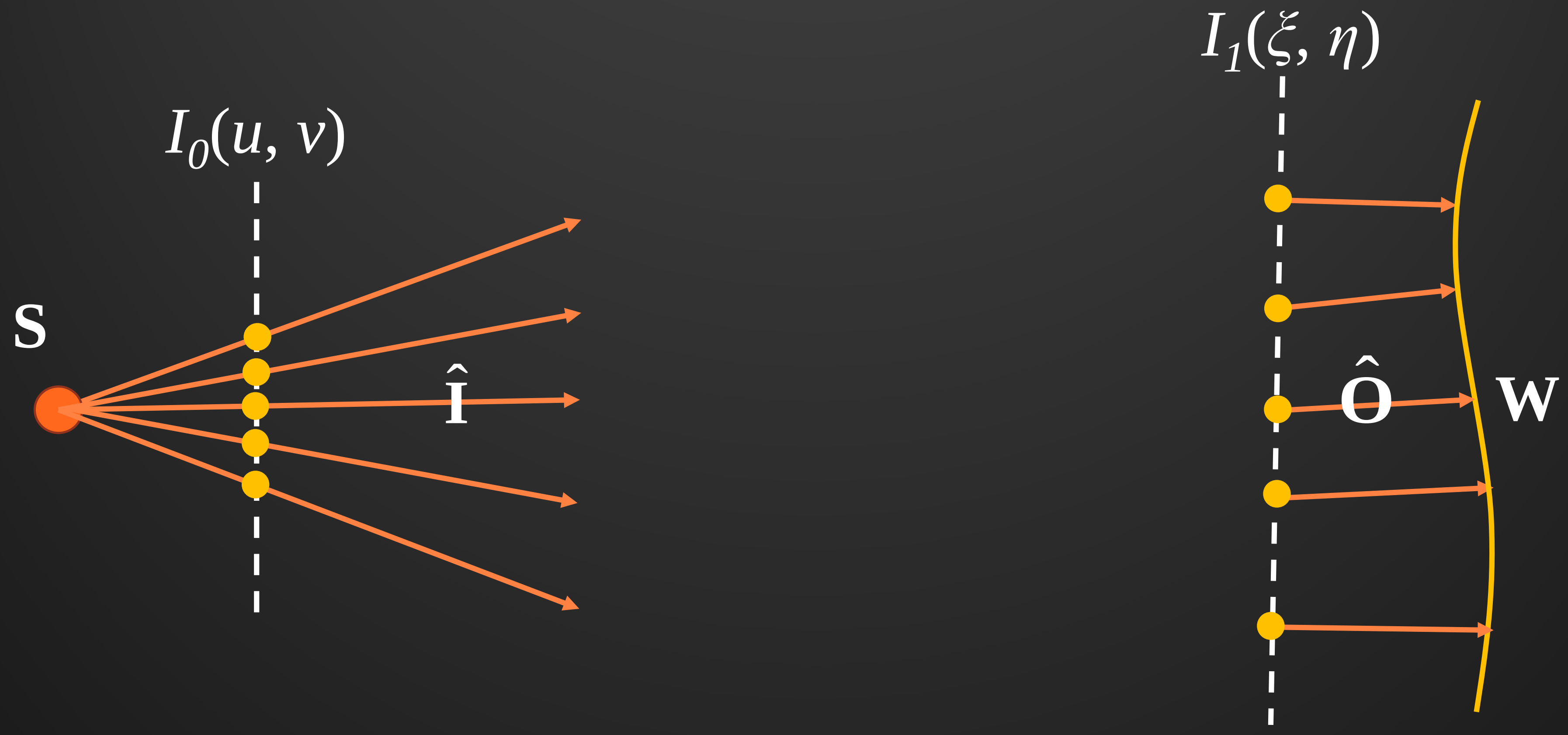
First-order PDE

- D. L. Shealy and S. Chao, “Geometric optics-based design of laser beam shapers,” Opt. Eng. 42, 3123–3138 (2003).
- Y. Ding, X. Liu, Z. R. Zheng, and P. F. Gu, “Freeform LED lens for uniform illumination,” Opt. Express 16, 12958-12966 (2008).

Least squares

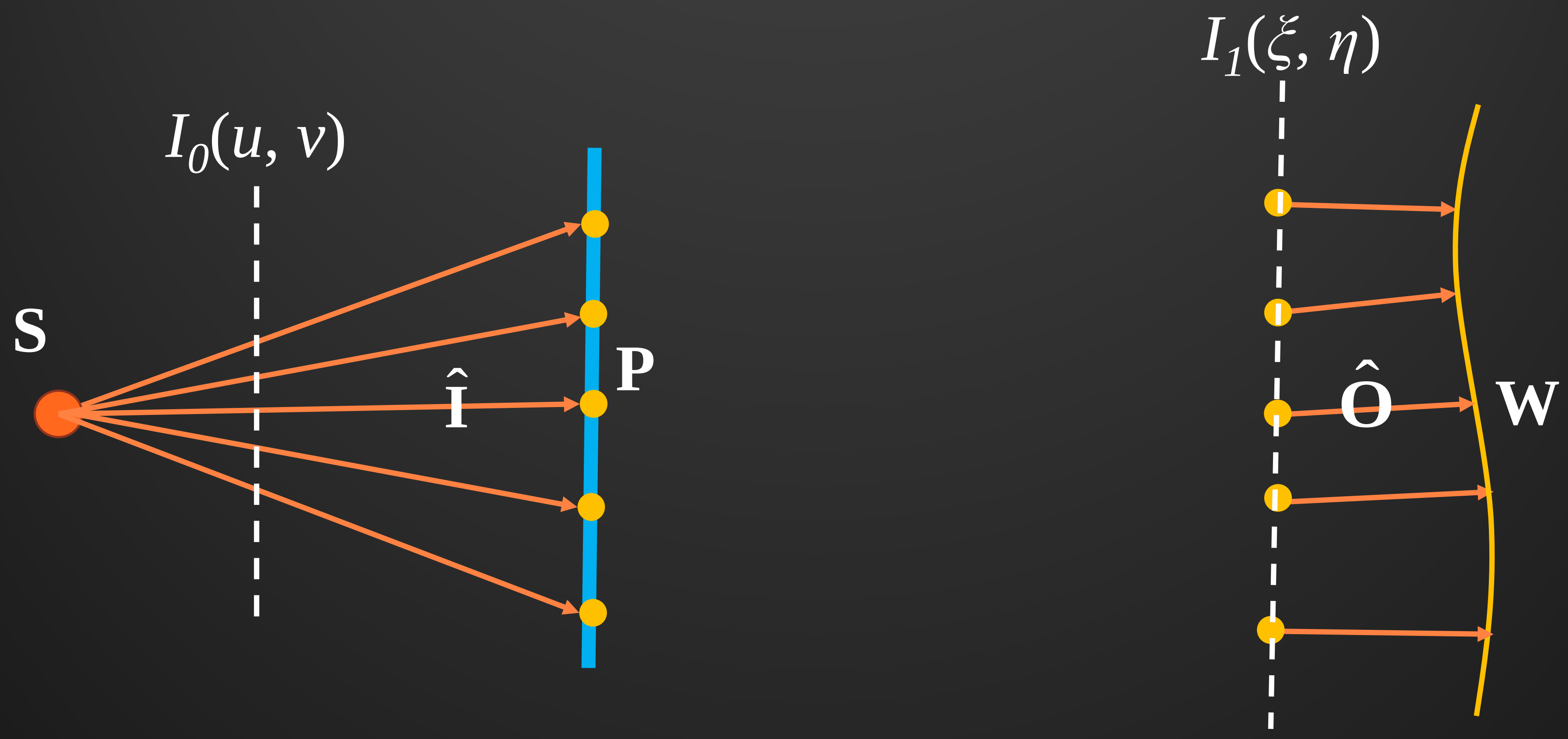
- A. Bruneton, A. Bäuerle, P. Loosen, and R. Wester, "Freeform lens for an efficient wall washer," Proc. SPIE 8167, 816707 (2011).
- A. Bäuerle, A. Bruneton, R. Wester, J. Stollenwerk, and P. Loosen, "Algorithm for irradiance tailoring using multiple freeform optical surfaces," Opt. Express 20, 14477-14485 (2012).
- A. Bruneton, A. Bäuerle, R. Wester, J. Stollenwerk, and P. Loosen, "High resolution irradiance tailoring using multiple freeform surfaces,“ Opt. Express 21, 10563-10571 (2013).
- Z. Feng, B. D. Froese, and R. Liang, "Freeform illumination optics construction following an optimal transport map," Appl. Opt. 55, 4301-4306 (2016)

ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



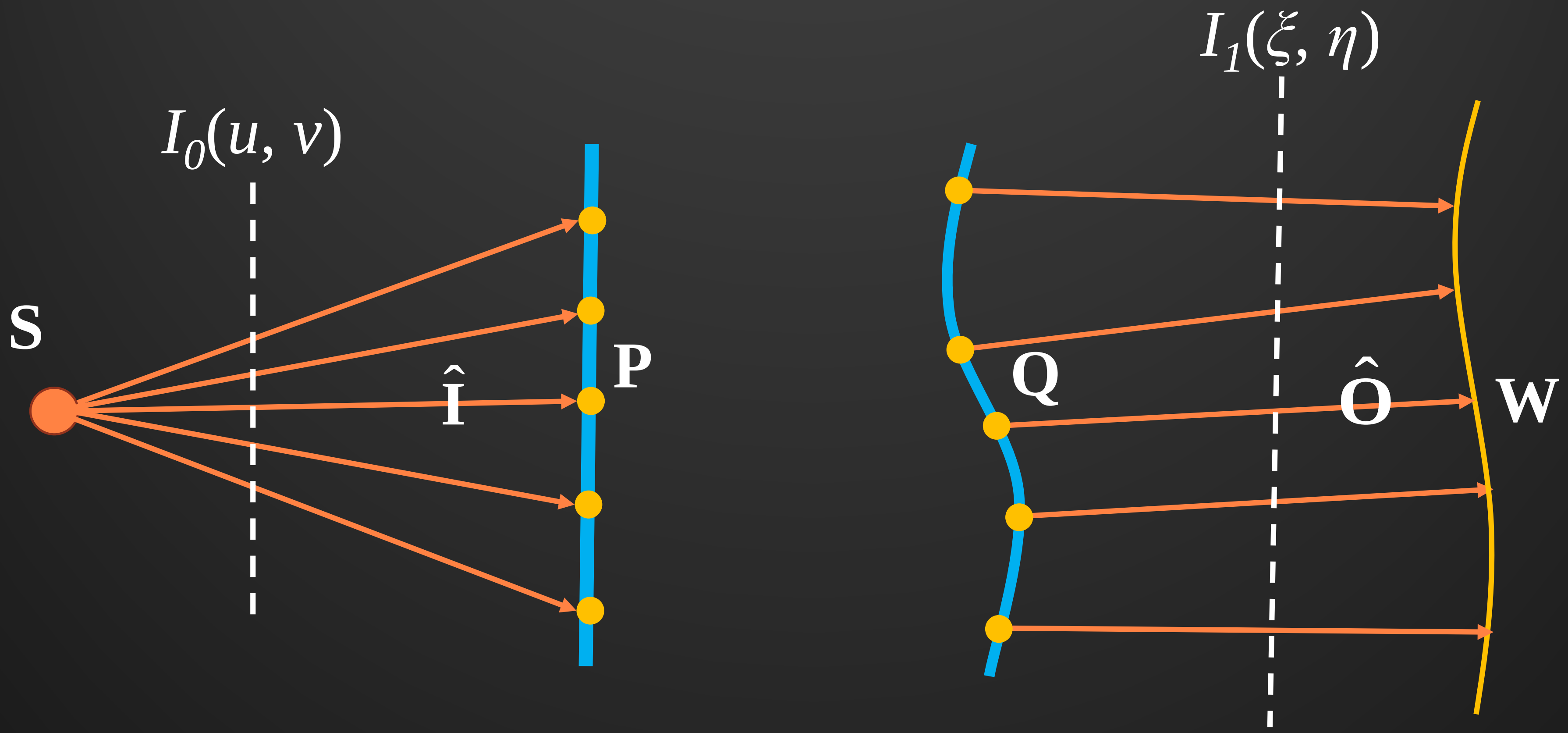
Step 1: Define an input ray sequence and output ray sequence based on the **optimal transport map**

ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



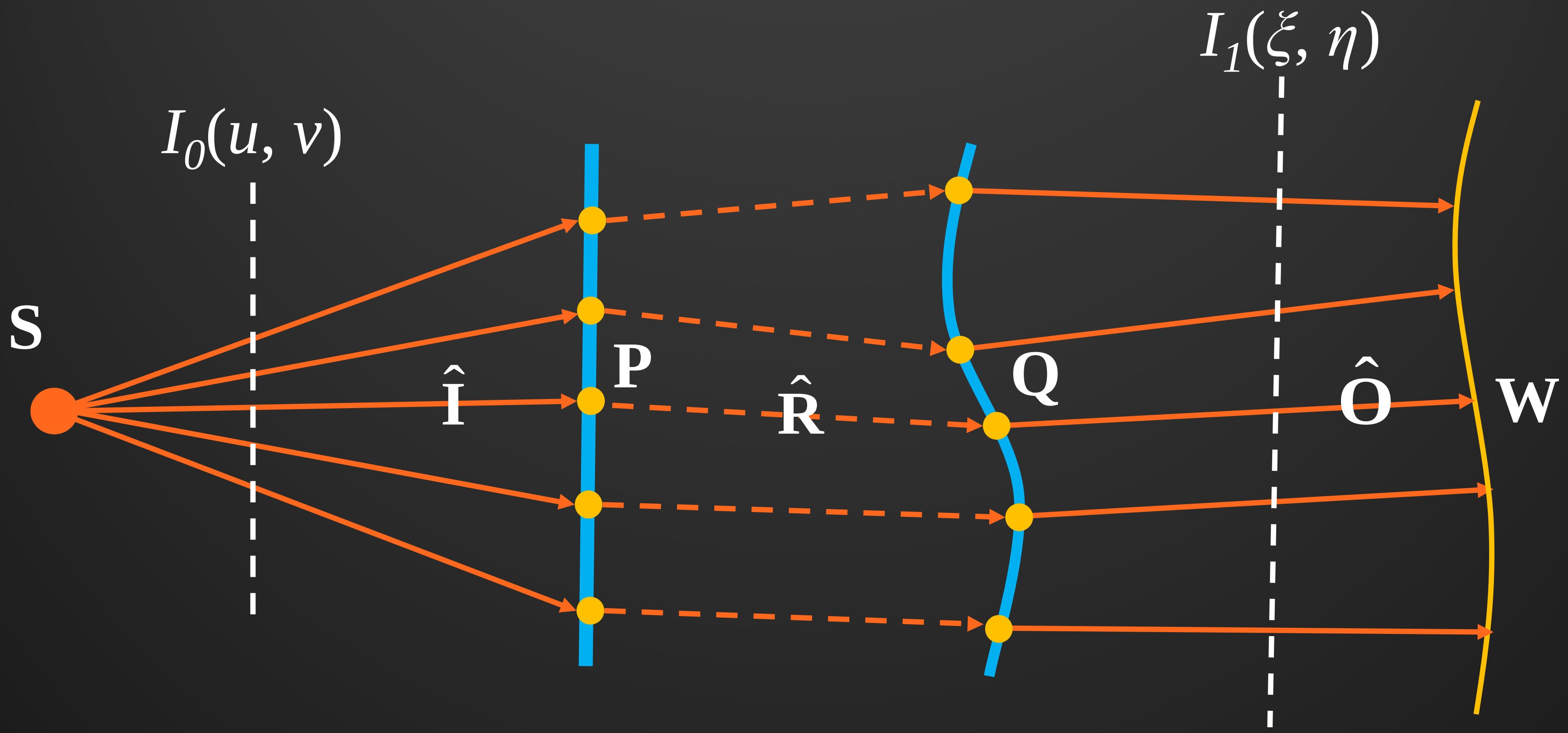
Step 2: Given an estimate of the first freeform surface

ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



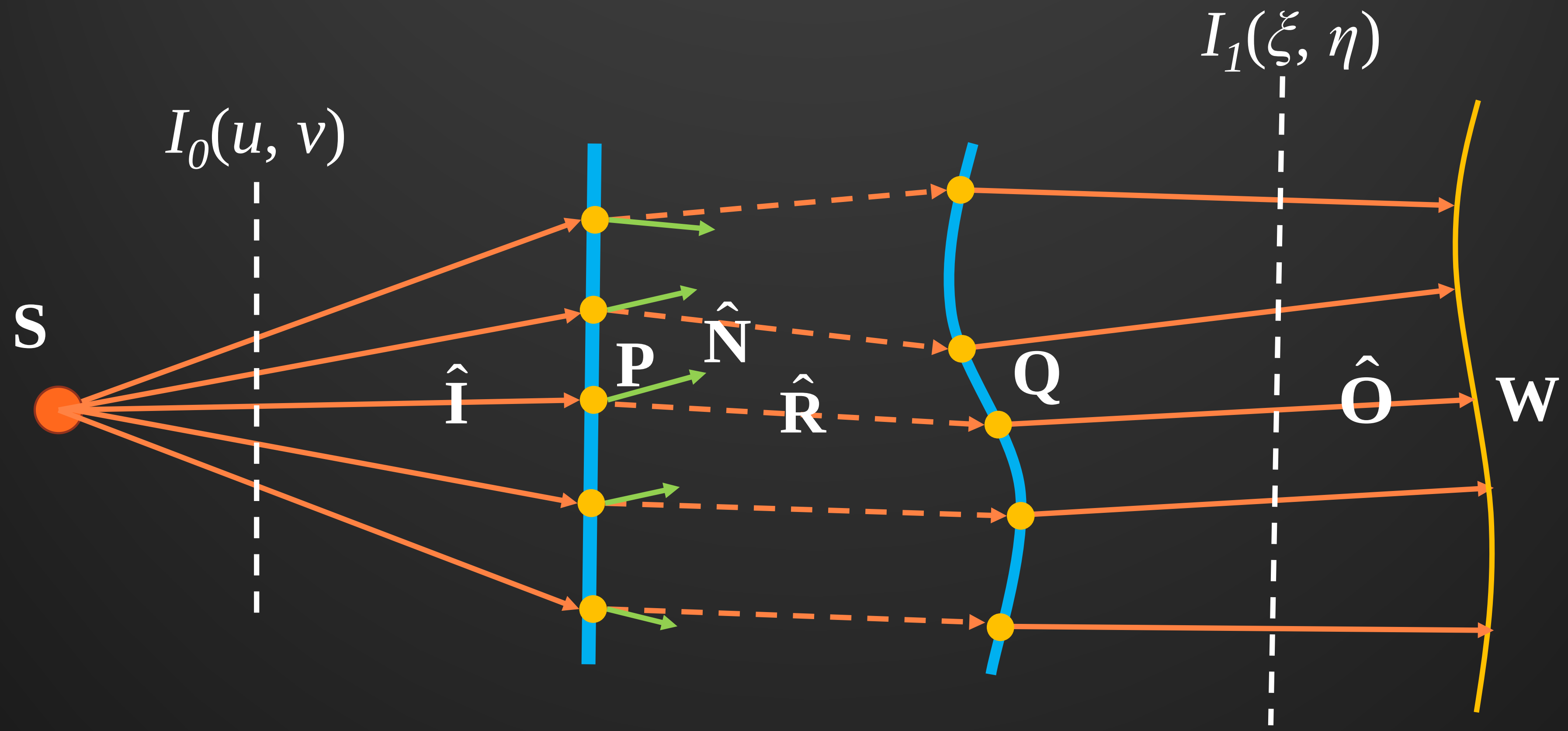
Step 3: Construct the second freeform surface based on the OPL constancy

ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



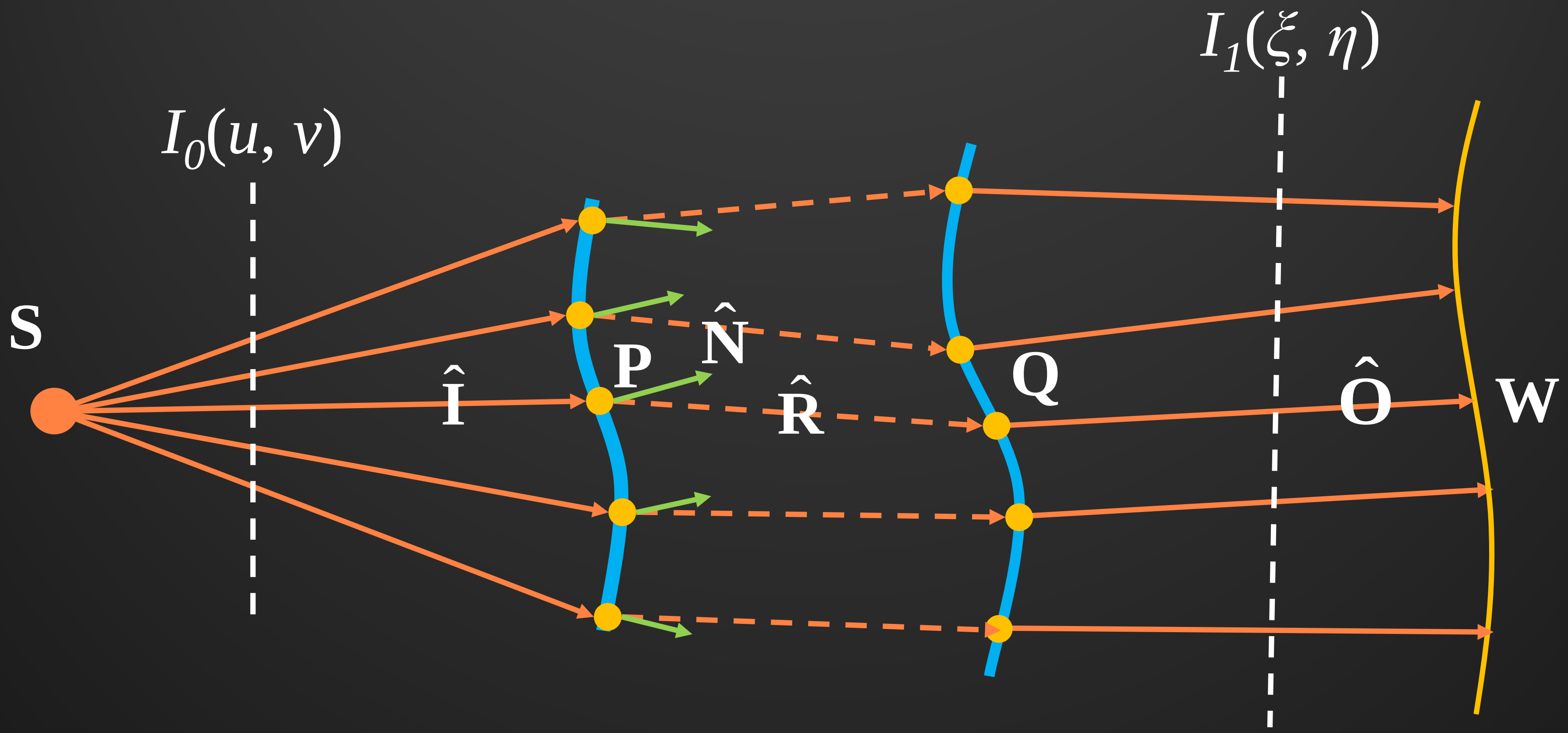
Step 4: Obtain a ray sequence between P and Q

ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



Step 5: Compute a normal field based on Snell’s law

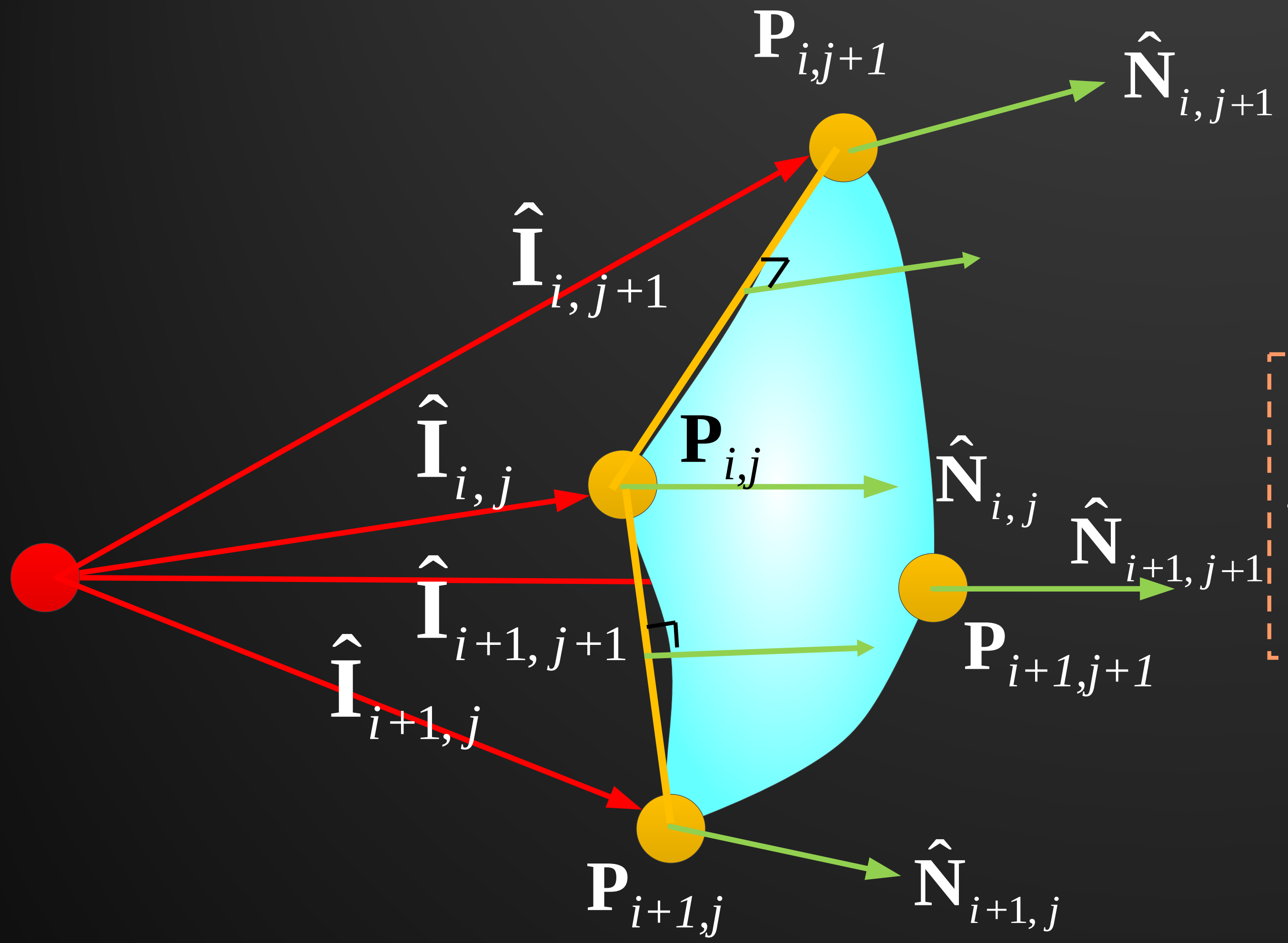
ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



