

# Spatiotemporal Dynamics of Optical Pulse Propagation in Multimode Fibers

Presented by:



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- Mission
  - To benefit YOU
  - Webinars, e-Presence, Publications, Tech Events, Business Events, Outreach
- Membership
  - OSA members can participate in 5 TGs
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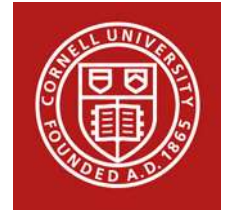
This is YOUR Technical Group!

**If you have an idea,  
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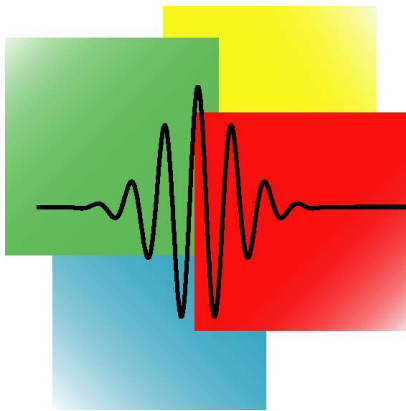
- Email: [TGNonlinearOptics@osa.org](mailto:TGNonlinearOptics@osa.org)
- Facebook: [osanonlinearoptics](https://www.facebook.com/osanonlinearoptics)

# Webinar series

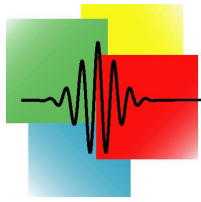
- Past seminars
  - Prof. Giulio Cerullo, “Ultrabroadband Optical Parametric Amplifiers: Toward Single-Cycle CEP-Controlled Pulses”
  - Prof. Takuro Ideguchi, “Molecular Spectroscopy with Optical Frequency Combs”
  - [www.osa.org/tgwebinars/#tab\\_ondemand](http://www.osa.org/tgwebinars/#tab_ondemand)
- Spatiotemporal Dynamics of Optical Pulse Propagation in Multimode Fibers
  - Prof. Frank Wise of Cornell University
  - Theoretical and experimental studies of the basic properties and spatiotemporal behavior of complex nonlinear dynamics in multimode fiber will be presented.



*Spatiotemporal Dynamics of Optical Pulse  
Propagation in Multimode Fibers*

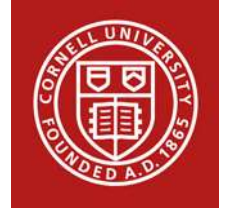


F. W. Wise  
*Department of Applied Physics  
Cornell University*



# *Spatiotemporal Dynamics...*

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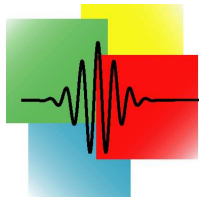


Introduction to nonlinear pulse propagation

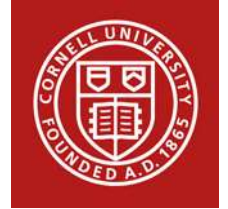
Recent progress in multimode nonlinear propagation

- Solitons in multimode GRIN fiber: formation and fission
- Multimode continuum generation
- Spatiotemporal dispersive waves
- Spatiotemporal modulation instability
- Beam self-cleaning

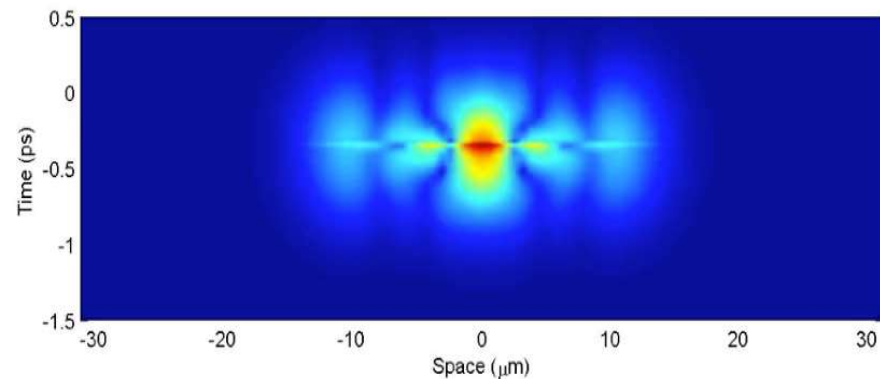
Future directions / toward applications



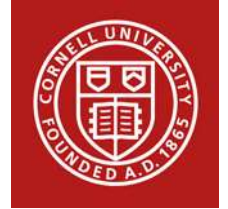
## *Spatiotemporal Dynamics...*



- Pulse propagation in multimode fiber is spatiotemporally complex
- 4D vector field



- Our job is to figure out basic processes, building blocks, and “rules”



# *Introduction to Nonlinear Wave Propagation*

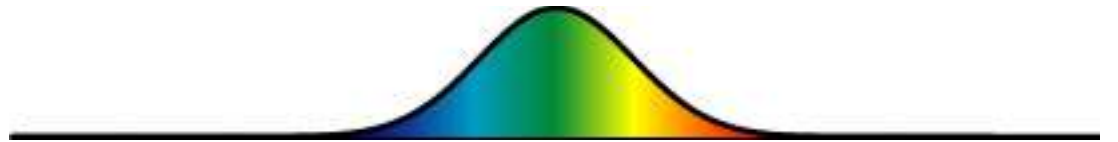
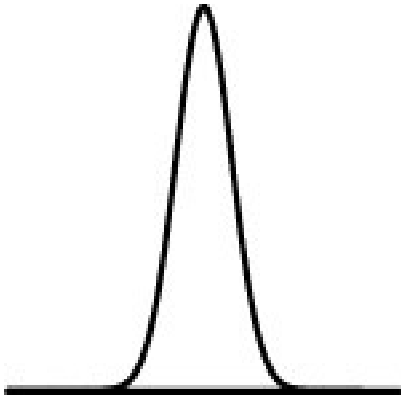




## Short pulses: dispersion

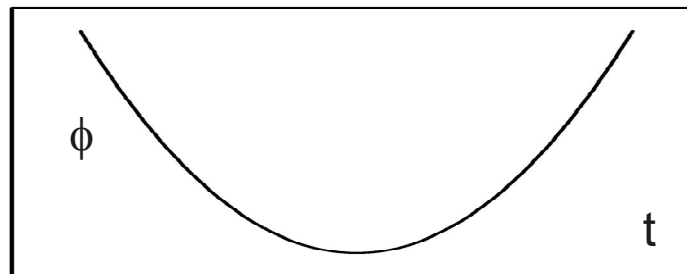
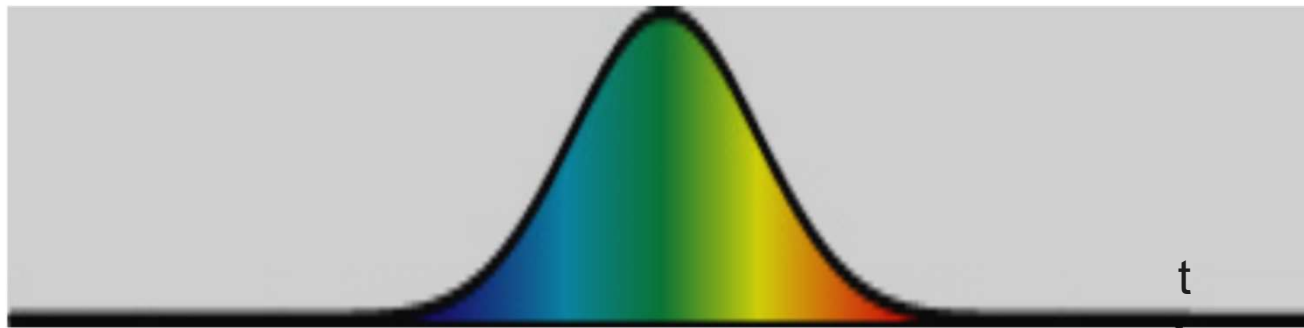


$$n = n(\omega)$$
$$v(\omega) = c/n(\omega)$$





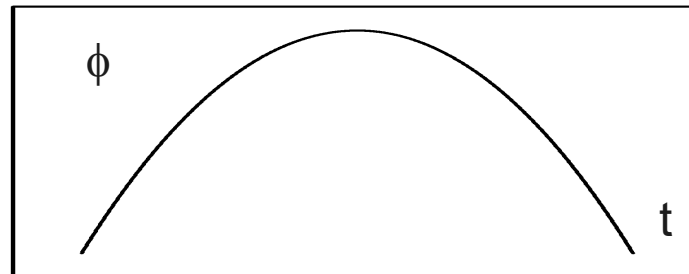
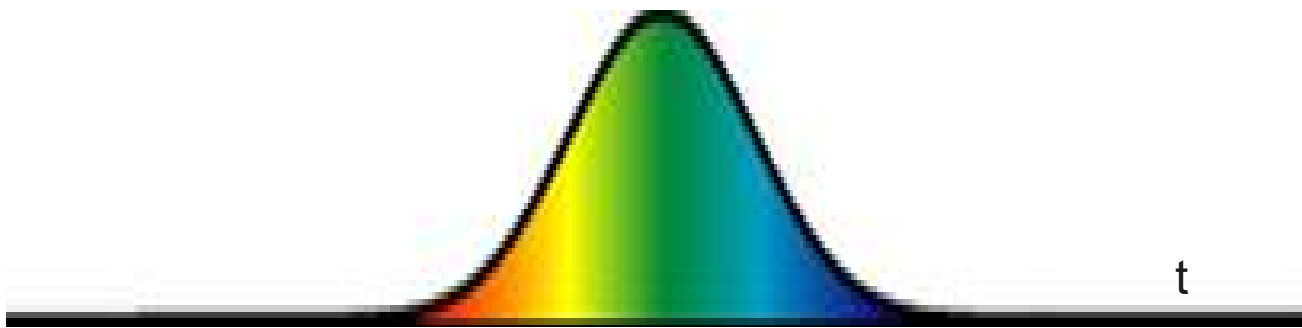
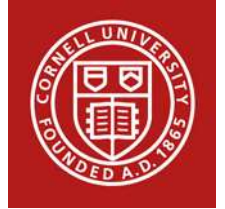
## *Dispersive phase accumulation*



*anomalous dispersion*  
 *$\lambda > 1300 \text{ nm}$  for silica*



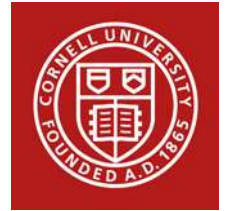
## *Dispersive phase accumulation*



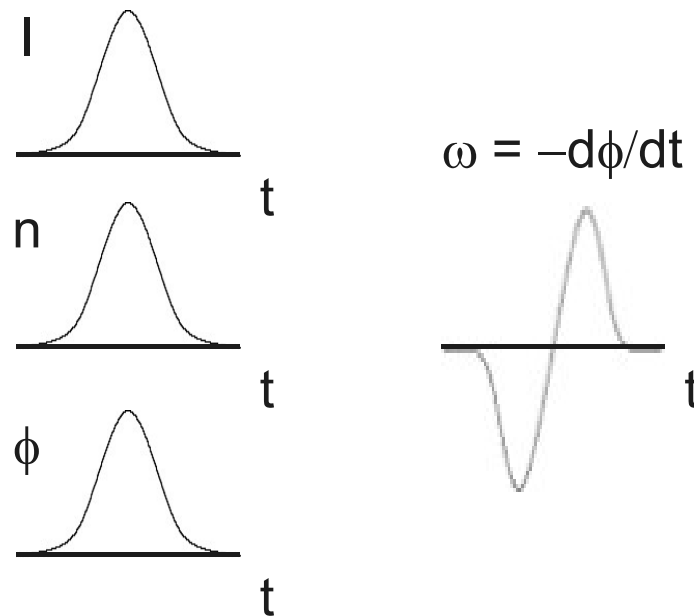
*normal dispersion*  
 $\lambda < 1300 \text{ nm}$  for silica



# Nonlinear propagation ( $\chi^{(3)}$ )



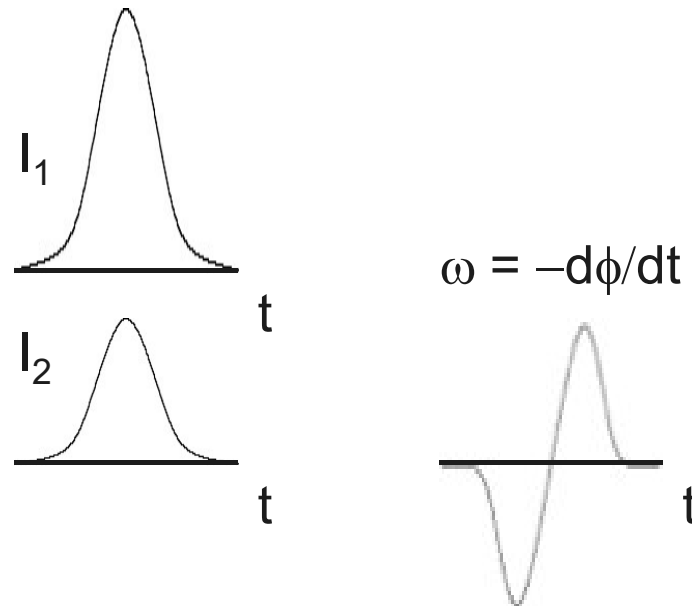
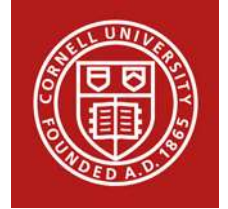
- $n = n_0 + n_2 I$



self-phase modulation produces new frequencies



# Nonlinear propagation ( $\chi^{(3)}$ )



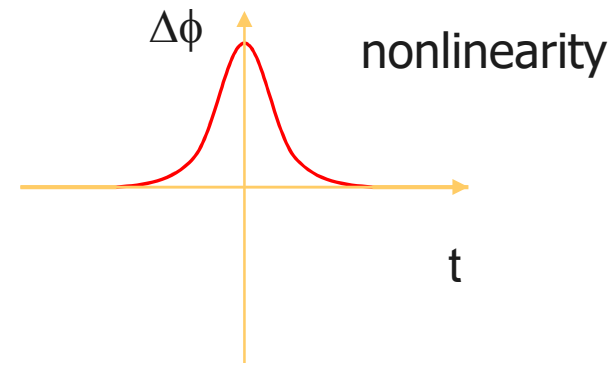
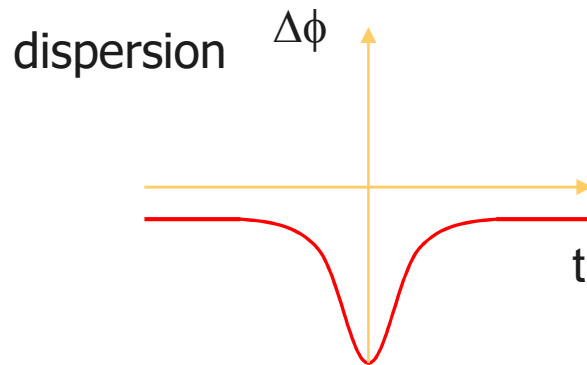
cross-phase modulation produces new frequencies



## Soliton formation



$$\frac{\partial A(z,t)}{\partial z} + i \frac{\beta^{(2)}}{2} \frac{\partial^2 A(z,t)}{\partial t^2} = i\gamma |A(z,t)|^2 A(z,t)$$



(anomalous) dispersion cancels nonlinearity for

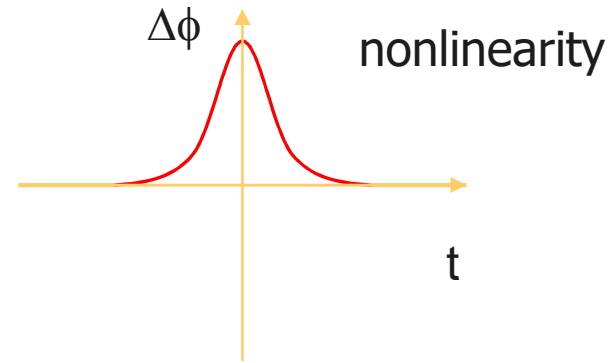
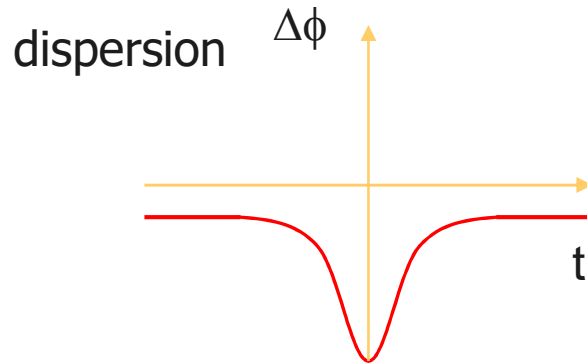
$$A(t) = A_0 \operatorname{sech}(t/\tau_p) \exp(iz/z_{sol})$$



# Soliton formation



$$\frac{\partial A(z,t)}{\partial z} + i \frac{\beta^{(2)}}{2} \frac{\partial^2 A(z,t)}{\partial t^2} = i\gamma |A(z,t)|^2 A(z,t)$$

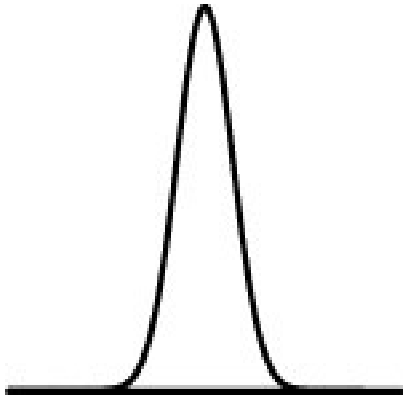
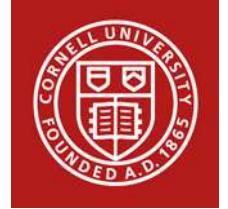


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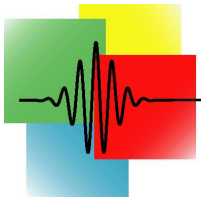
$$A_0 \tau_p = \sqrt{\frac{\beta_2}{\gamma}}$$



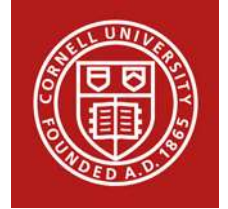
# *Soliton formation*



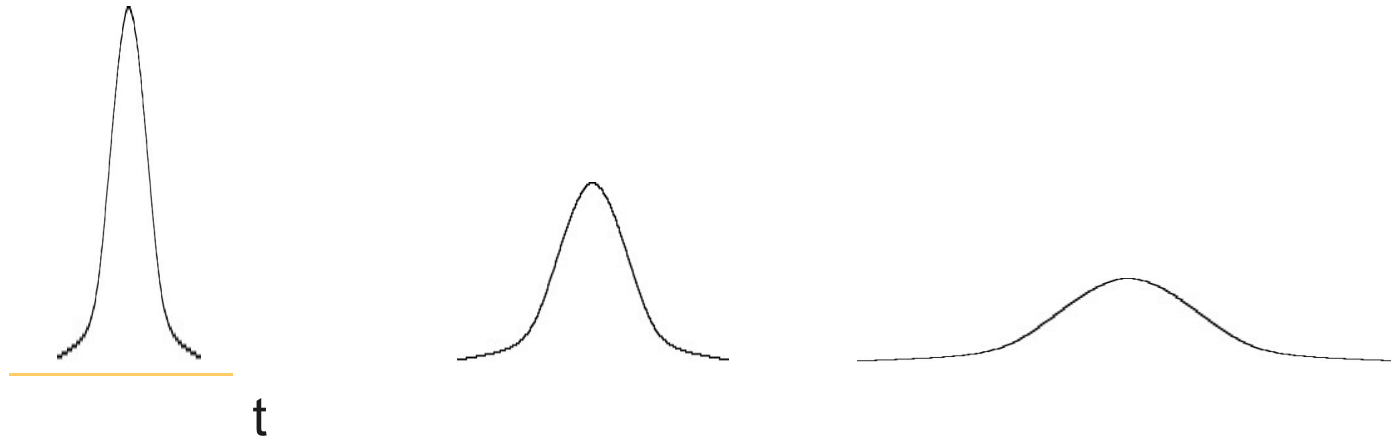




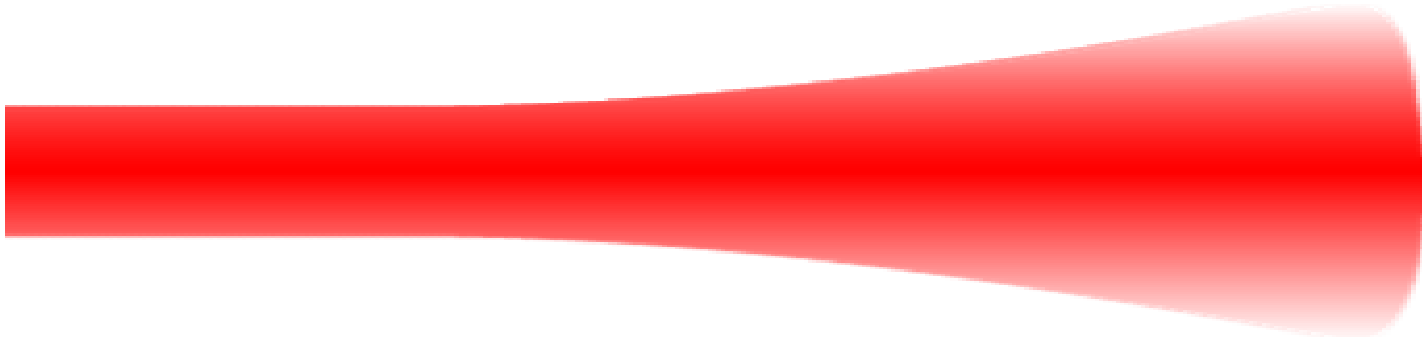
## *Linear wave propagation*



- pulse spreads owing to group-velocity dispersion

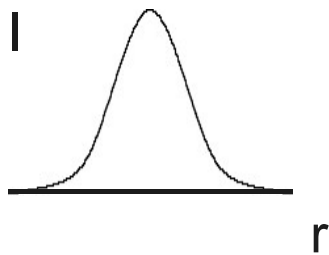
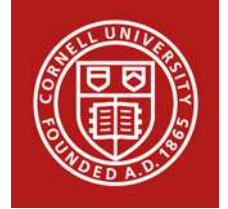


- beam spreads owing to diffraction





## Nonlinear propagation ( $\chi^{(3)}$ )

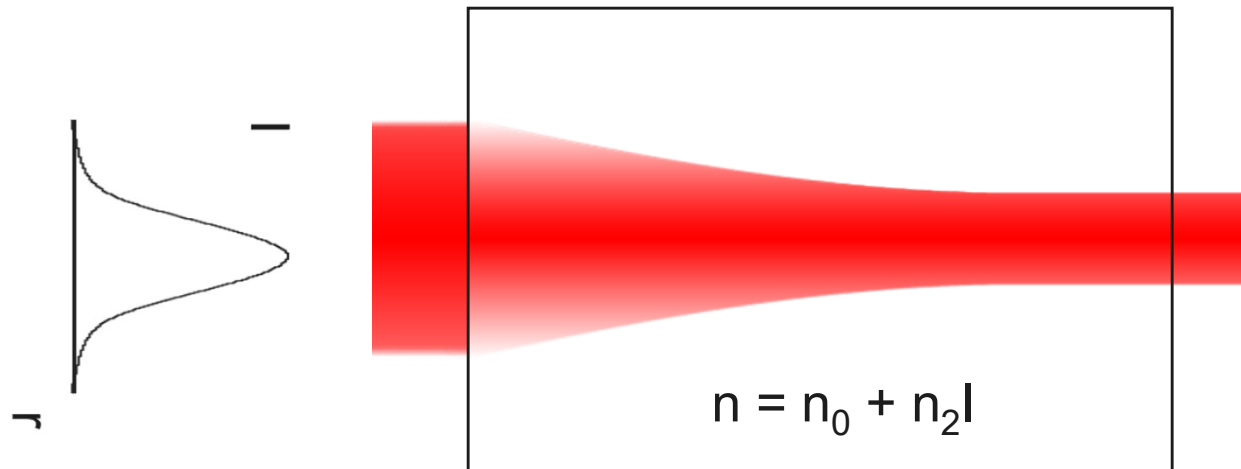
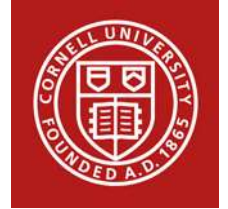


$$n = n_0 + n_2 I$$

nonlinear phase shift produces self-focusing



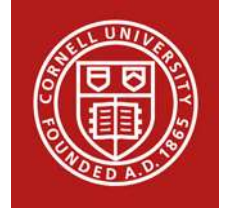
## Nonlinear propagation ( $\chi^{(3)}$ )



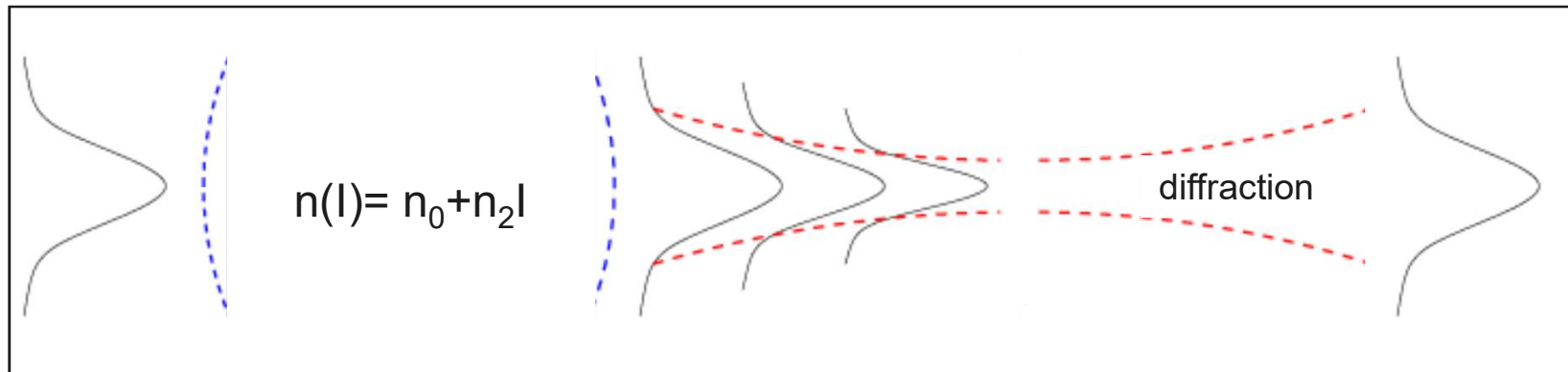
nonlinear phase shift produces self-focusing

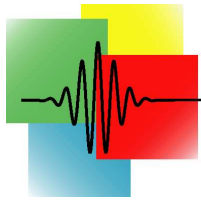


## Critical power

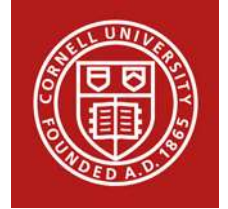


- diffraction balances self-focusing for  
 $P = P_{cr} \sim 5 \text{ MW}$  in glass



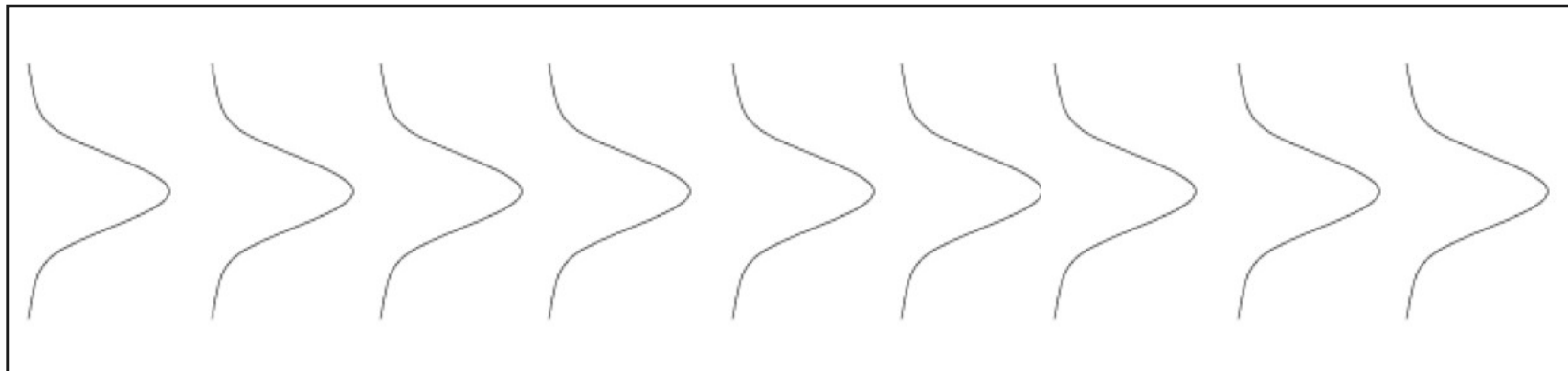


## Critical power



- diffraction balances self-focusing for

$$P = P_{cr} \sim 5 \text{ MW in glass}$$



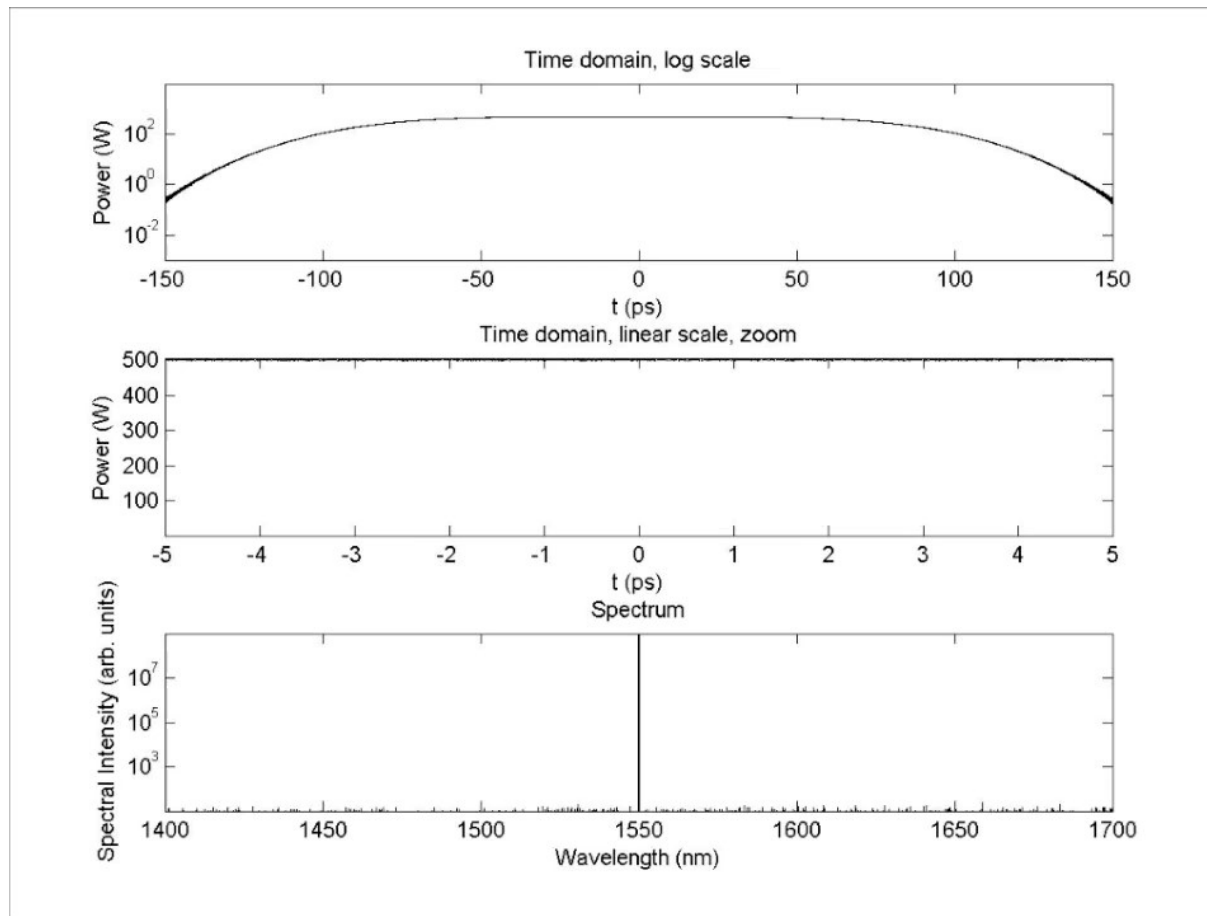
- 2D: unstable against collapse



# Why are solitons so important?

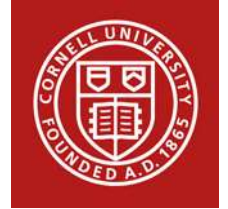


- A continuous wave breaks into temporal components



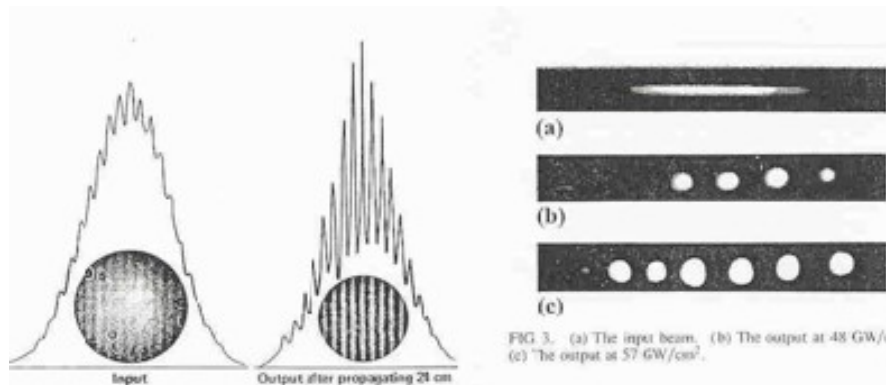


## Why are solitons so important?



- In general, waves in nonlinear media are unstable

### Modulation Instability



- A beam breaks into its component solitons
- Stable products of instability are “eigenmodes” of nonlinear systems



## *Why are solitons so important?*

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*If they exist*, solitons are important

- as stable wave packets (sometimes nonlinear attractors)
- as components of arbitrary fields

In 1D solitons underlie

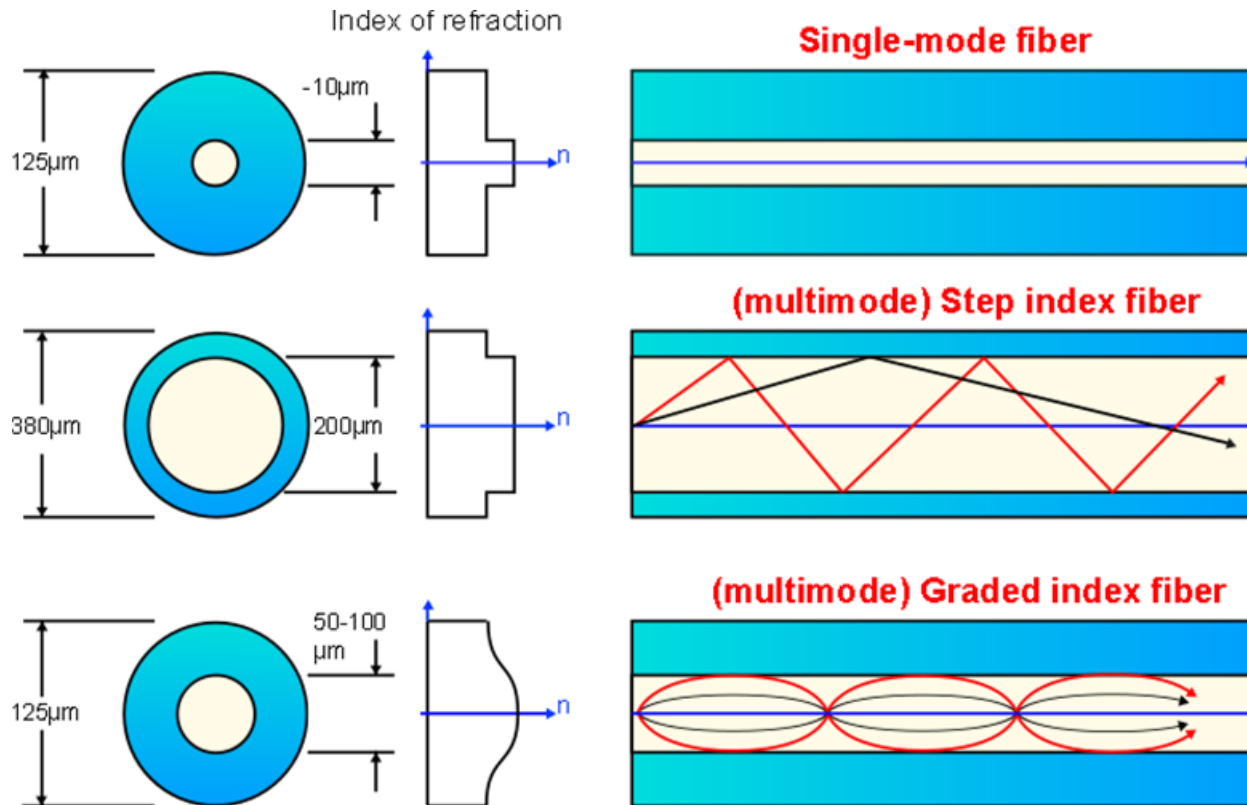
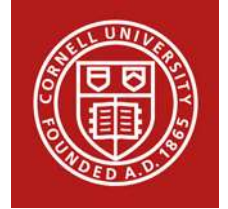
- modelocked lasers
- continuum generation
- breathers, Peregrine soliton
- rogue waves
- ...