Step 6: Reconstruct the first freeform surface from the normal field

Return to Step 3, and repeat the above process…

ITERATIVE LEAST SQUARES SURFACE CONSTRUCTION



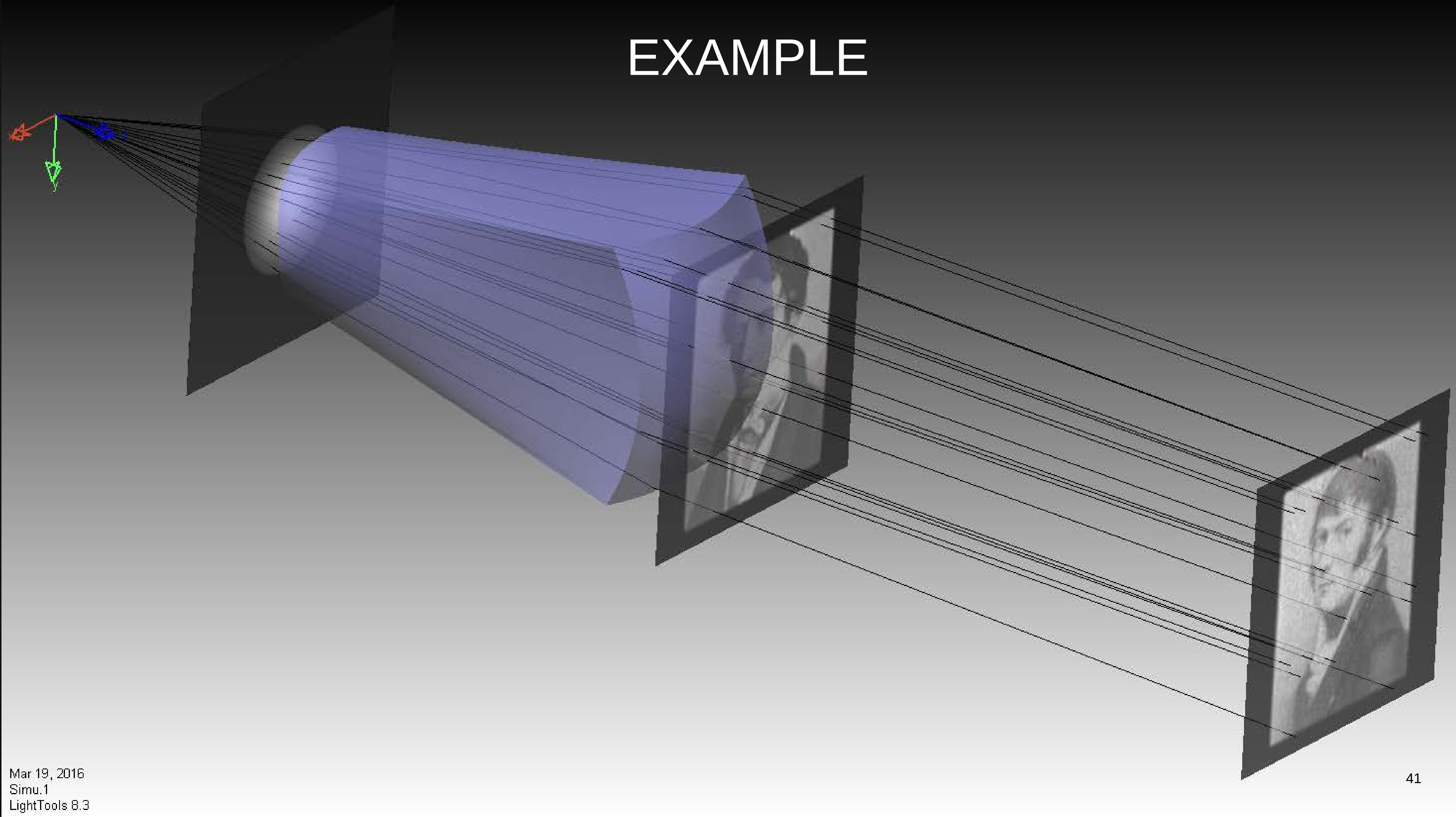
$$\hat{\mathbf{N}} \cdot (\nabla \times \hat{\mathbf{N}}) \neq 0$$

$$\begin{cases} (\mathbf{P}_{i,j+1} - \mathbf{P}_{i,j}) \cdot (\hat{\mathbf{N}}_{i,j+1} + \hat{\mathbf{N}}_{i,j}) = 0 \\ (\mathbf{P}_{i+1,j} - \mathbf{P}_{i,j}) \cdot (\hat{\mathbf{N}}_{i+1,j} + \hat{\mathbf{N}}_{i,j}) = 0 \end{cases}$$