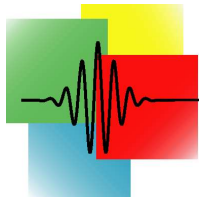
2D and 3D: solitons are unstable



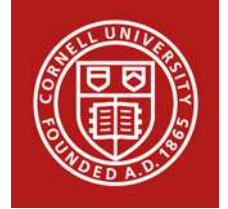


# Multimode waveguides: between 1- and 3-D



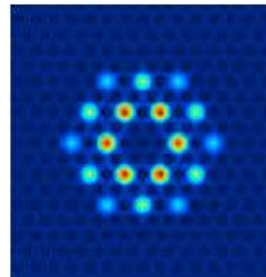


## *Why study propagation in multimode fiber now?*



- Little work on multimode nonlinear pulse propagation before 2013
- Recent theoretical, computational advances  
e.g., transfer matrix, principal modes,...

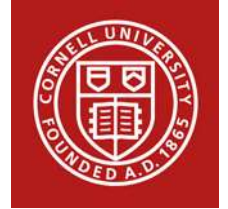
- Relevance to multicore fibers



*Huang et al., Opt Exp 2014*



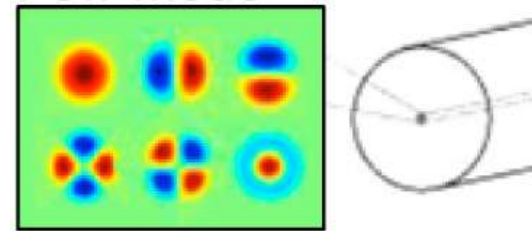
# Why study propagation in multimode fiber now?



- Laser/ amplifier / transmission applications



- Spatial division multiplexing in telecom



*Agrell et al., J Opt 2016*

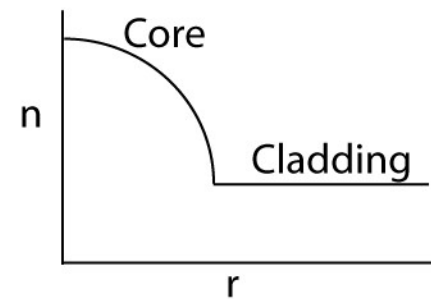
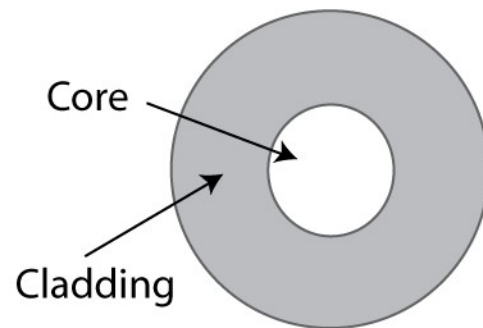
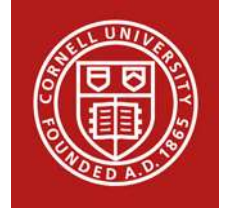
- Imaging through multimode fiber/  
complex media



*Ploschner et al., Nature Photon 2015*



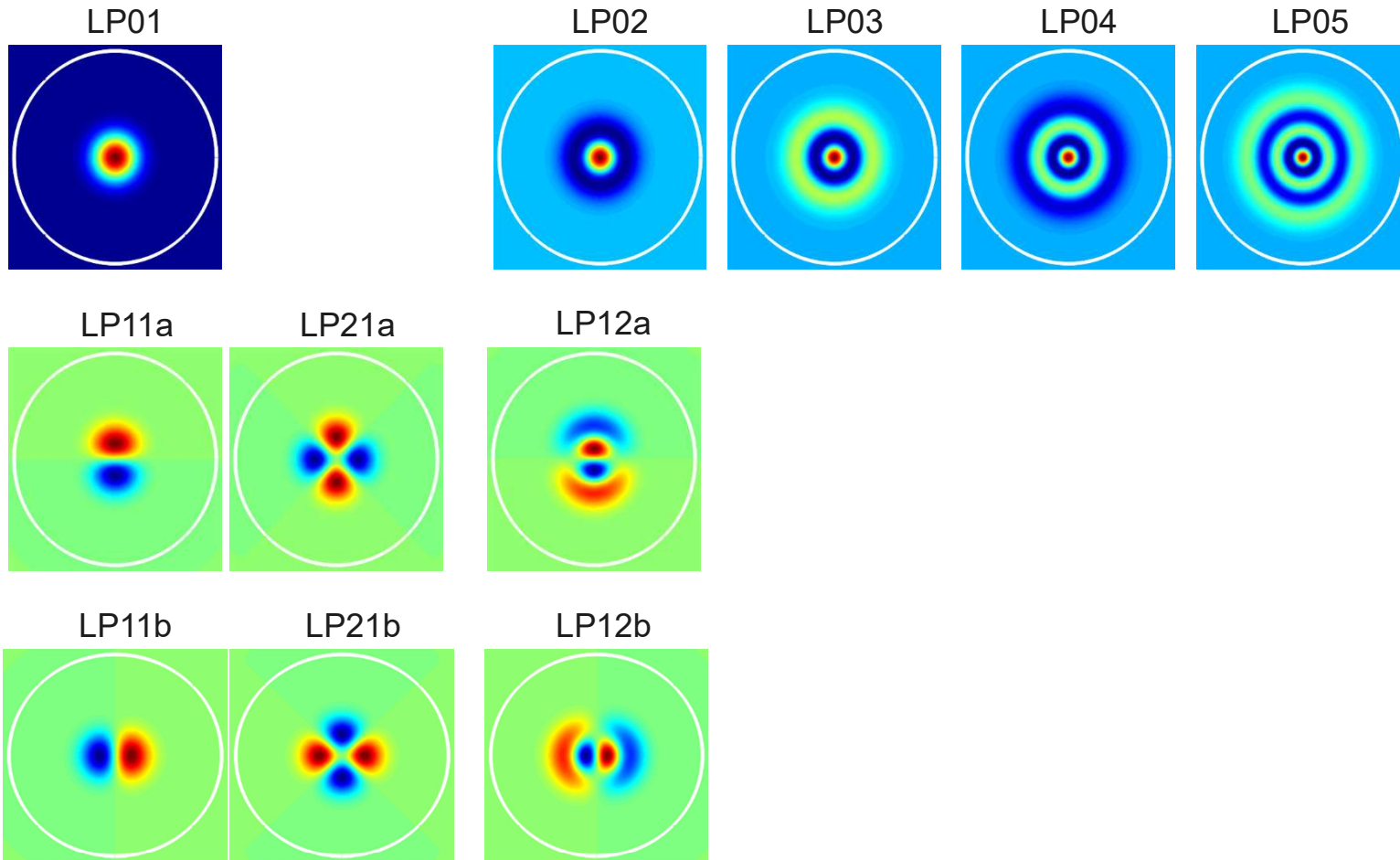
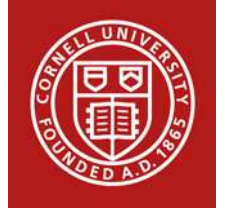
## Graded-index (GRIN) multimode fiber

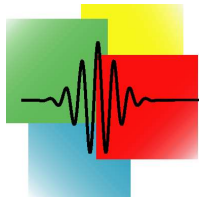


$$\begin{aligned}n^2(\rho) &= n_0^2 \left[ 1 - 2\Delta \left( \frac{\rho}{R} \right)^\alpha \right], \quad \rho \leq R \\ &= n_0^2(1 - 2\Delta), \quad \rho > R\end{aligned}$$



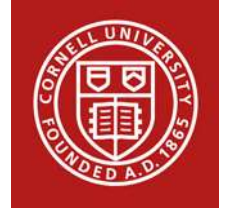
# Modes of GRIN fiber



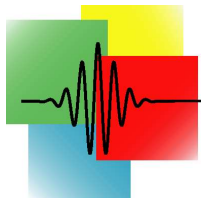
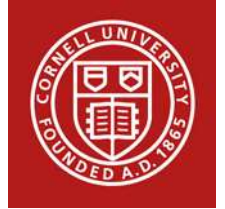


## *Modes of GRIN fiber*

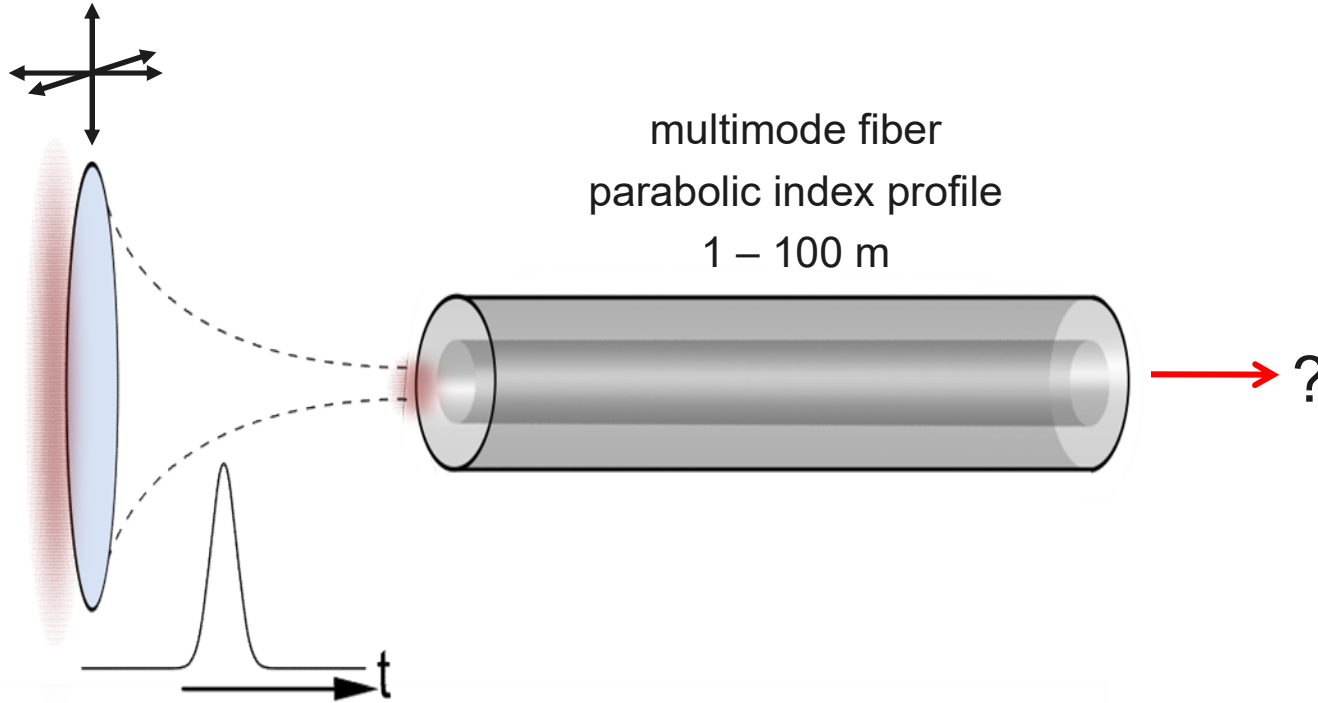
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- Propagation constants equally-spaced
- Velocities of modes vary much less than in step-index fiber



# Experiments

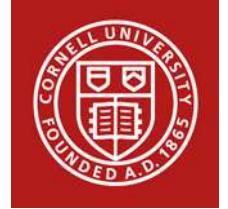


fs or ns pulses  
energy up to  $1 \mu\text{J}$   
peak power kW to MW

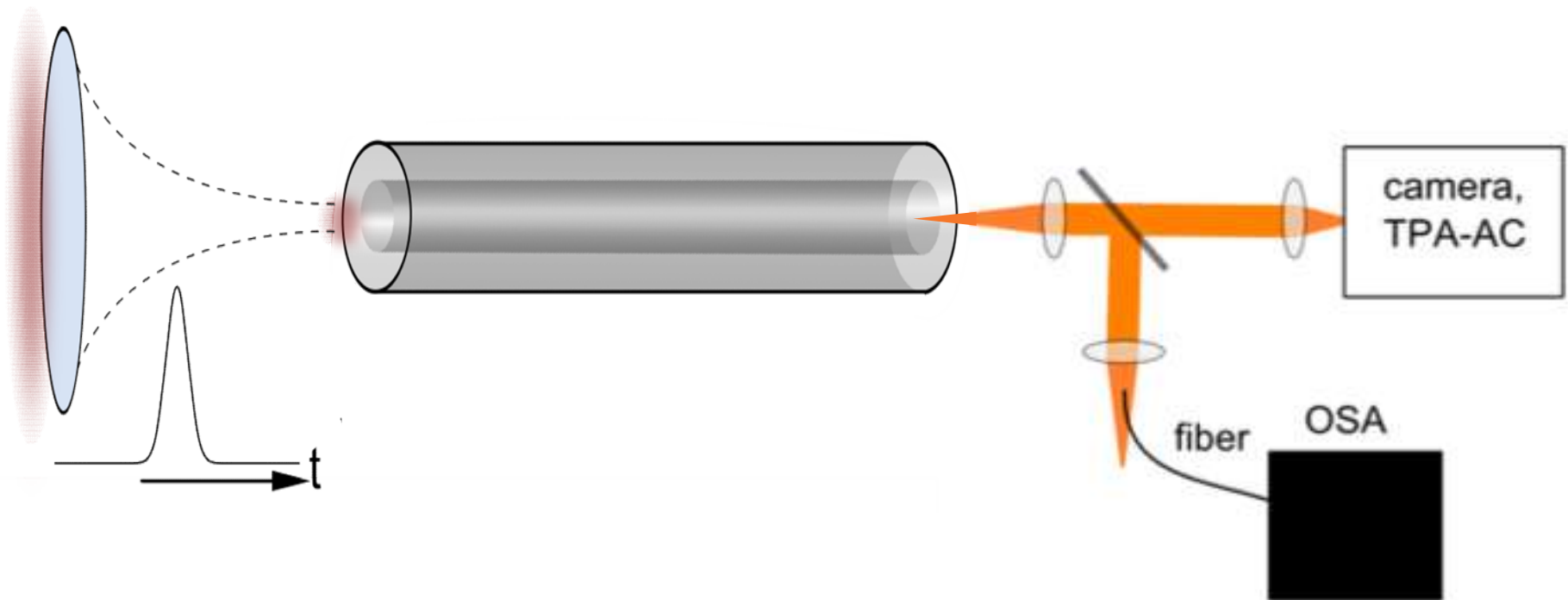
1550 nm  
1050 nm  
532 nm



## What should we measure?



- Broadband space-time diagnostic does not exist



- Record overall average spectrum to compare to calculated
- Image near-field on autocorrelator
- Compute spatiotemporal autocorrelation for comparison

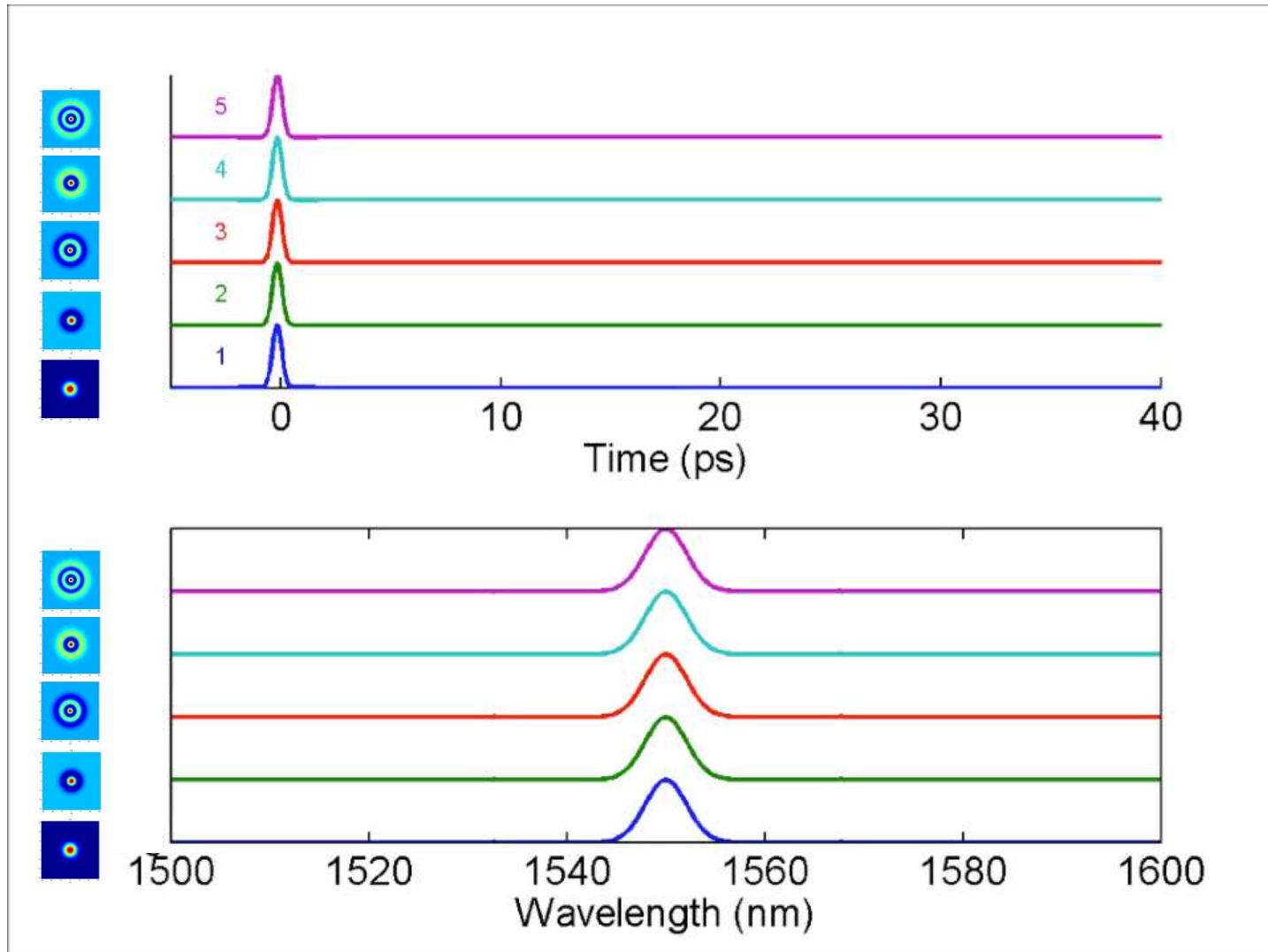




## *Multimode Solitons*

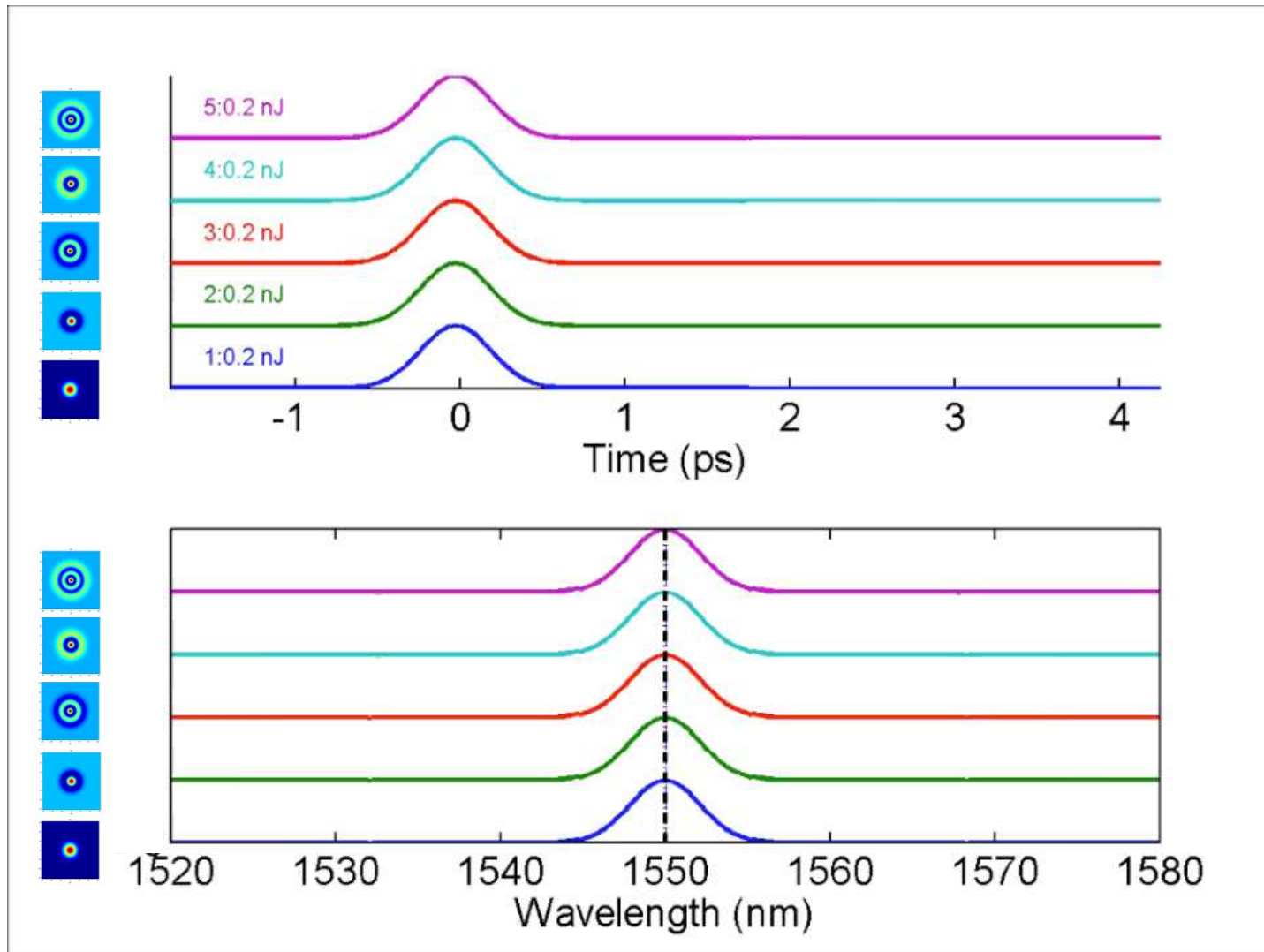
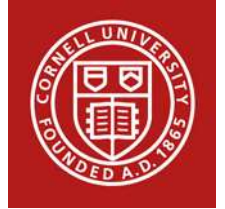


# Linear propagation





# Multimode soliton formation

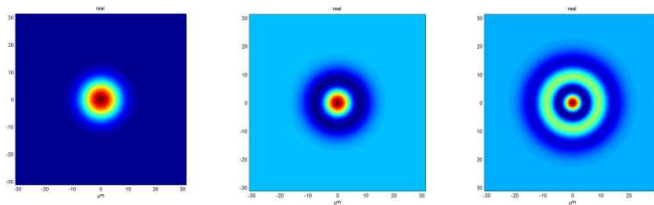
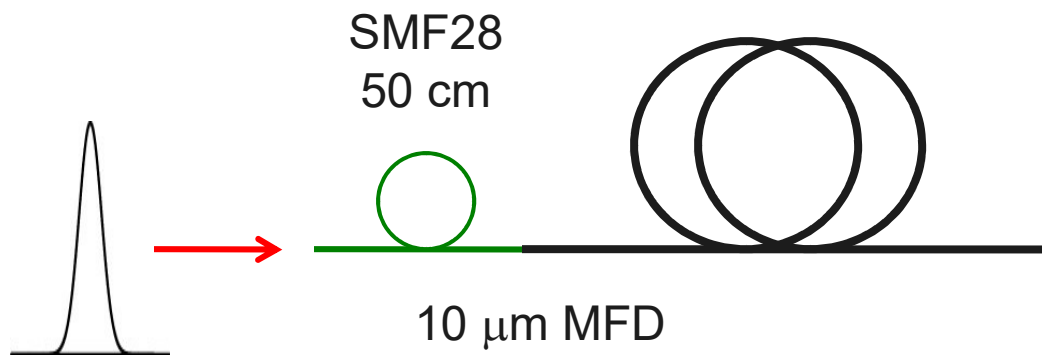




## First steps: 3 modes



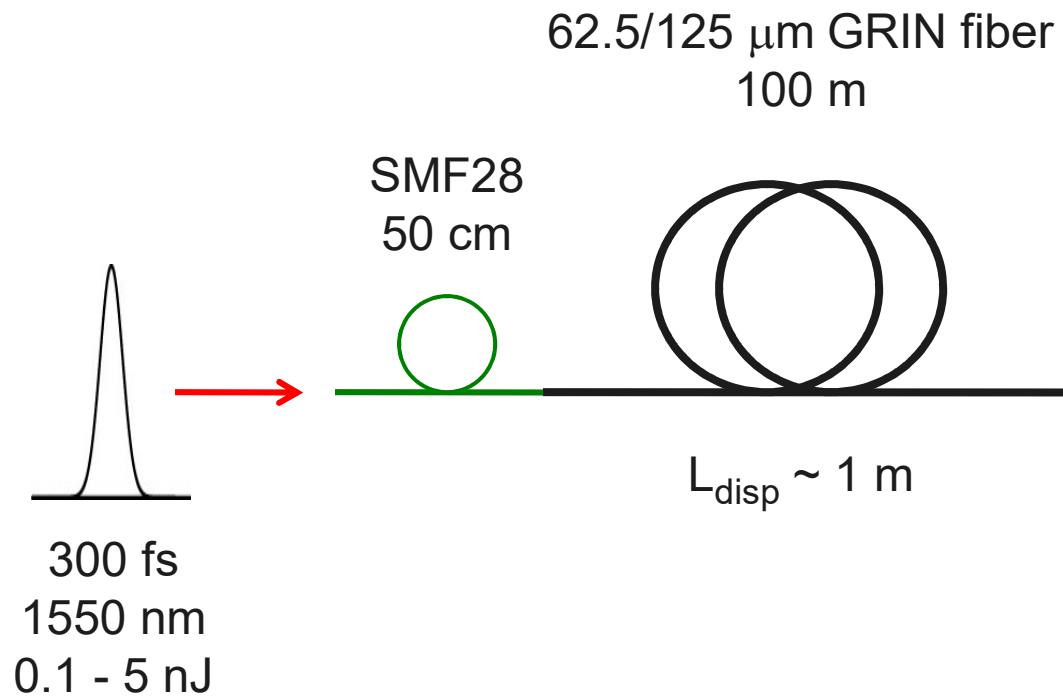
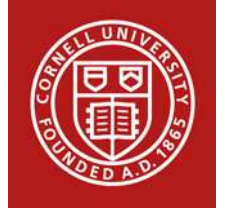
62.5/125  $\mu\text{m}$  GRIN fiber  
supports  $\sim 100$  modes



- Excite 3 lowest modes

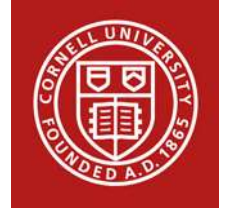


# Experiment

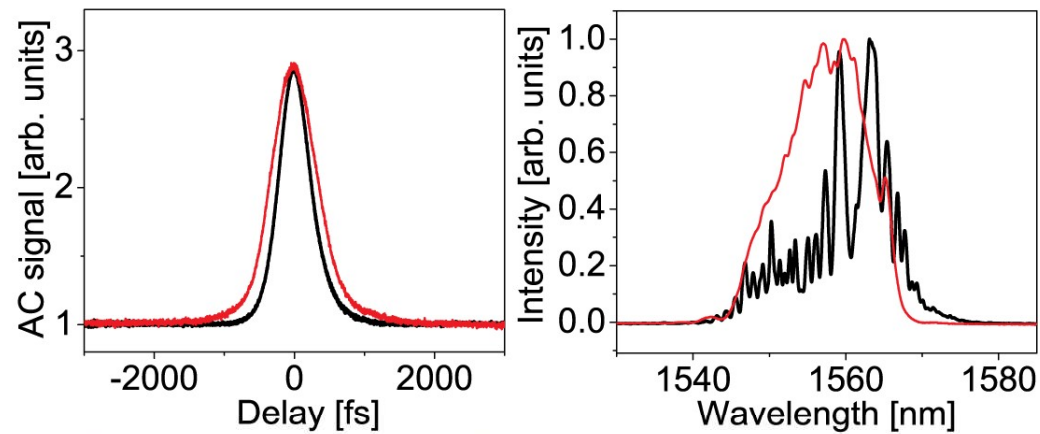




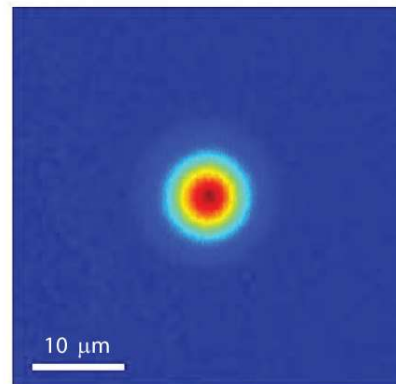
## Experimental results



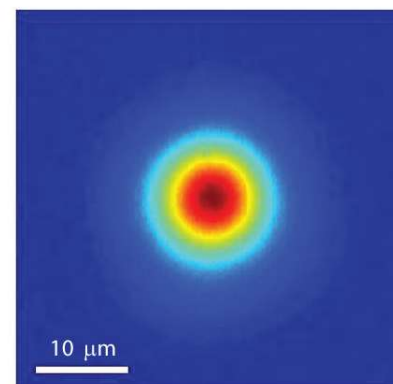
- For  $E < 0.1$  nJ pulse disperses
- 0.5 nJ pulse energy



input  
output



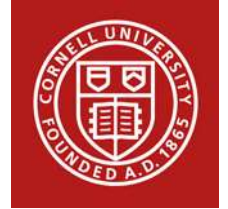
input



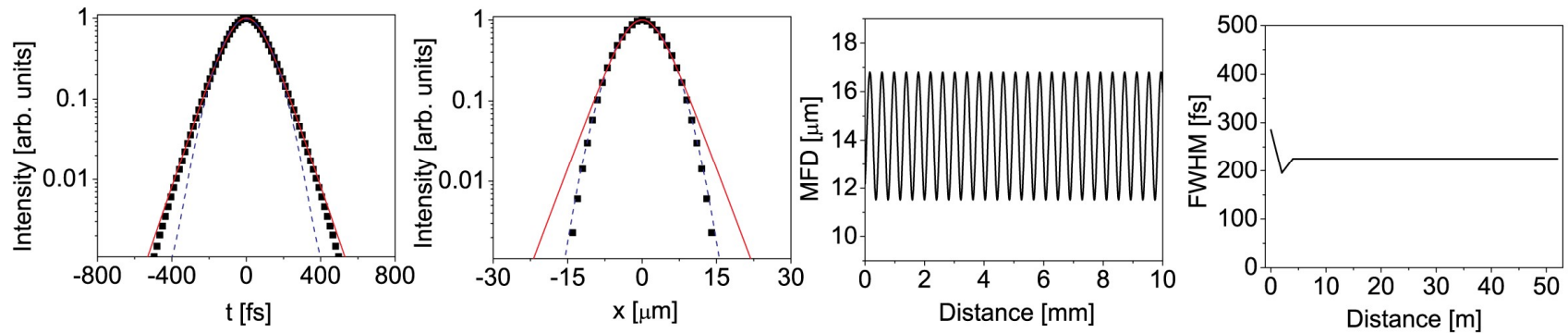
output



## 3 modes: theory



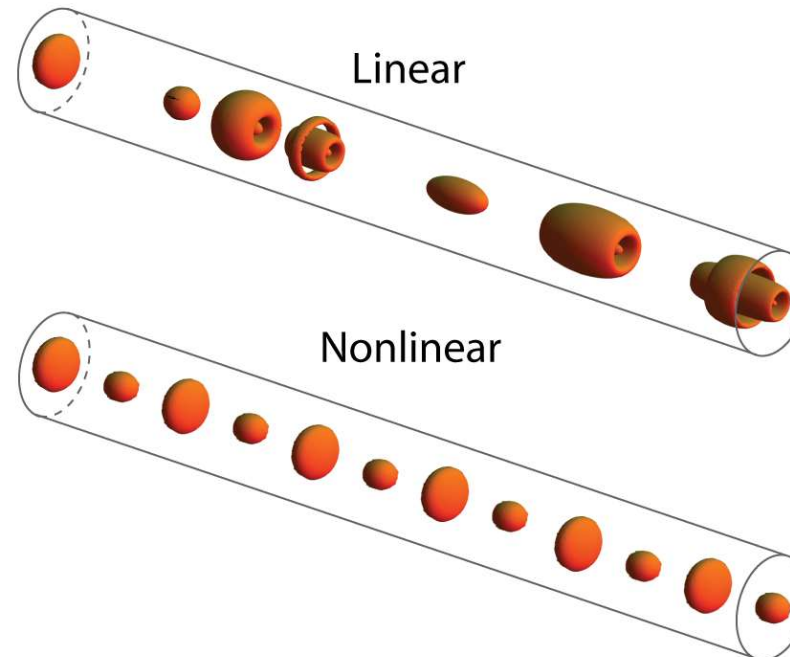
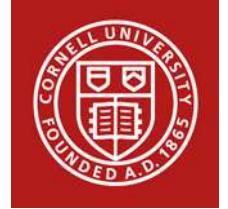
- Launch 0.5 nJ / 300 fs



- Coupled-mode theory and beam-propagation give similar results



## Intuitive picture



Renninger *et al.*, *Nature Commun* 2013

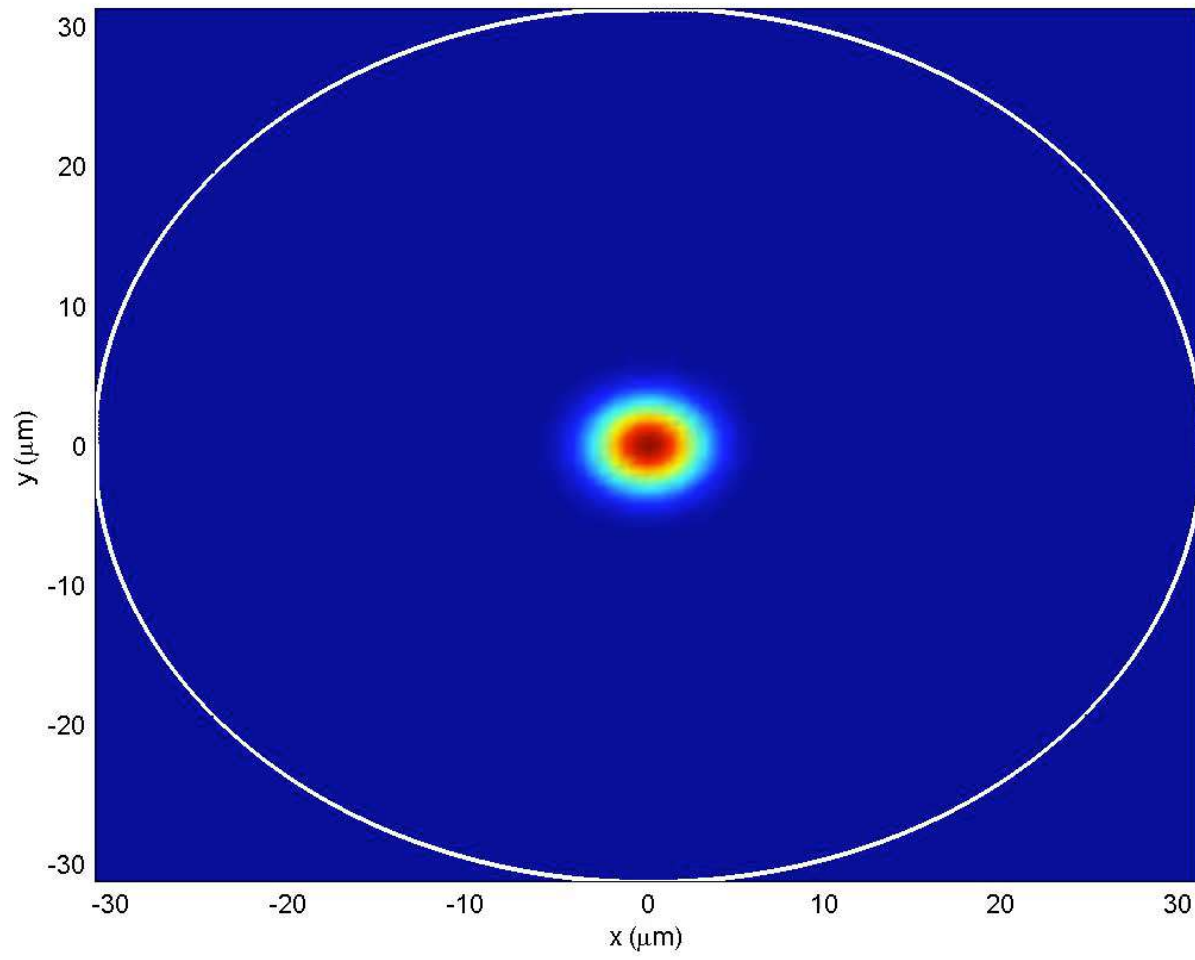
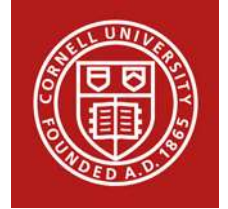
- Solitons with more modes require greater nonlinear phase / energy
- Solitons with up to 10 modes generated