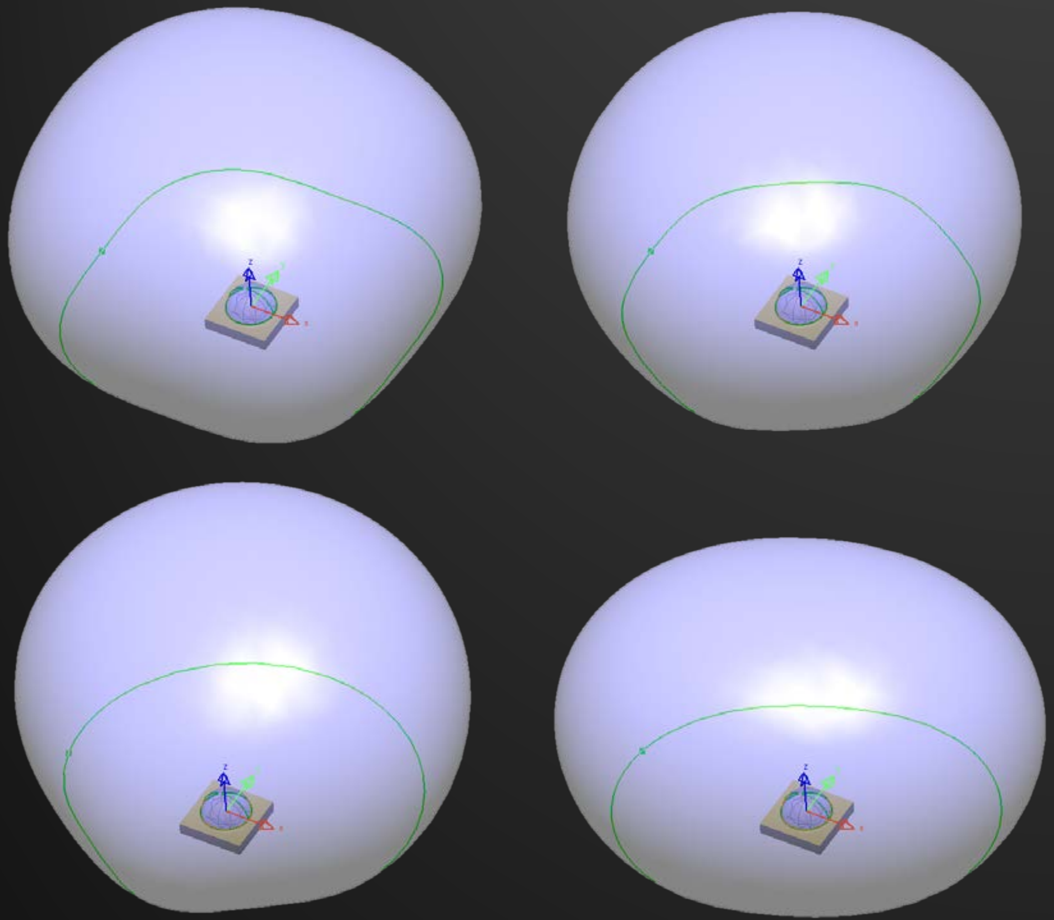
LEAST SQUARES SOLUTION

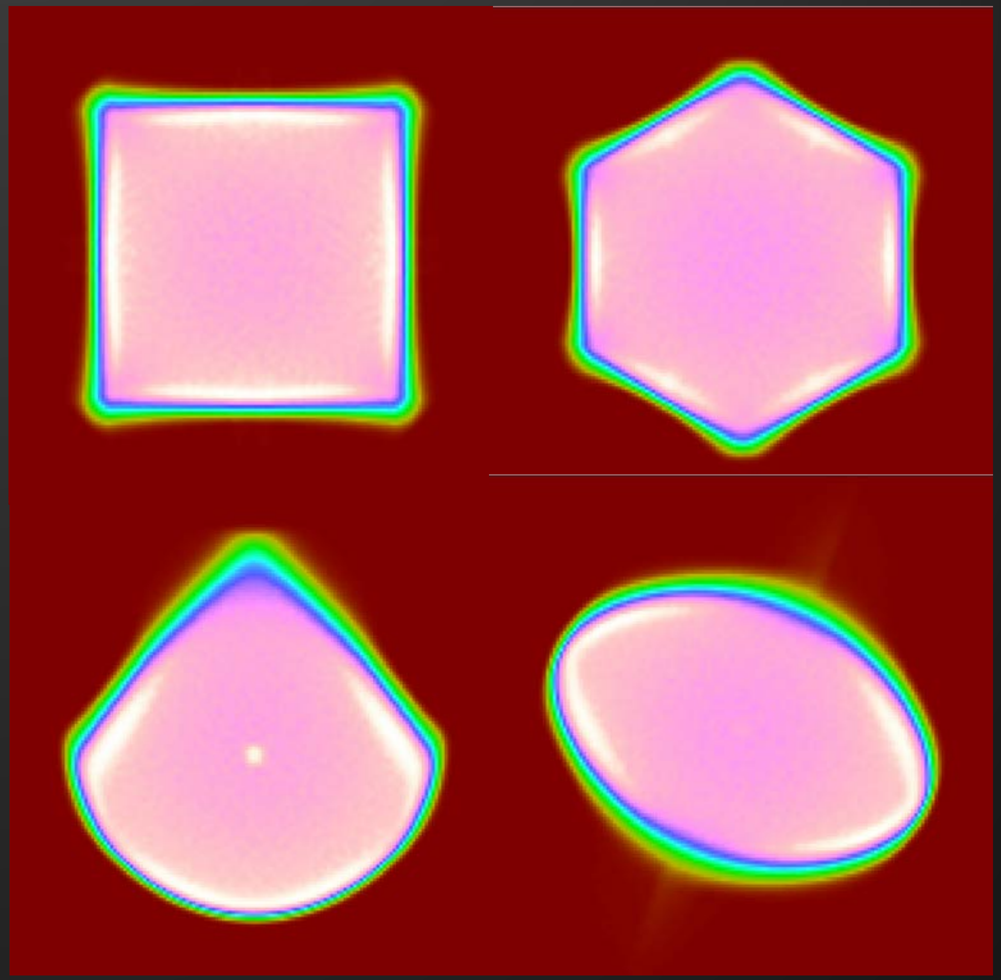
- Zexin Feng, Brittany D. Froese, and Rongguang Liang, "Freeform illumination lens construction following an optimal transport map," Appl. Opt. 55, 4301-4306 (2016)

EXAMPLE



EXAMPLE





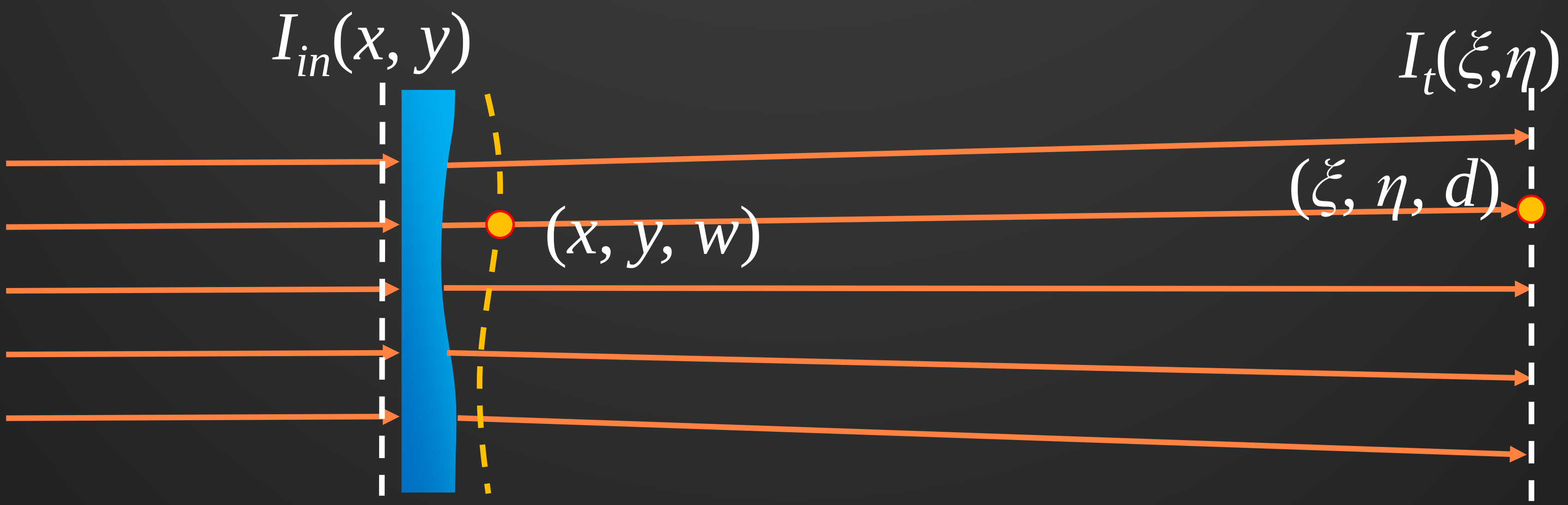
3000 mm

Central Thickness ≈ 15 mm
Target Distance = 1000 mm

Zexin Feng, Brittany D. Froese, and Rongguang Liang, "Freeform illumination lens construction following an optimal transport map," Appl. Opt. 55, 4301-4306 (2016)

IV. Design Methods Under Paraxial and Thin Lens Approximations

GEOMETRIC OPTICS METHOD



Fermat's principle or the method of the stationary phase:

$$\frac{\partial w}{\partial x} = -\frac{\xi - x}{d}, \quad \frac{\partial w}{\partial y} = -\frac{\eta - y}{d}$$

- F. M. Dickey and H. C. Holswade, Laser Beam Shaping: Theory and Techniques (Marcel Dekker, 2000).
- O. Bryngdahl, "Geometrical transformations in optics," J. Opt. Soc. Am. 64, 1092-1099 (1974).

A SIMPLE DERIVATION PROCESS

Fermat's Principle

$$\frac{\partial w}{\partial x} = -\frac{\xi - x}{d}, \quad \frac{\partial w}{\partial y} = -\frac{\eta - y}{d}$$

$$\xi = x - d\frac{\partial w}{\partial x}, \quad \eta = y - d\frac{\partial w}{\partial y}$$

Energy Conservation

$$I_0(u,v) = I_1(\xi,\eta)\left|\frac{\partial \xi}{\partial u}\frac{\partial \eta}{\partial v} - \frac{\partial \eta}{\partial u}\frac{\partial \xi}{\partial v}\right|$$

+

Surface Continuity

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x}$$

- F. M. Dickey and H. C. Holswade, Laser Beam Shaping: Theory and Techniques (Marcel Dekker, 2000).