Wright *et al.*, *Opt Exp* 2015



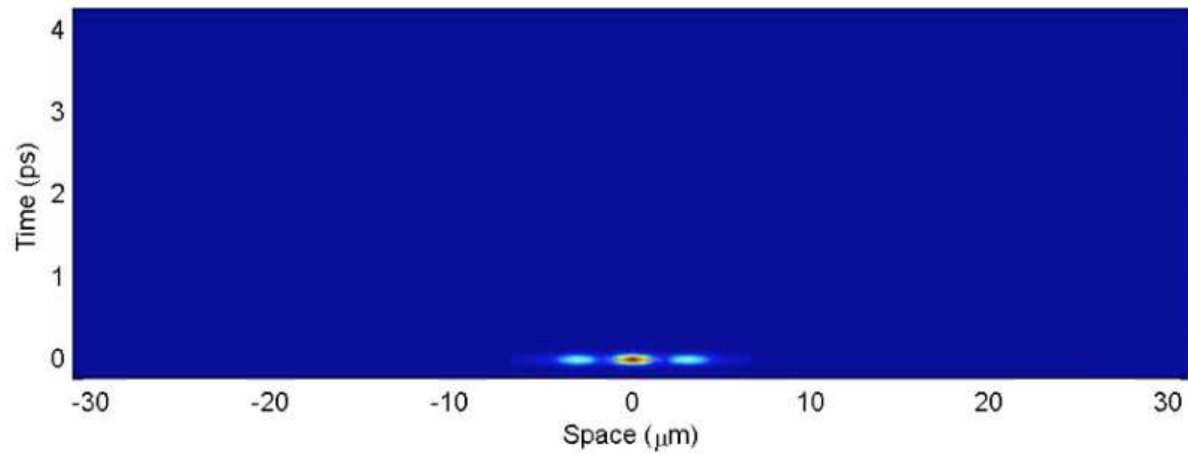
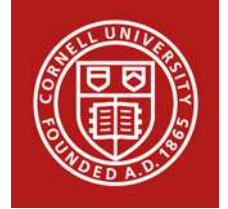


# Multimode soliton formation



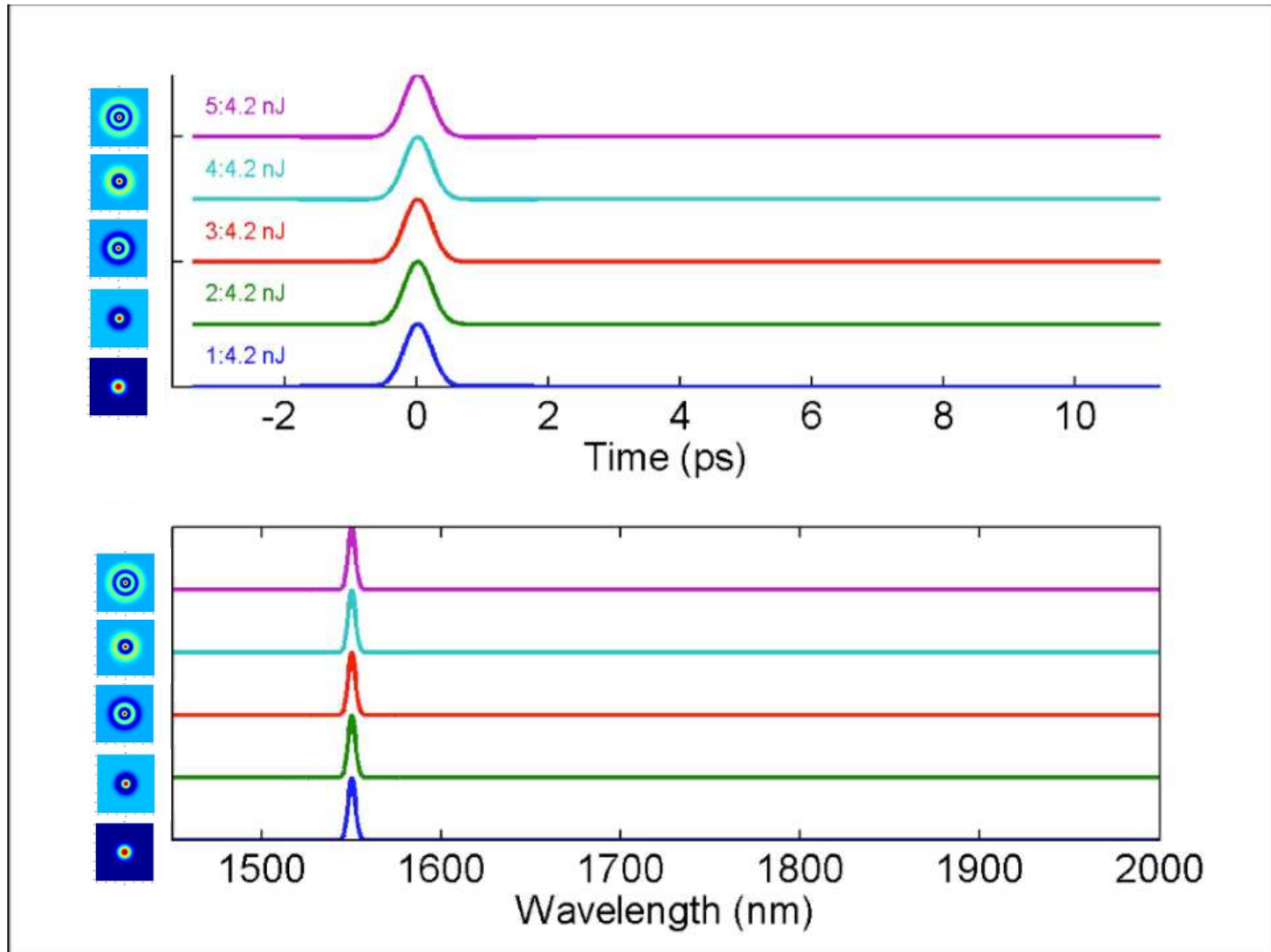
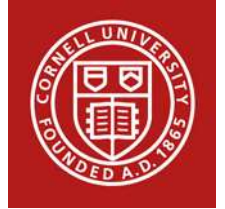


# *Multimode soliton formation*



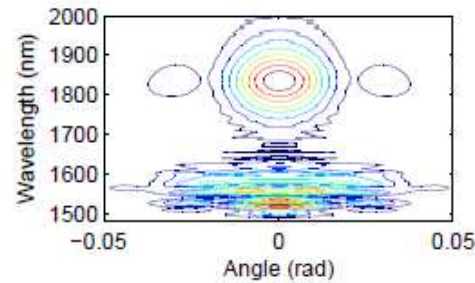
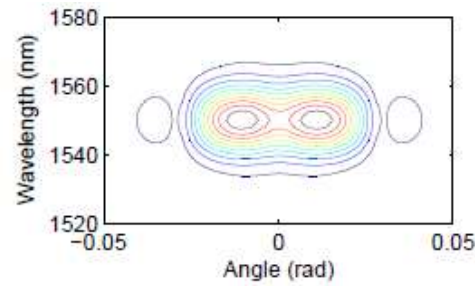
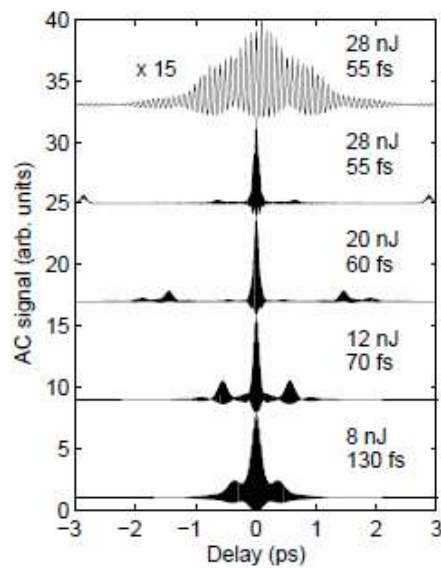
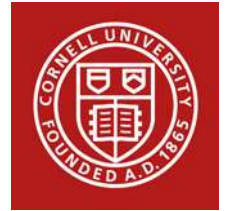


# Multimode soliton fission



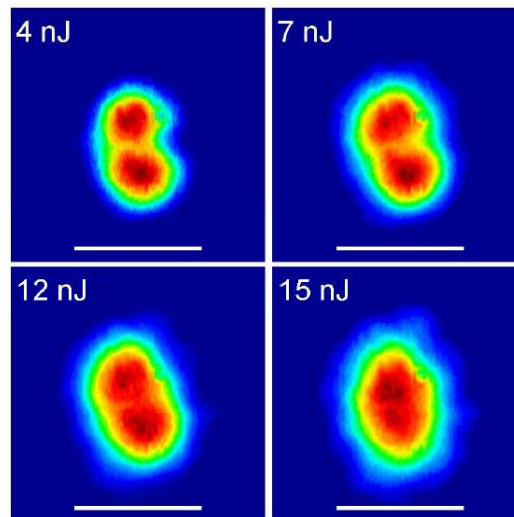
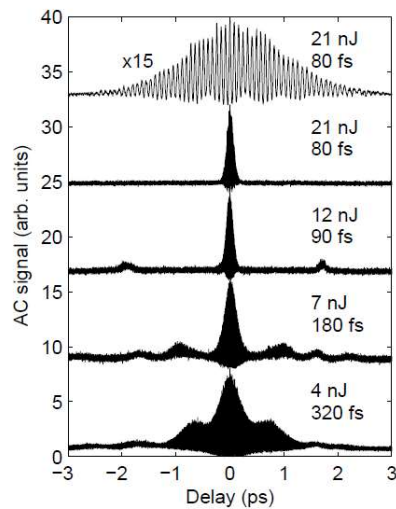


# Multimode soliton fission: experiment



## Simulation

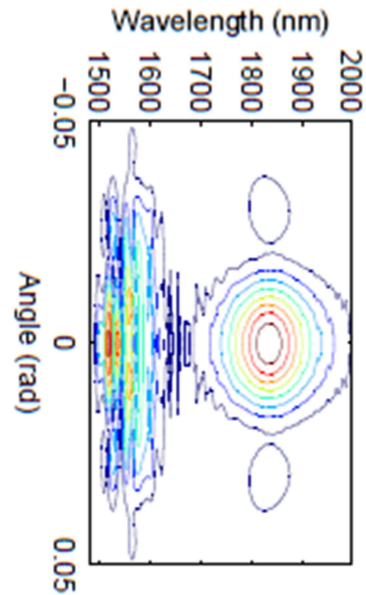
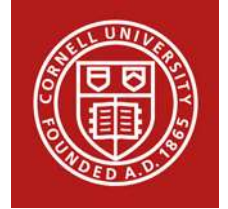
- Smaller peaks in AC from less-localized modes
- Intermodal energy transfer during, after fission



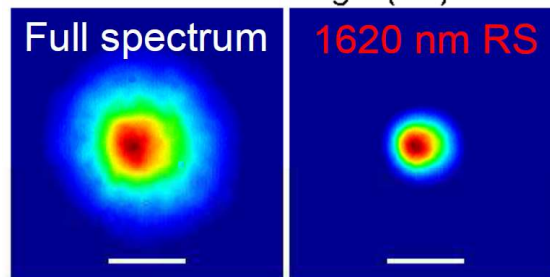
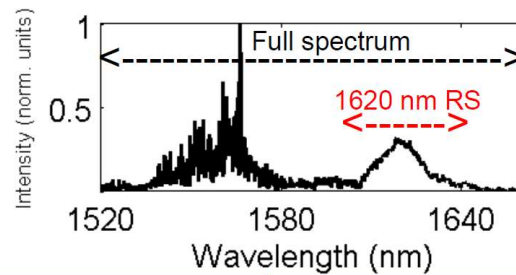
## Experiment



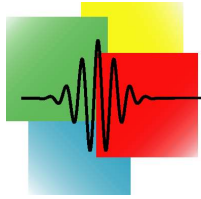
# Multimode soliton fission: experiment



Simulation

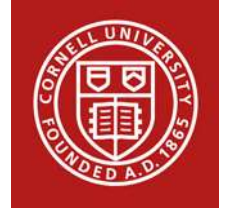


Experiment



## *Multimode soliton fission*

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- Fission produces multiple MM solitons and MM dispersive waves
- Fission is spatiotemporal
- Raman “focuses” energy into the low-order mode

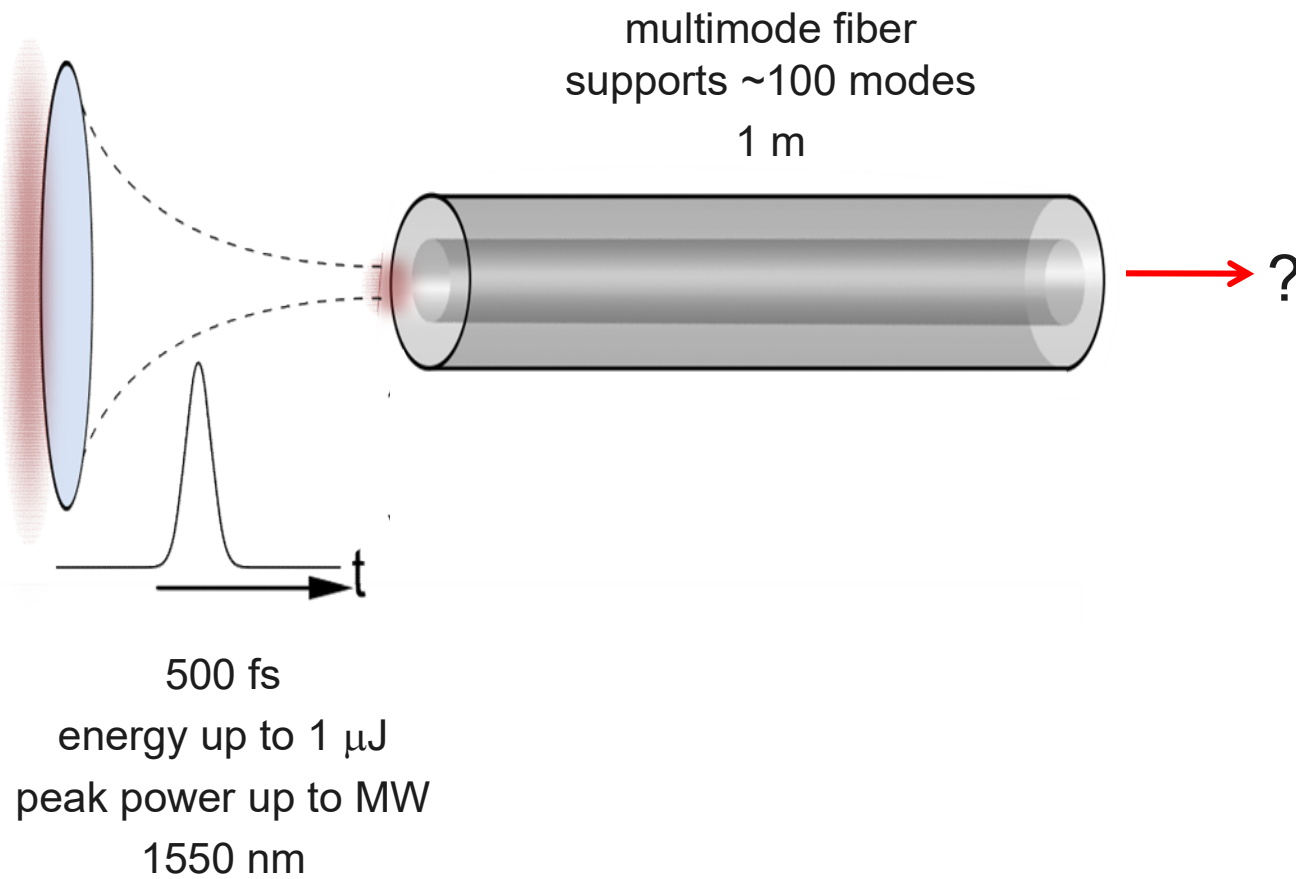
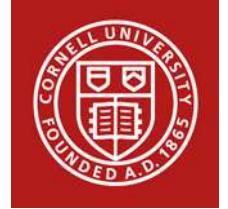
*Wright et al., Opt Express 2015*



## *Continuum Generation*



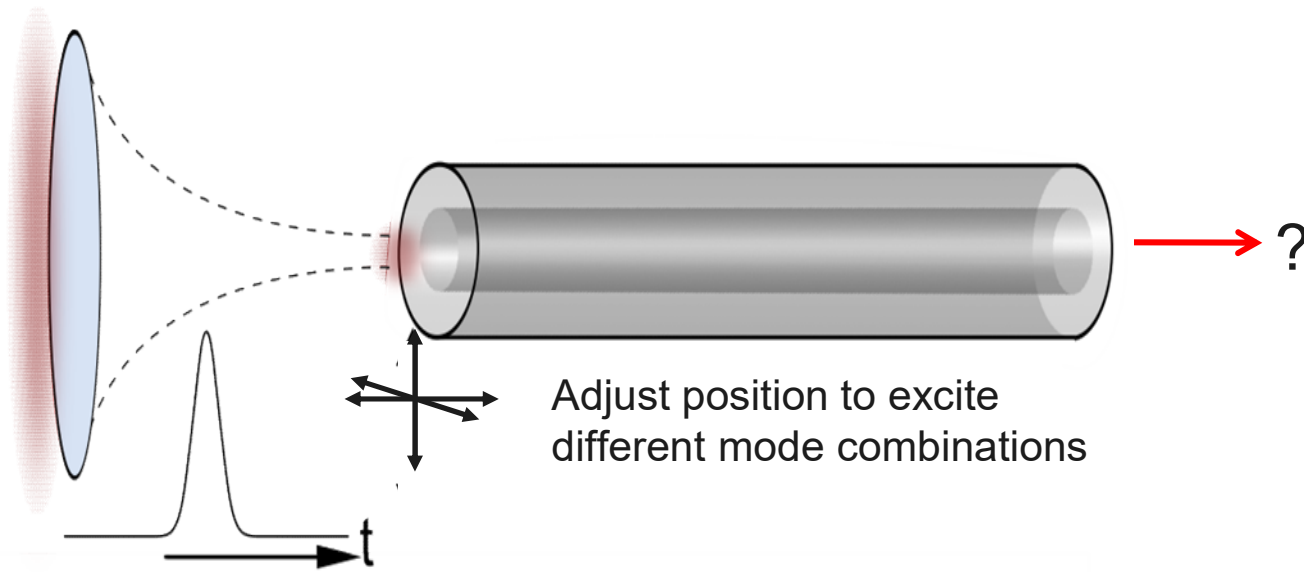
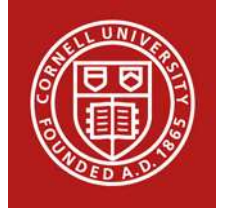
# Experiments







# Experiments

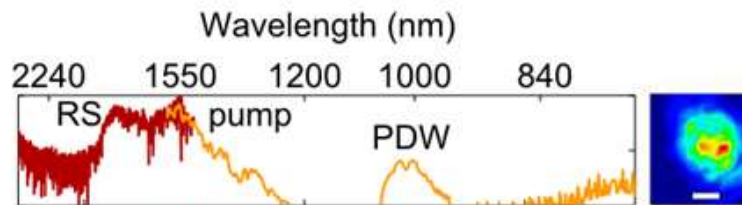




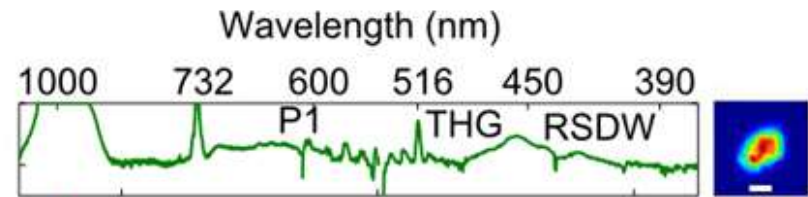
# *Spatial conditions determine the continuum*



Typical  
IR

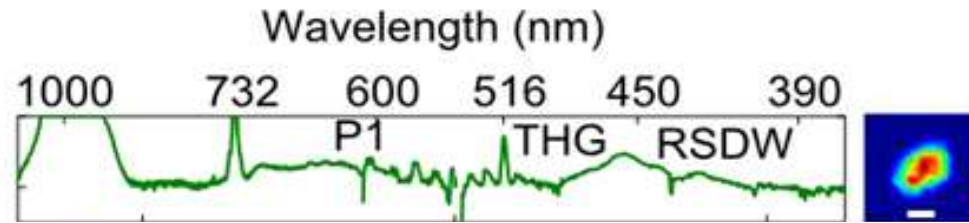
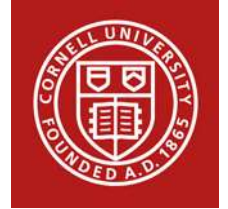


Typical  
visible



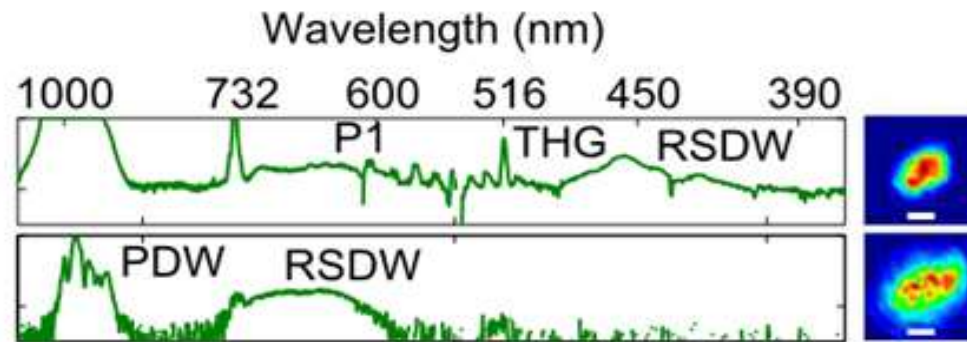
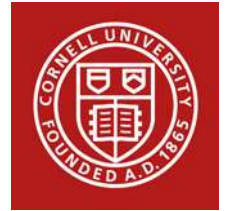


# *Spatial conditions determine the continuum*



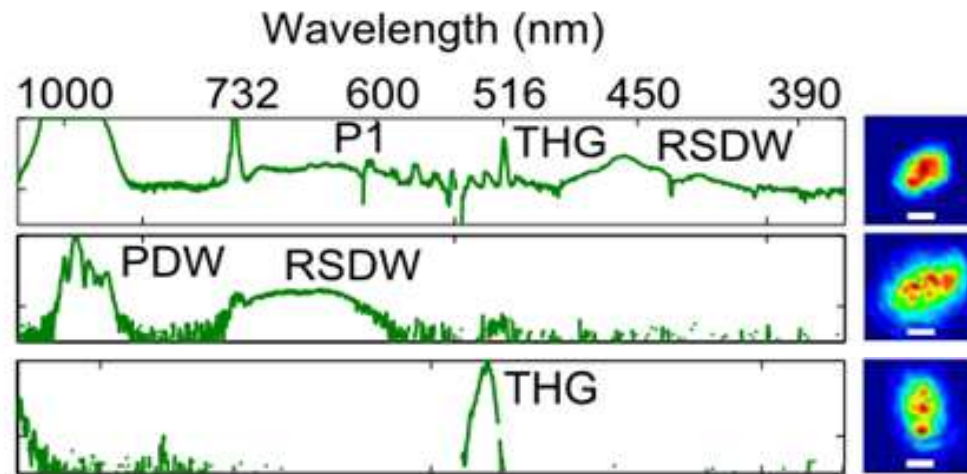
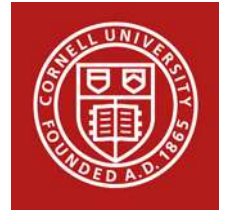


# *Spatial conditions determine the continuum*



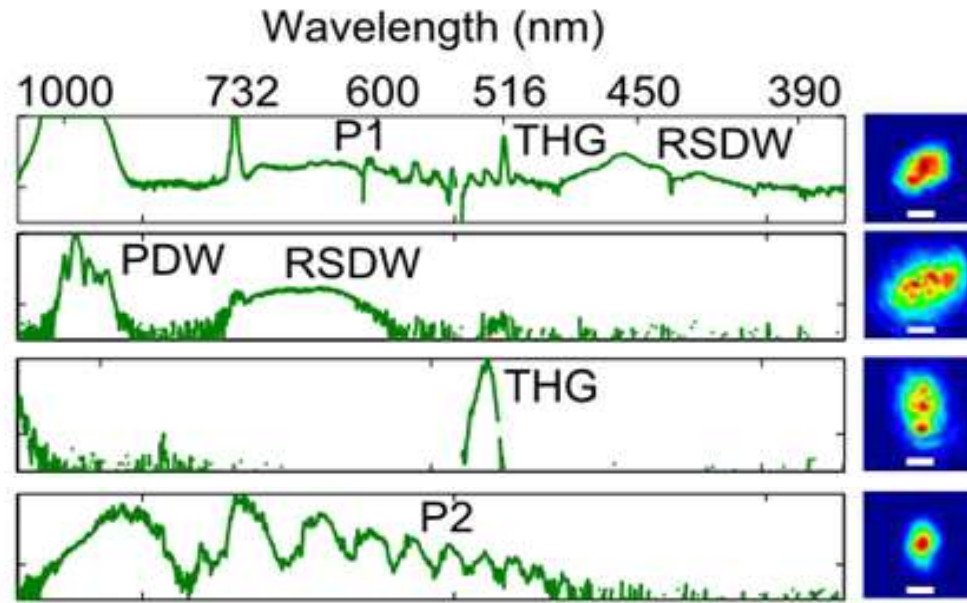
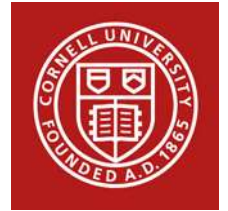


# *Spatial conditions determine the continuum*



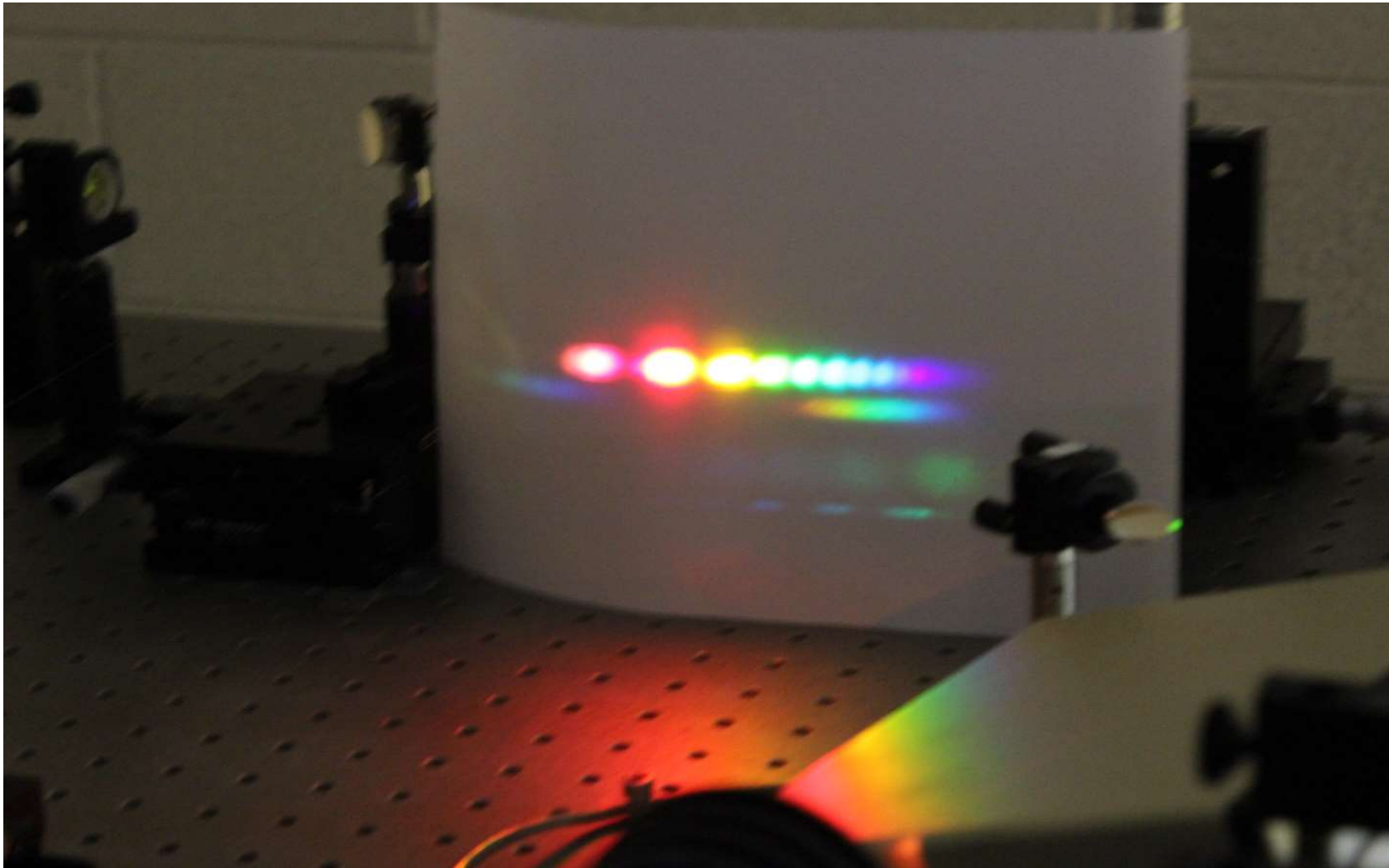


# *Spatial conditions determine the continuum*





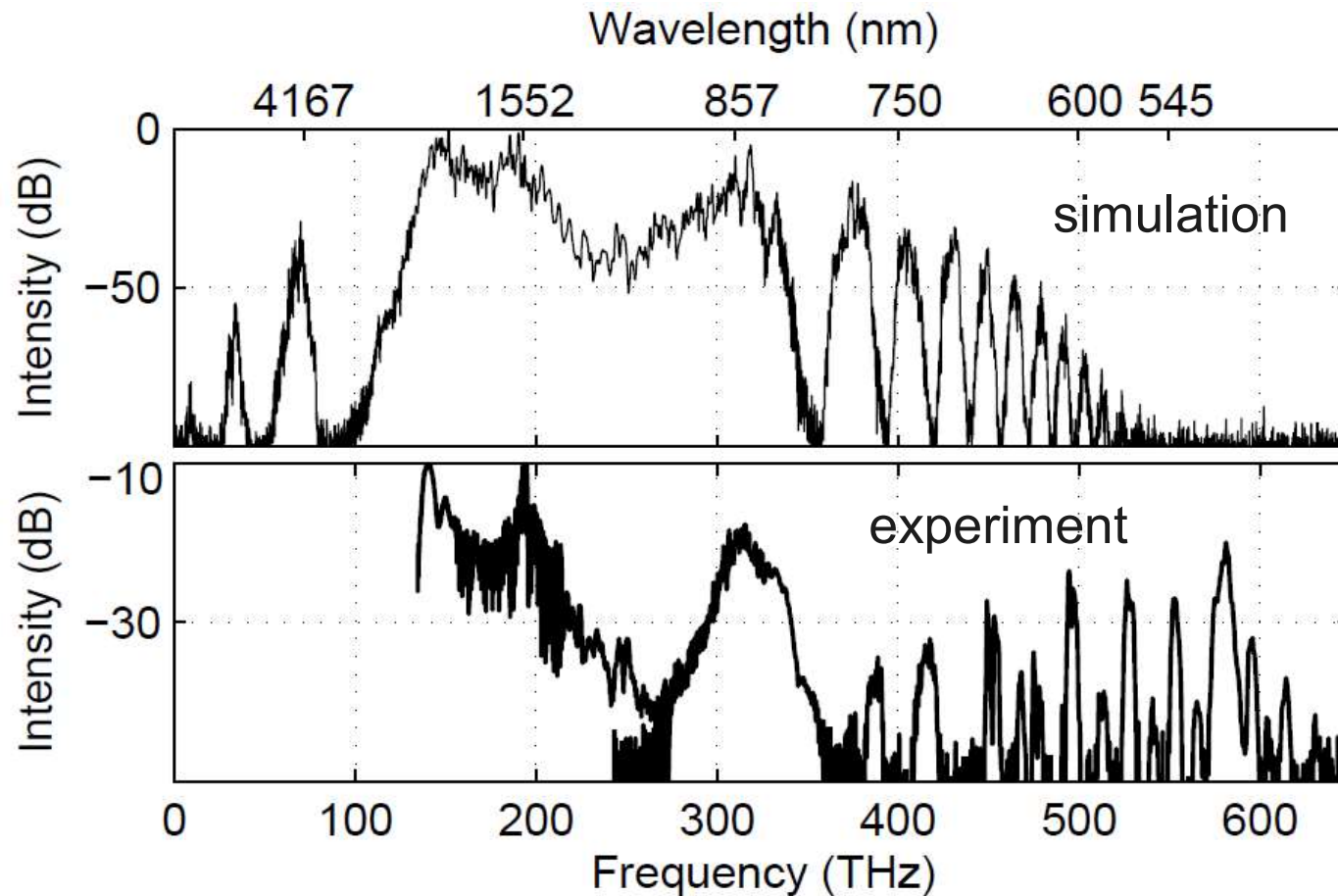
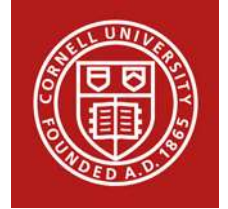
# *Spatial conditions determine the continuum*







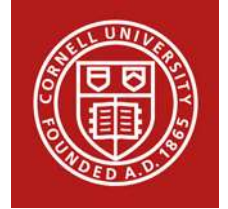
# What is the origin of bright visible peaks?