THE RESULTING EQUATION

$$\begin{cases} \dfrac{\partial^2\varphi}{\partial x^2}\dfrac{\partial^2\varphi}{\partial y^2} - \left(\dfrac{\partial^2\varphi}{\partial x\partial y}\right)^2 = \dfrac{I_0(x,y)}{I_1(\nabla\varphi)} \\ \nabla\varphi : \Omega_0 \to \Omega_1 \end{cases}$$

Where: $\varphi = -\dfrac{1}{d}w - \dfrac{1}{2}(x^2 + y^2)$

Standard Monge–Ampère Equation

A MUCH SIMPLER CASE: VARIABLE SPARABLE

Variable Separation

$$I_0(x,y)dxdy = I_1(\xi,\eta)d\xi d\eta$$

$$\xi = \xi(x),\ \ \eta = \eta(y)$$

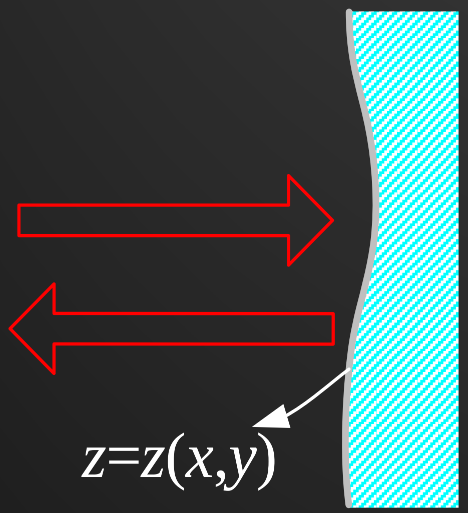
Wavefront Integration

$$\begin{cases} \dfrac{\partial w}{\partial x} = -\dfrac{1}{d}(\xi - x) \\ \dfrac{\partial w}{\partial y} = -\dfrac{1}{d}(\eta - y) \end{cases}$$

- O. Bryngdahl, “Geometrical transformations in optics,” J. Opt. Soc. Am. 64, 1092-1099 (1974).
- K. Nemoto, et al., , “Laser beam-forming by deformable mirror,” Proc. SPIE **2119**, 155-161 (1994).
- Y. Arieli, N. Eisenberg, A. Lewis, and I. Glaser, “Geometrical transformation approach to optical two-dimensional beam shaping,” Appl. Opt. 36, 9129–9131 (1997).
- H. Aagedal, M. Schmid, S. Egner, J. Müller-Quade, T. Beth, and F. Wyrowski, “ Analytical beam shaping with application to laser-diode arrays,” J. Opt. Soc. Am. A 14, 1549-1553 (1997)
- Z. Zeng, N. Ling, and W. Jiang, “The investigation of controlling laser focal profile by deformable mirror and wave-front sensor,”Journal of Modern Optics 46, 341-348 (1999).

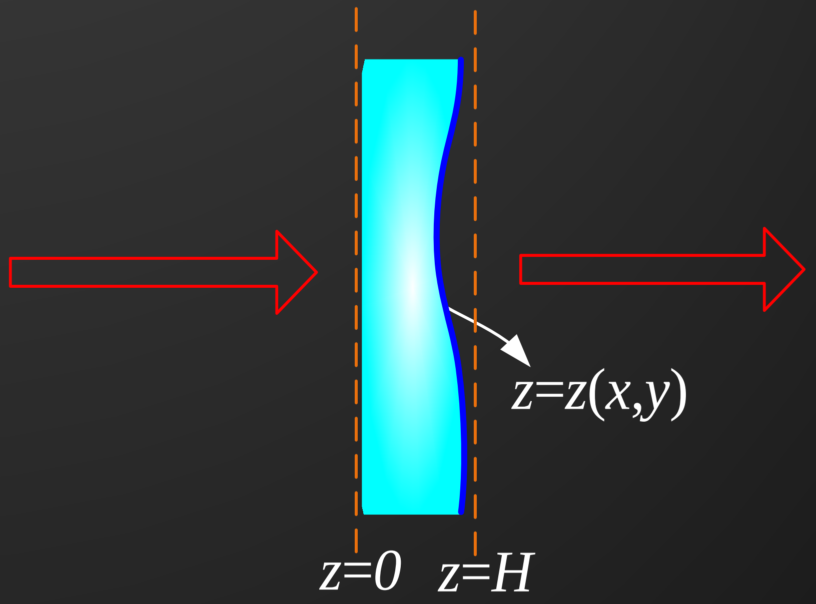
DIRECT CACULATION OF THE FREEFORM SURFACE FROM WAVEFRONT

Freeform Mirror



$$w_{mirror} = -2z(x, y)$$

Freeform Lens



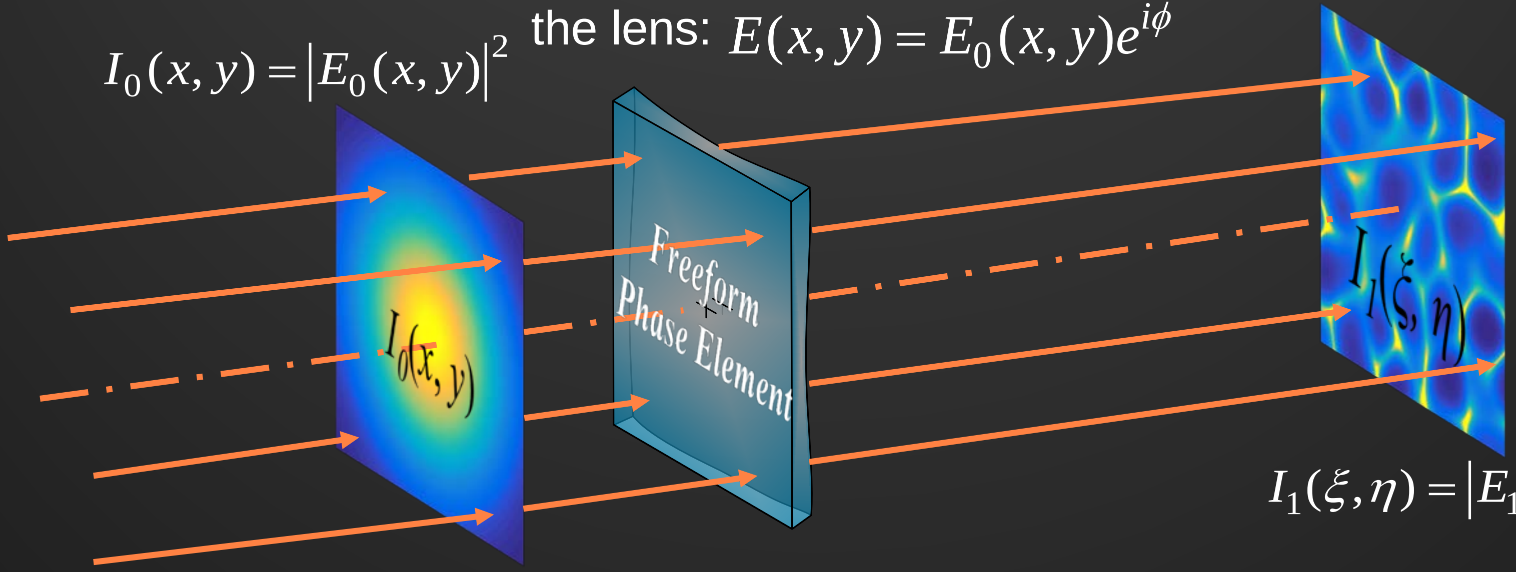
$$w_{lens} = -nz(x, y) - [H - z(x, y)]$$

J. W. Goodman, Introduction to Fourier Optics, 2nd ed. (McGraw-Hill, 1996)

CONSIDERING DIFFRACTION

The optical field immediately after the lens: $E(x,y)=E_0(x,y)e^{i\phi}$

$I_0(x,y)=|E_0(x,y)|^2$



$I_1(\xi,\eta)=|E_1(\xi,\eta)|^2$

Fresnel diffraction equation

$$E_1(\xi,\eta)=\frac{e^{ikd}}{i\lambda d}e^{i\frac{k}{2d}(\xi^2+\eta^2)}\iint E(x,y)e^{i\frac{k}{2d}(x^2+y^2-2\xi x-2\eta y)}dxdy$$

$$=\frac{e^{ikd}}{i\lambda d}e^{i\frac{k}{2d}(\xi^2+\eta^2)}FT\left[E(x,y)e^{i\frac{k}{2d}(x^2+y^2)}\right]$$

Iterative Fourier Transform Algorithms

$$E_1(\xi,\eta)=\frac{e^{ikd}}{i\lambda d}e^{i\frac{k}{2d}(\xi^2+\eta^2)}\,\mathrm{FT}\left[E(x,y)e^{i\frac{k}{2d}(x^2+y^2)}\right]$$

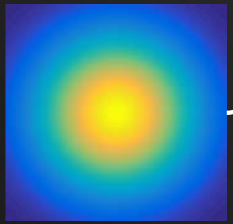
Forward Fourier transform

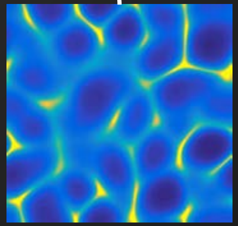
$$E(x,y)=\sqrt{I_0(x,y)}\,e^{i\phi}$$

Remain ϕ

Remain ϕ_1

$$E_1(\xi,\eta)=\sqrt{I_1(\xi,\eta)}\,e^{i\phi_1}$$





Inverse Fourier transform

$$E(x,y)=\frac{e^{-ikd}}{i\lambda d}e^{-i\frac{k}{2d}(x^2+y^2)}\,\mathrm{FT}^{-1}\left[E_1(\xi,\eta)e^{-i\frac{k}{2d}(\xi^2+\eta^2)}\right]$$

Gerchberg–Saxton (GS) algorithm

- R. W. Gerchberg and W. O. Saxton, “A practical algorithm for the determination of phase from image and diffraction plane pictures,” Optik 35, 237–246 (1972).
- J. R. Fienup, “Iterative method applied to image reconstruction and to computer-generated holograms,” Opt. Eng. 19, 193297 (1980).
- O. Ripoll, V. Kettunen, and H. P. Herzig, “Review of iterative Fourier transform algorithms for beam shaping applications,” Opt. Eng. 43, 2549–2556 (2004).
- C. Béchet, A. Guesalaga, B. Neichel, et al., “Beam shaping for laser-based adaptive optics in astronomy,” Opt. Express 22, 12994-13013 (2014) .

COMPOSITE METHODS

Geometrics methods

Initial phase estimate

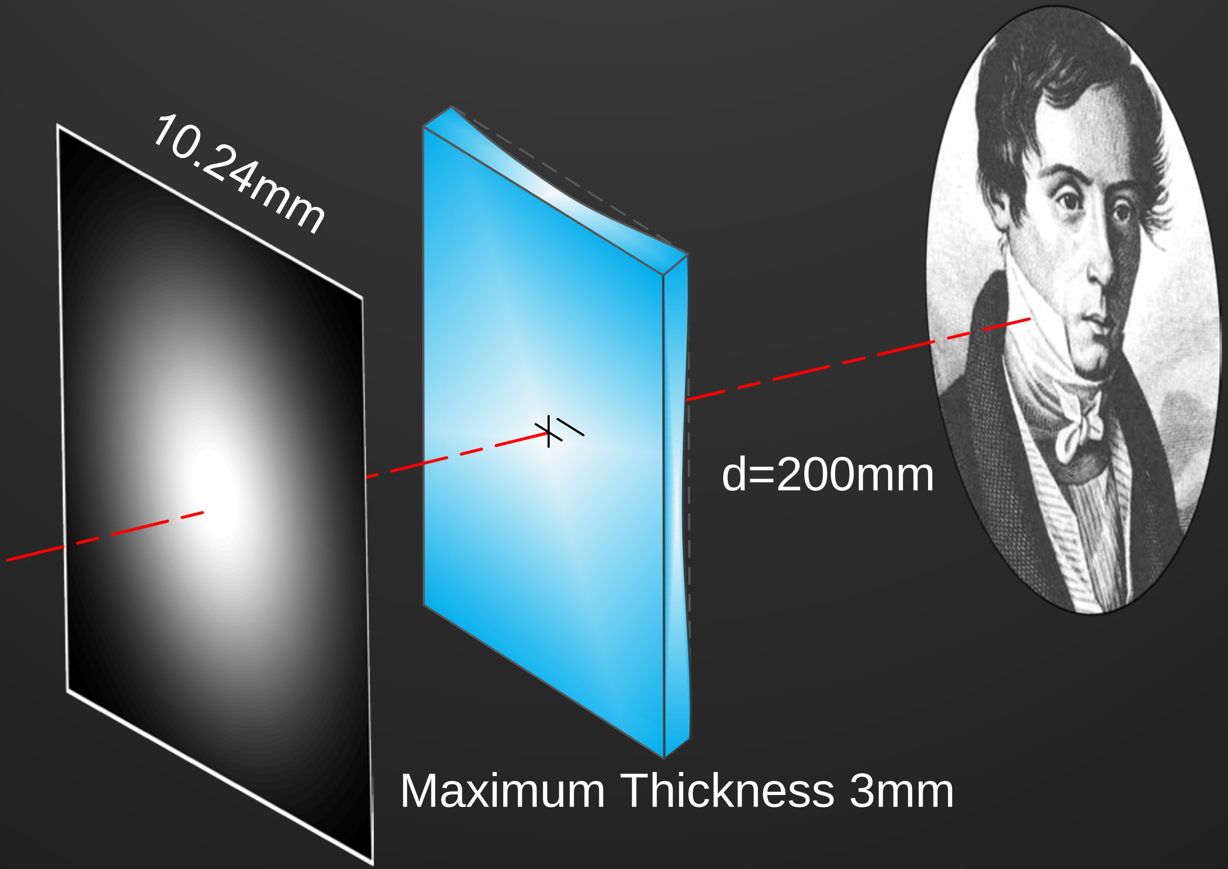
↓

Iterative Fourier Transform Algorithms

GS or variants of GS

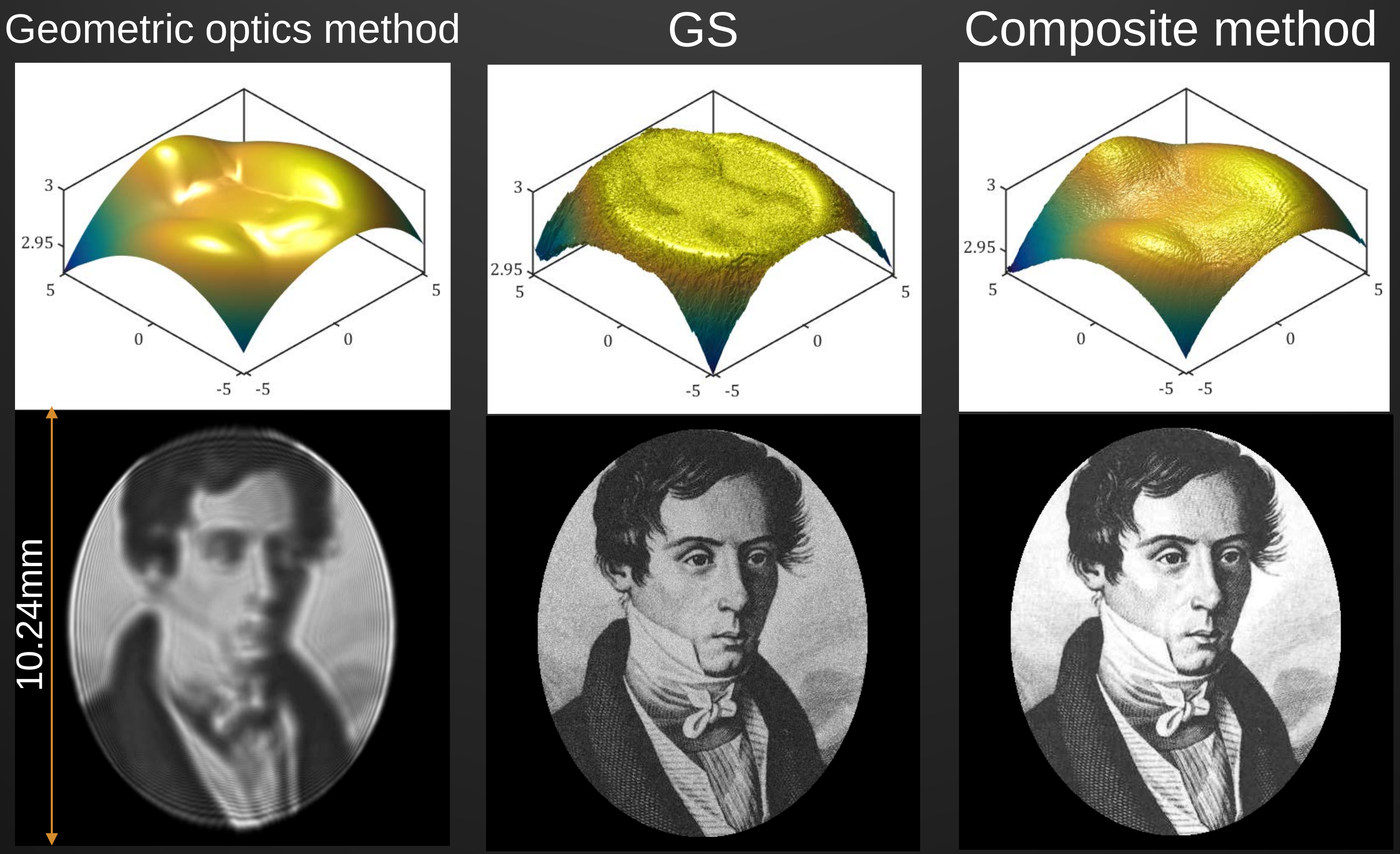
- M. T. Eismann, A. M. Tai, and J. N. Cederquist, "Iterative design of a holographic beamformer," Appl. Opt. 28, 2641–2650 (1989).
- X. Tan, B. Gu, G. Yang, and B. Dong, “Diffractive phase elements for beam shaping: a new design method,” Appl. Opt. 34, 1314–1320 (1995).
- X. Deng, D. Fan, Y. Qiu, and Y. Li, “Pure-phase plates for superGaussian focal-plane irradiance profile generations of extremely high order,” Opt. Lett. 21, 1963–1965 (1996).
- J. S. Liu and M. R. Taghizadeh, “Iterative algorithm for the design of diffractive phase elements for laser beam shaping,” Opt. Lett. 27,1463–1465 (2002).
- Z. Feng, et al., “A composite method for high resolution freeform optical beam shaping,” Appl. Opt. 54, 9364–9369 (2015).