## Perturbation of solitons (1D tutorial)



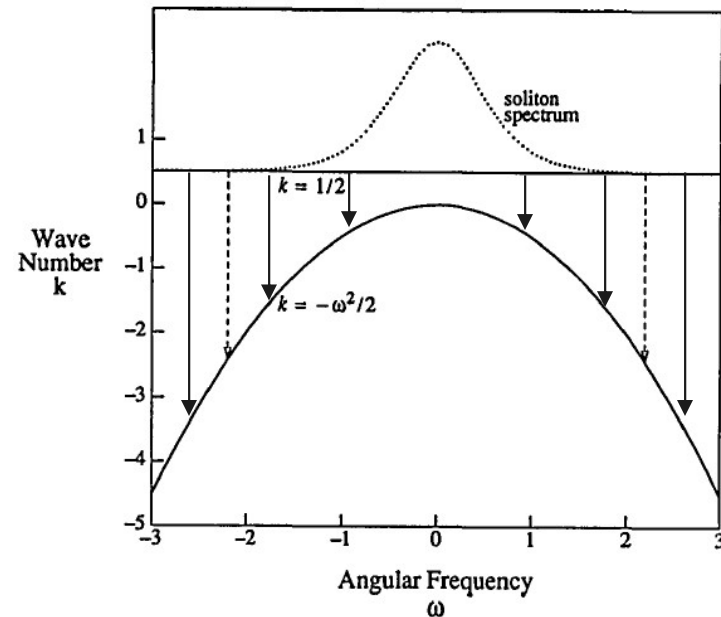
- Perturbed soliton adjusts to reach  $A_0\tau_p = \sqrt{\frac{\beta_2}{\gamma}}$

and radiates dispersive wave

- Periodic perturbation (period =  $Z_c$ )  
Resonant energy transfer when wave vectors match

$$(k_{sol} - k_{dis}) = 2m\pi/Z_c$$

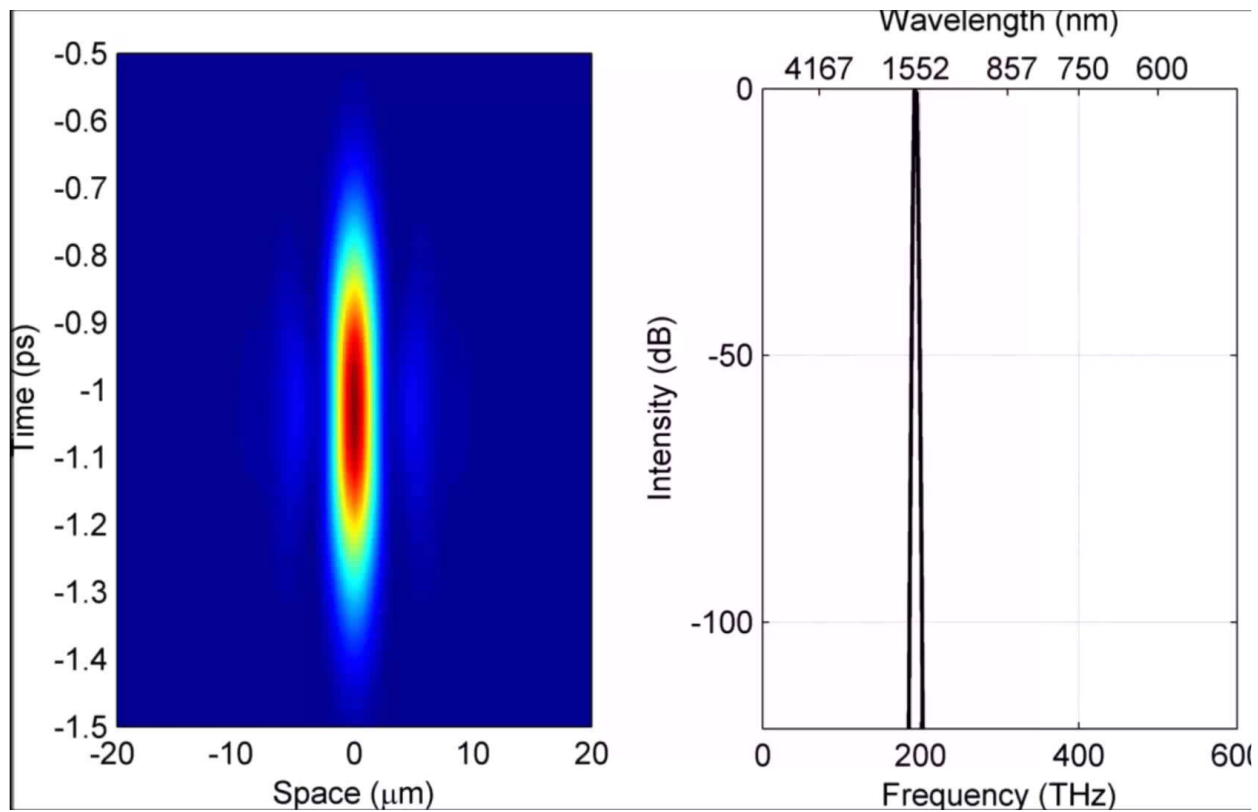
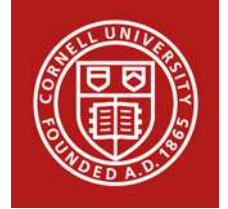
$$\Omega_{res} = \frac{1}{\tau} \sqrt{\frac{8Z_0m}{Z_c} - 1}$$



Gordon, *J Opt Soc Am B* 1992

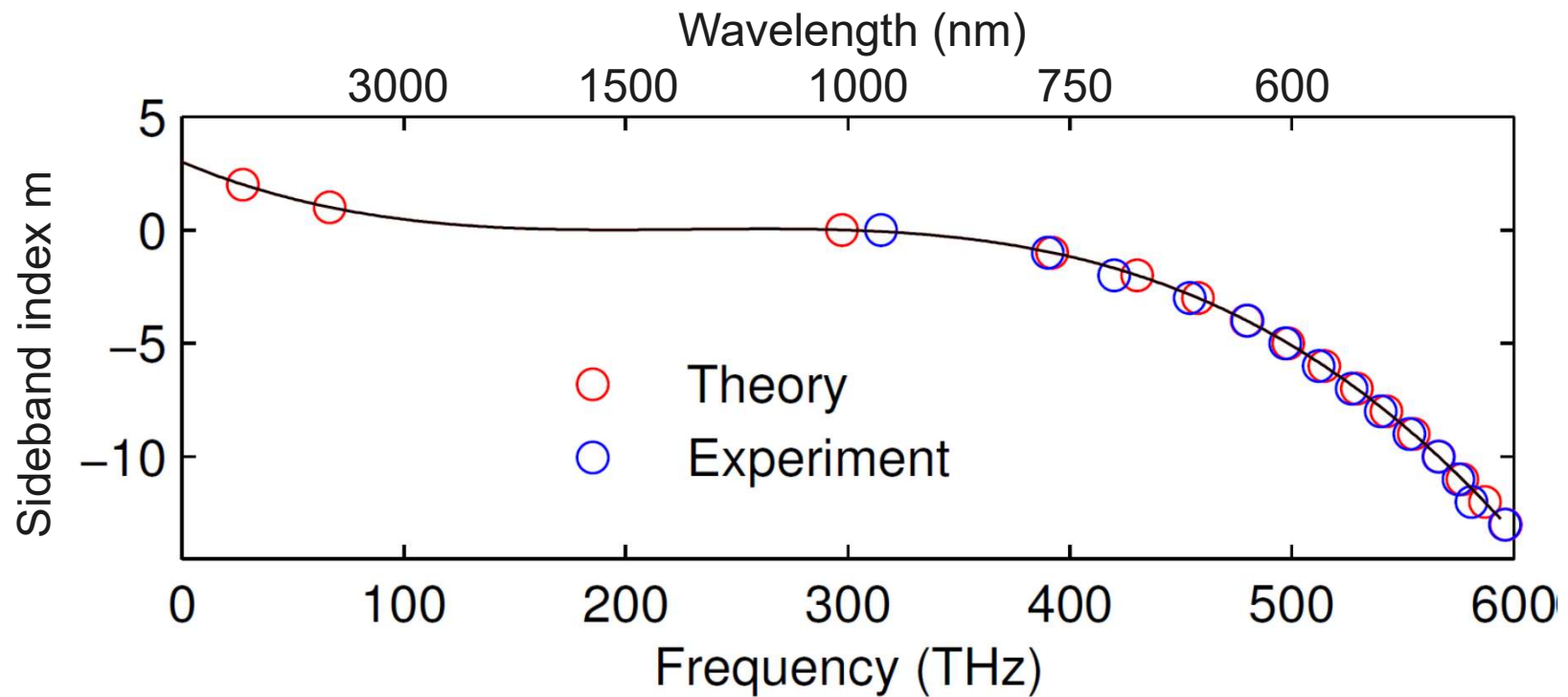
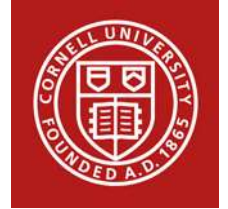


# *Spatiotemporal oscillations*





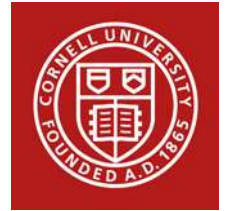
## Theory and experiment

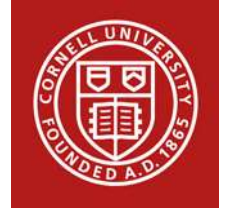


- Simulation, experiment and analytic theory agree well



*Oscillations about equilibrium as an instability:  
why more degrees of freedom matters*

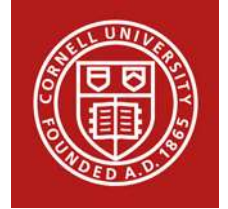




- Continuum is controllable through launched spatial modes
- Spatiotemporal oscillation leads to the generation of multimode dispersive waves
- Phenomenon understood in terms of multimode soliton dynamics

*Wright et al., Nature Photon 2015*

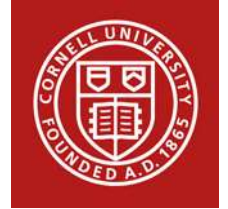
*Wright et al., Phys Rev Lett 2015*



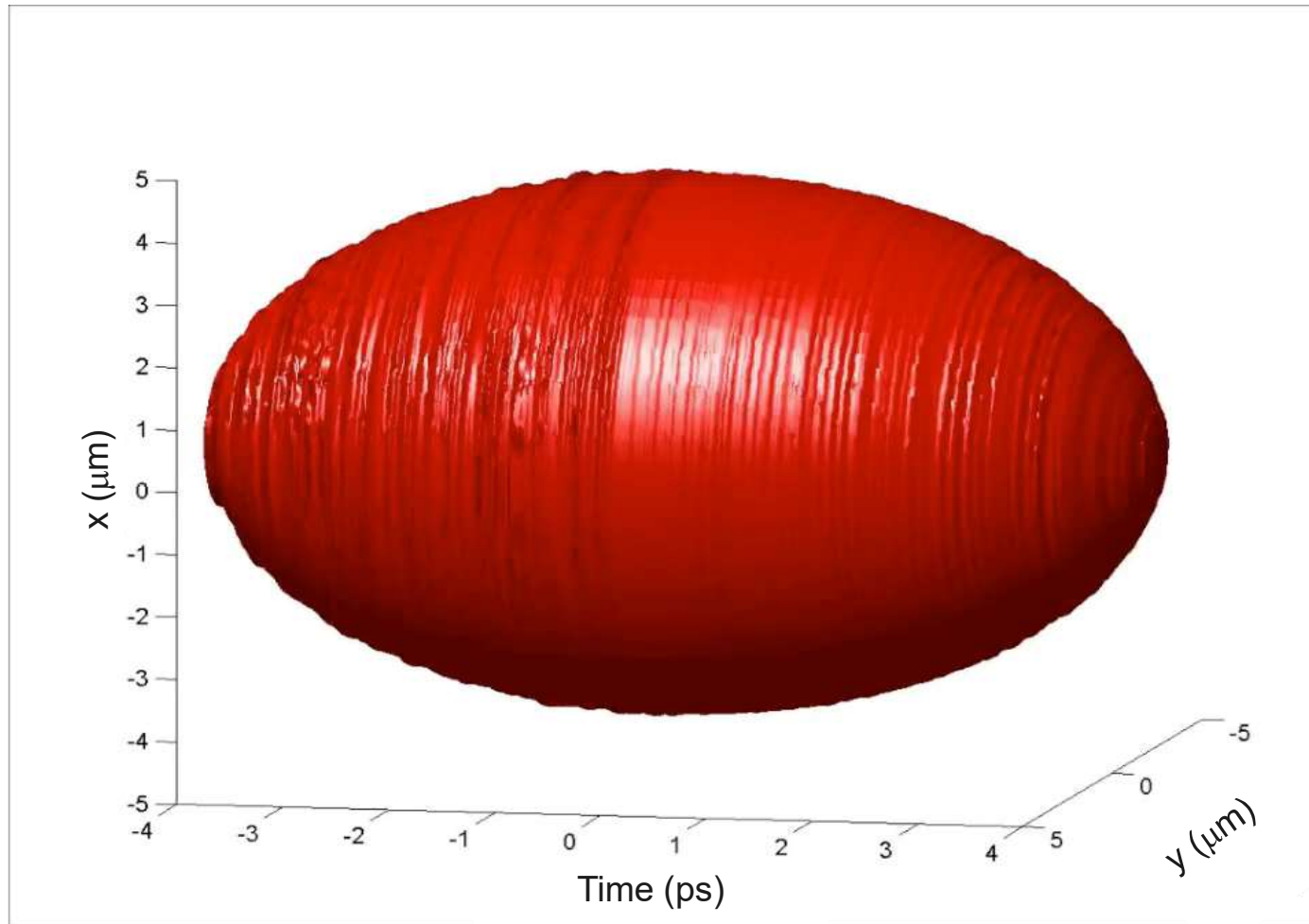
# *Spatiotemporal Modulation Instability*



# *Spatiotemporal modulation instability*

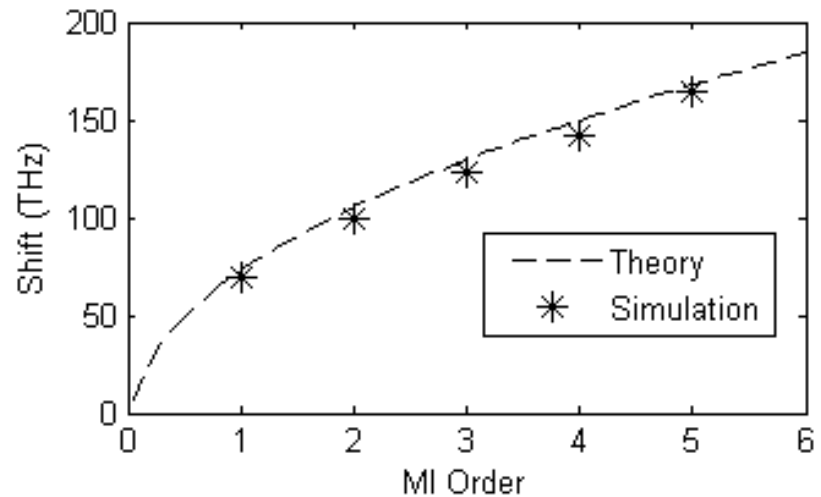
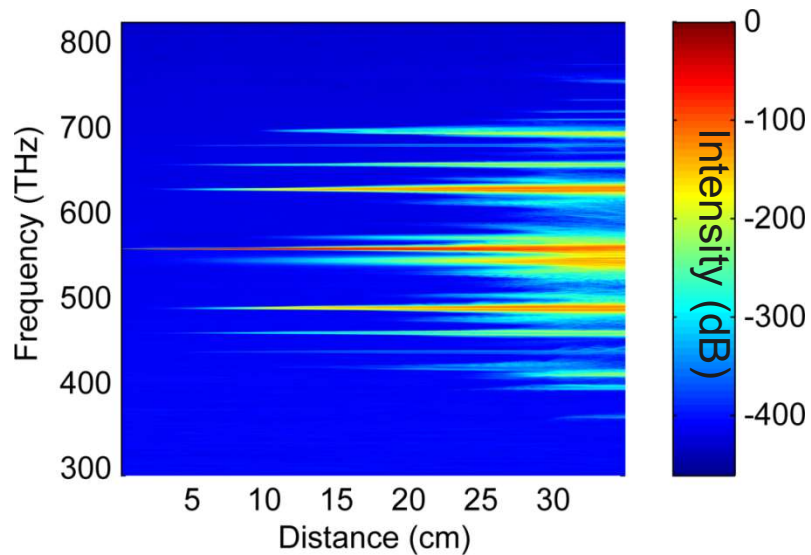
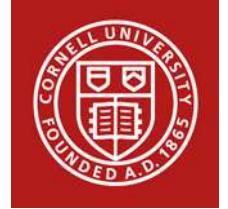


- Launch continuous wave or long pulse at normal dispersion





# Spatiotemporal modulation instability



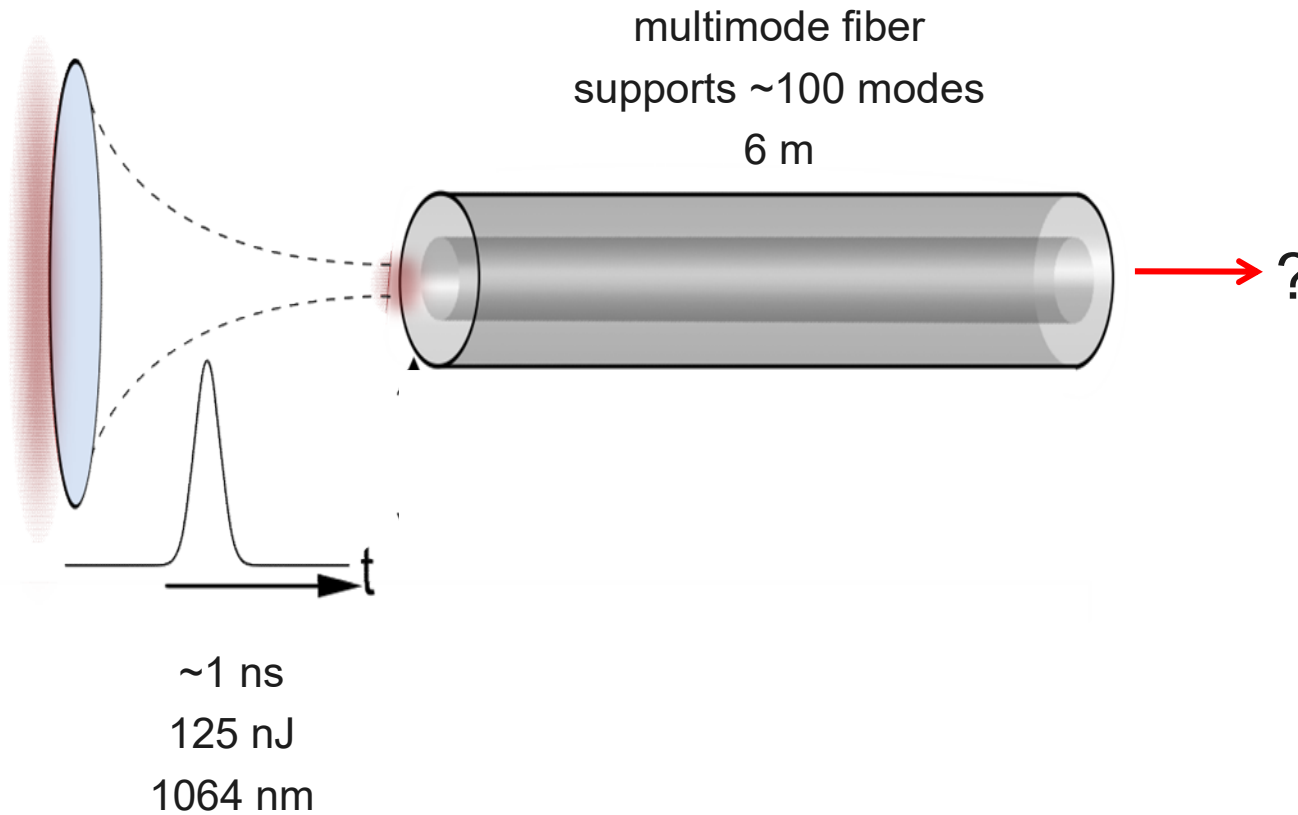
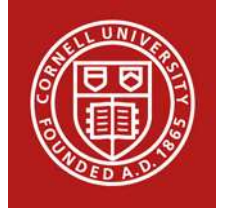
- Periodic self-imaging plays a role
- Instability occurs for either sign of dispersion