EXAMPLE



Z. Feng, et al., “A composite method for high resolution freeform optical beam shaping,” Appl. Opt. 54, 9364–9369 (2015).

SIMULATION RESULTS



Z. Feng, et al., “A composite method for high resolution freeform optical beam shaping,” Appl. Opt. 54, 9364–9369 (2015).

V. Conclusions & Outlook

CONCLUSION

Freeform optics design for Illumination & Beam Shaping is a very difficult inverse problem

$$A\left[\frac{\partial^2\rho}{\partial u^2}\frac{\partial^2\rho}{\partial v^2}-\left(\frac{\partial^2\rho}{\partial u\partial v}\right)^2\right]+B\frac{\partial^2\rho}{\partial u^2}+2C\frac{\partial^2\rho}{\partial u\partial v}+D\frac{\partial^2\rho}{\partial v^2}+E=0$$

CONCLUSION

Aside from the direct determination methods, we can use simplified design methods e.g., ray mapping methods

$$\frac{\partial^2\phi}{\partial u^2}\frac{\partial^2\phi}{\partial v^2}-\left(\frac{\partial^2\phi}{\partial u\partial v}\right)^2=\frac{I_0(u,v)}{I_1(\nabla\phi)}$$

CONCLUSION

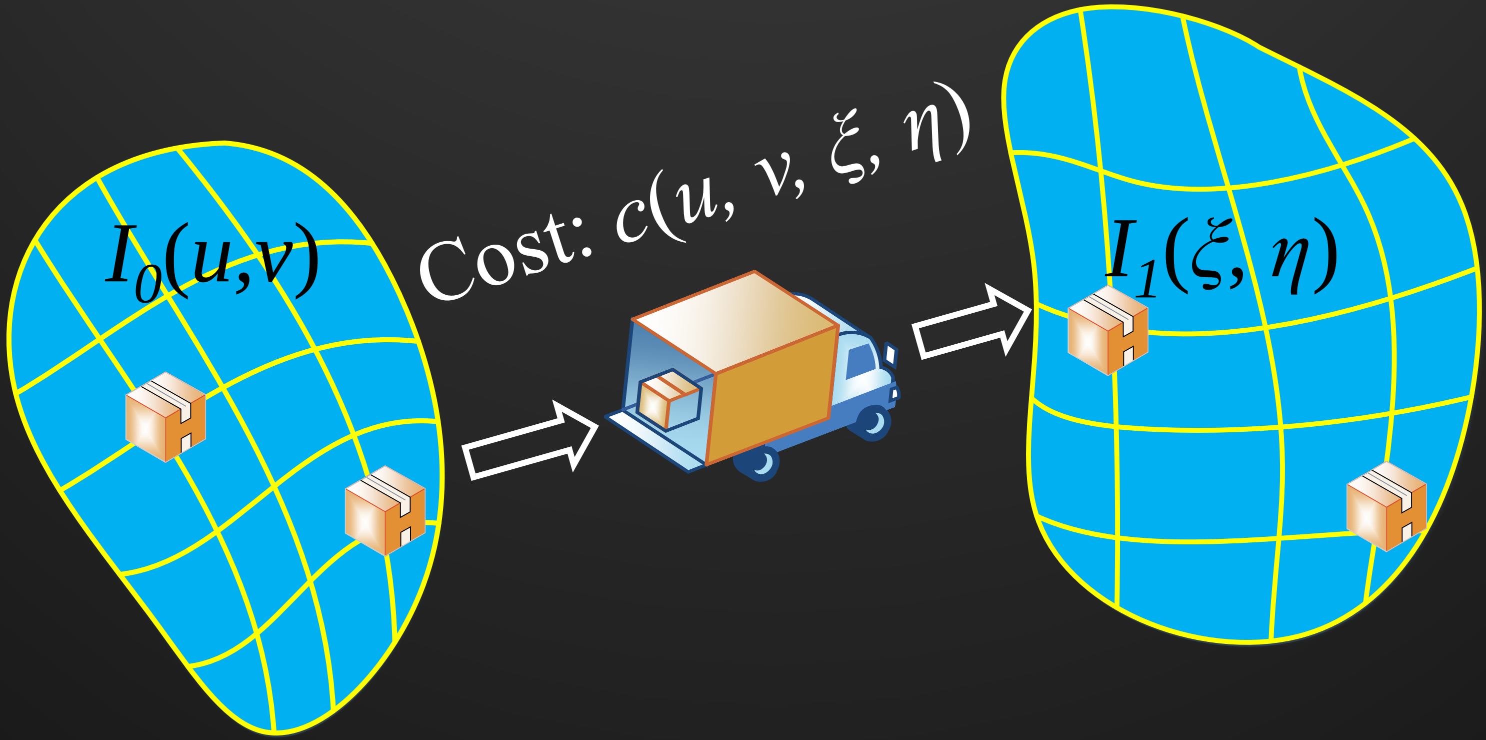
The design problem under paraxial and thin lens approximations can also lead to the solution of a standard MA equation, and IFTAs can be used to reduce the diffraction effects

$$\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi}{\partial y^2}-\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^2=\frac{I_0(x,y)}{I_1\left(\nabla\varphi\right)}$$

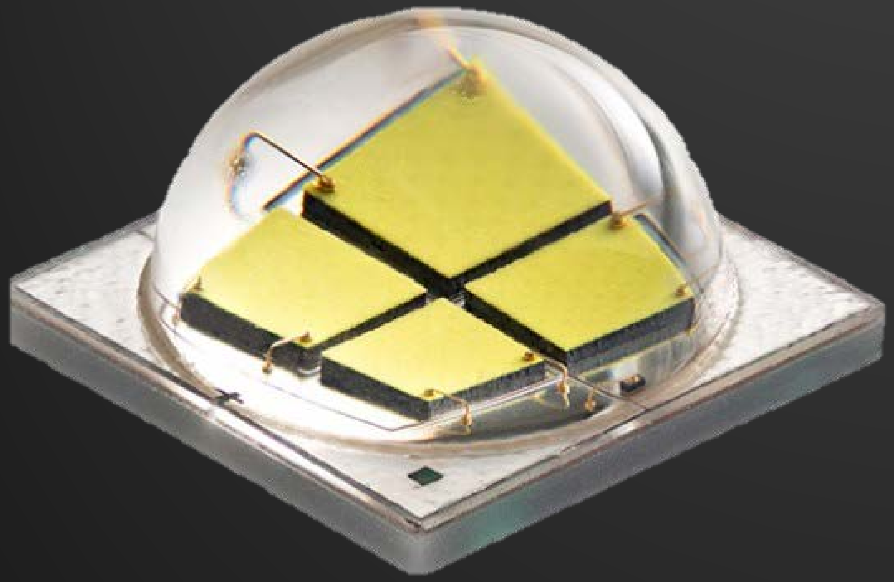
+ IFTA

OUTLOOK

New optimal transport cost function



OUTLOOK



Extended light source

- K. Wang, F. Chen, Z. Liu, X. Luo, and S. Liu, "Design of compact freeform lens for application specific light-emitting diode packaging," Opt. Express 18, 413-425 (2010).
- Z. Feng, Yi Luo, and Yanjun Han, "Design of LED freeform optical system for road lighting with high uminance/illuminance ratio," Opt. Express 18, 22020-22031 (2010).
- Y. Luo, Z. Feng, Y. Han, and H. Li, "Design of compact and smooth free-form optical system with uniform illuminance for LED source," Opt. Express 18, 9055-9063 (2010).
- L. Cao, Y. Luo, Y. Han, and Z. Feng, "Reflector design for large-size spherical surface sources," Opt. Eng. 50, 023001 (2011).
- R. Wester, G. Müller, A. Völl, M. Berens, J. Stollenwerk, and P. Loosen, “Designing optical free-form surfaces for extended sources,” Opt. Express 22, A552-A560 (2014).
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- X. Mao, H. Li, Y. Han, and Y. Luo, "Two-step design method for highly compact three-dimensional freeform optical system for LED surface light source," Opt. Express 22, A1491-A1506 (2014)
- K. Wang, Y. Han, H. Li, Y. Luo, C. Sun, Z. Hao, B. Xiong, J. Wang, and L. Wang,”Design of high-compactness freeform optical surfaces via energy accumulating optimization. Opt. Express, “ 2016, 24(26): A1489-A1504
- M. Brand and A. Aksoylar, “Sharp images from freeform optics and extended light sources,”Frontiers in Optics, 2016: FW5H.

Thank you for your attention!