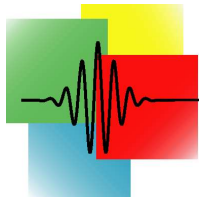
*Longhi, Opt Lett 2003*  
*Matera et al., Opt Lett 1993*  
*Nazemosadat et al., JOSA B 2016*



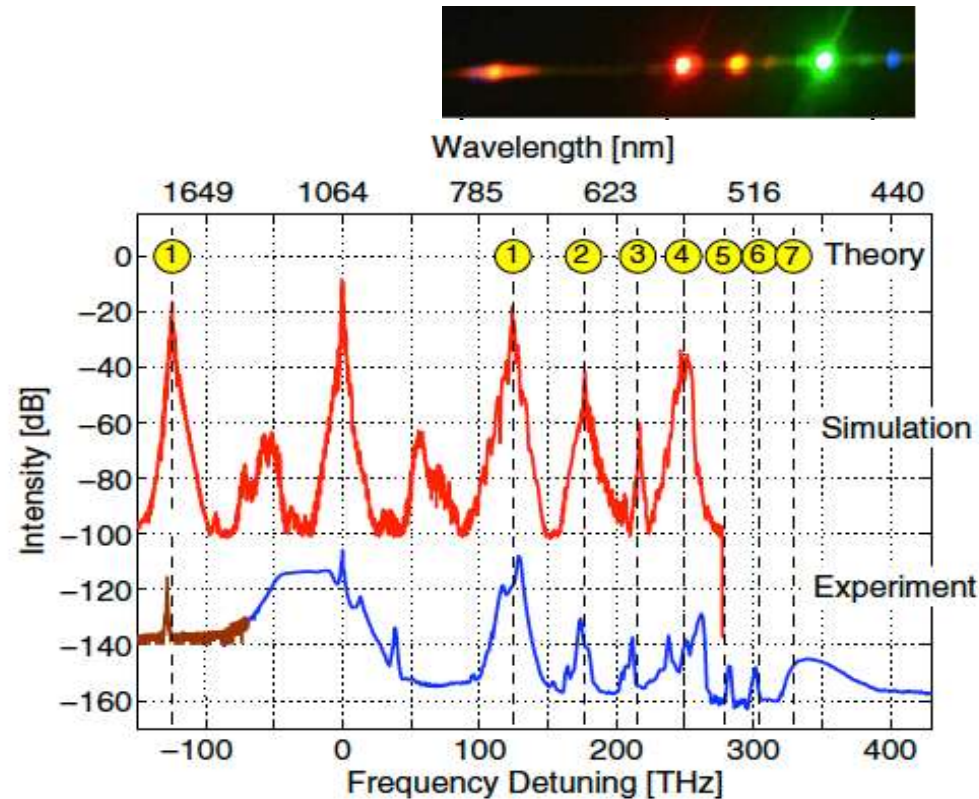
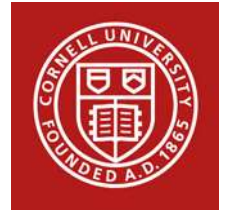


# Spatiotemporal MI in GRIN fiber

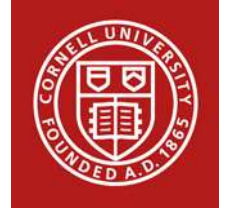




# Spatiotemporal MI in GRIN fiber



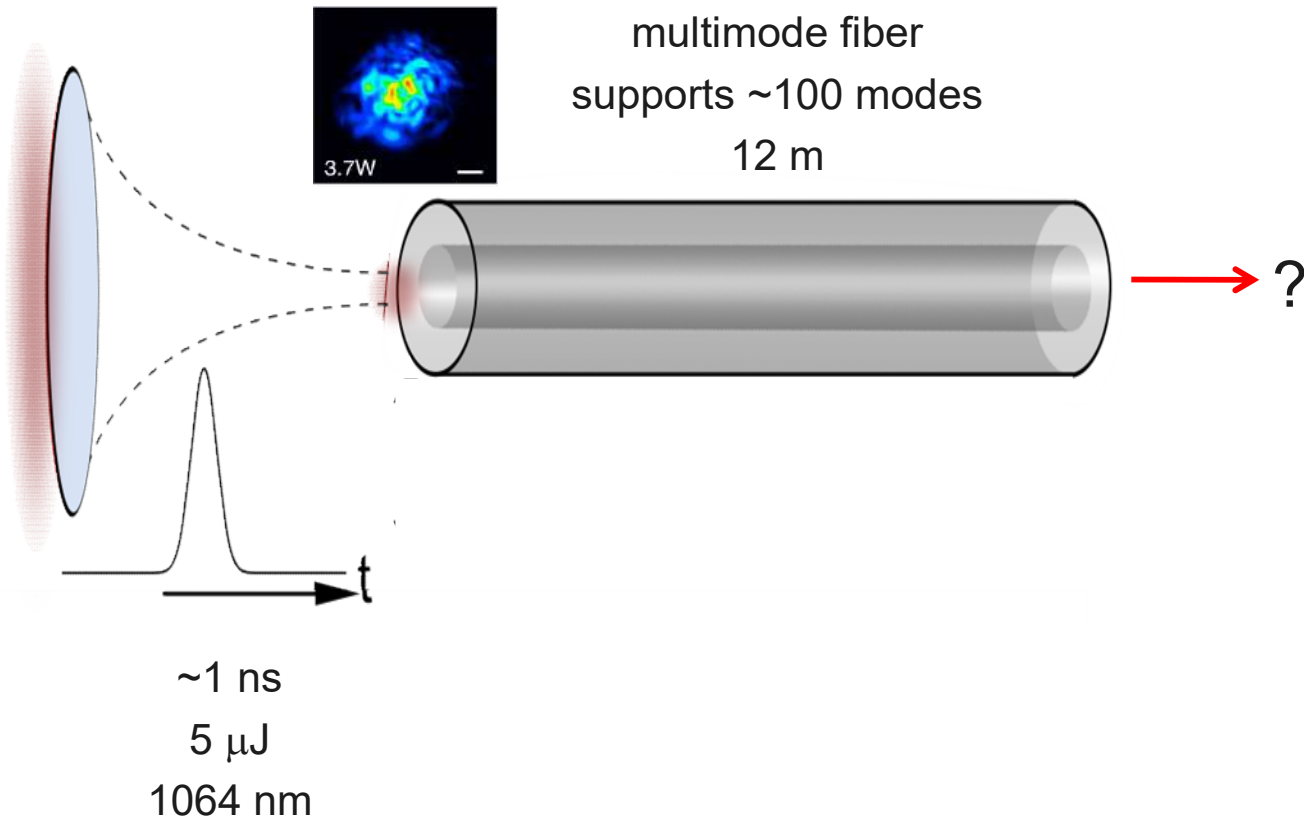
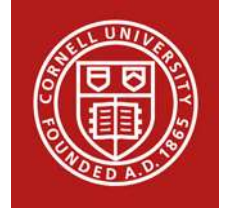
- Geometric parametric instability: periodic self-imaging of field allows quasi-phase-matching of 4WM sidebands



## *Beam Self-Cleaning in Multimode Fiber*

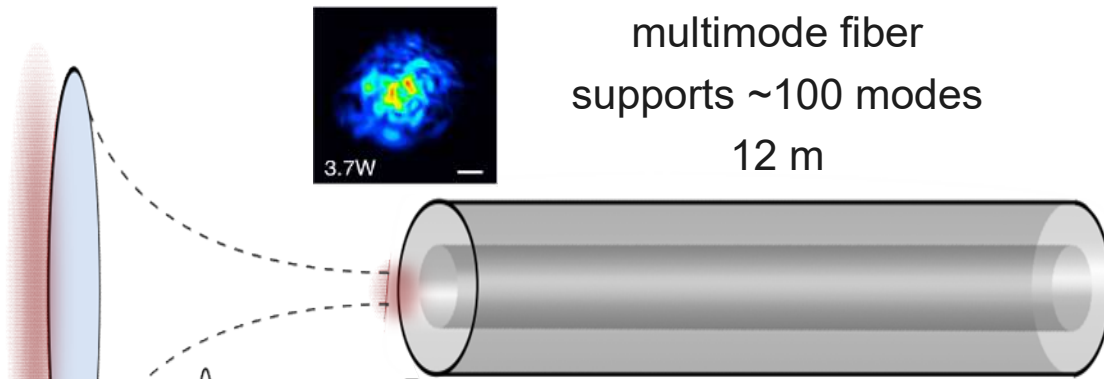
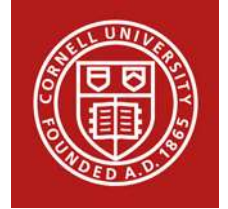


# Beam self-cleaning in GRIN fiber

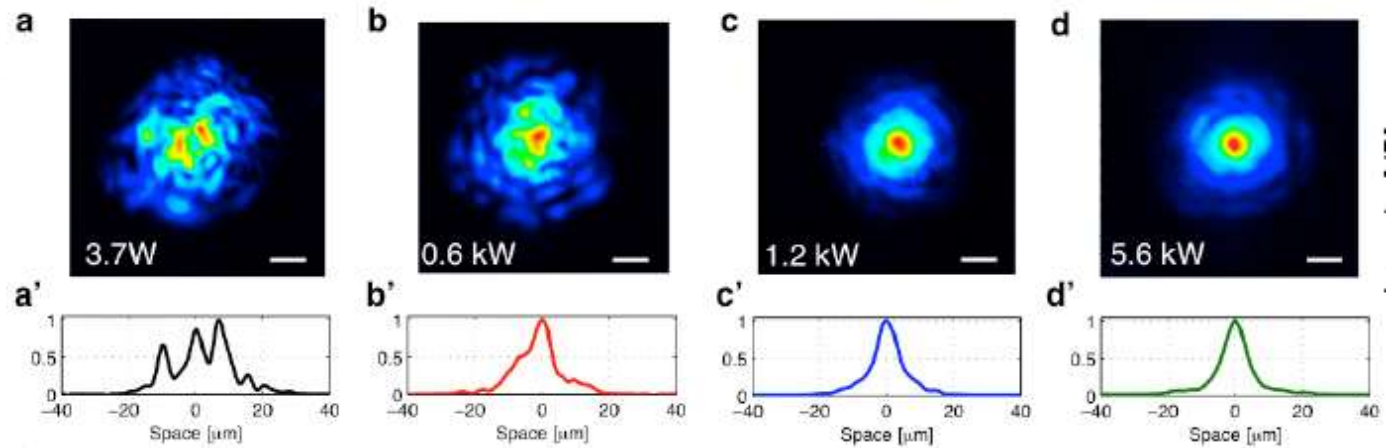




# Beam self-cleaning in GRIN fiber

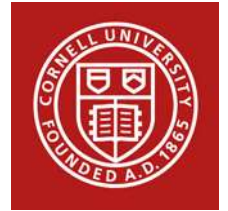


~1 ns  
5  $\mu$ J  
1064 nm

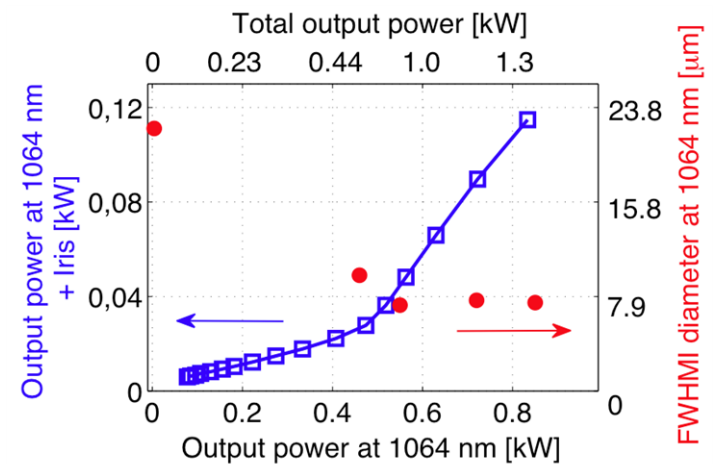
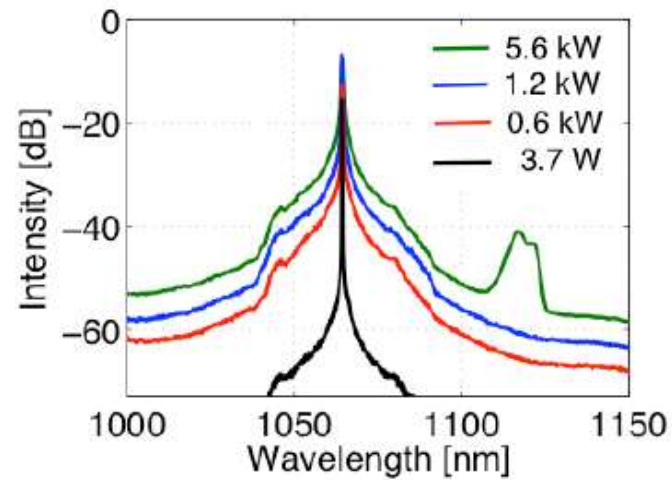




## Beam self-cleaning in GRIN fiber

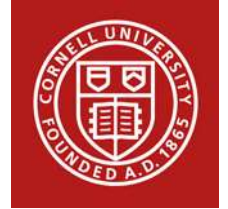


- $P \ll P_{cr}$
- Negligible dissipation
- Spatial coherence enhancement

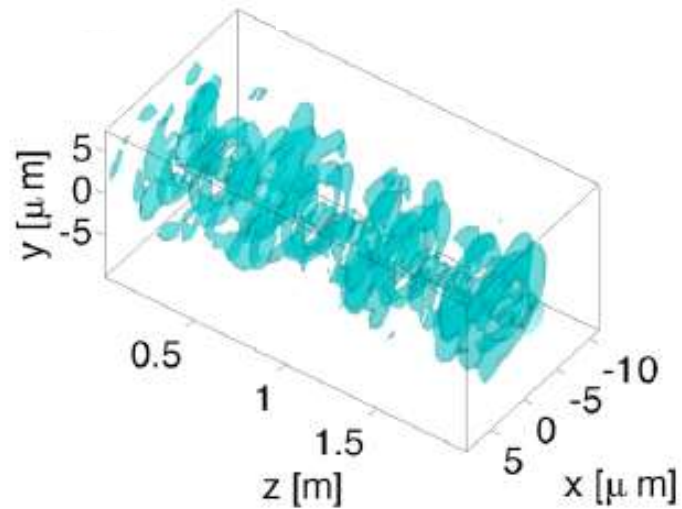




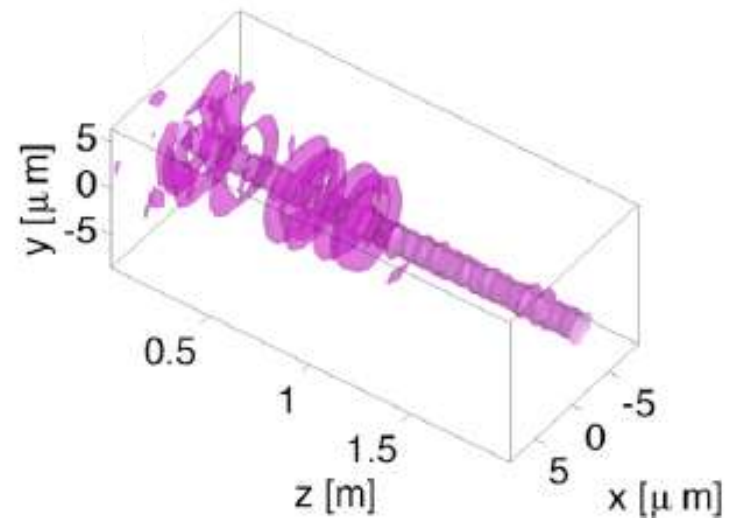
# Beam self-cleaning in GRIN fiber



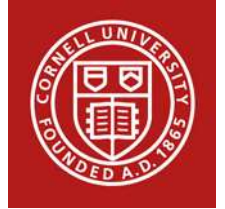
## Linear dynamics



## Nonlinear dynamics

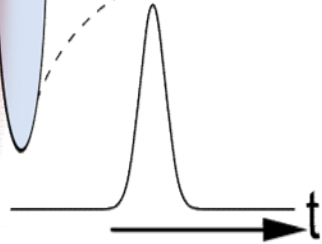
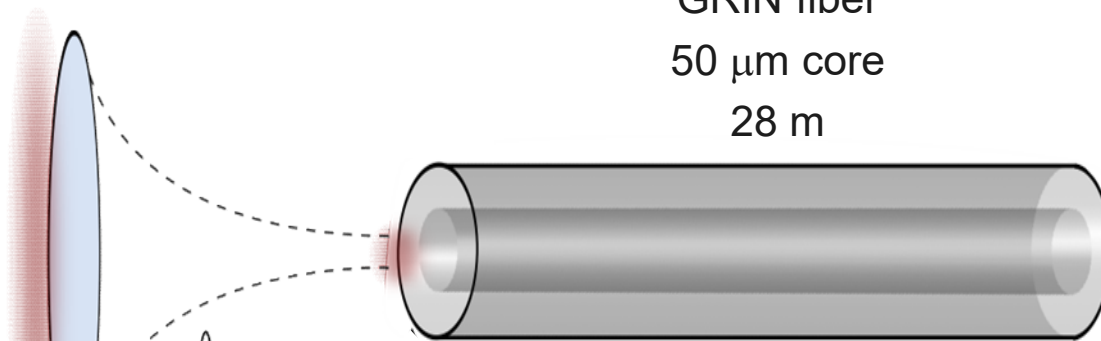


- Simulations show that Kerr nonlinearity underlies self-cleaning

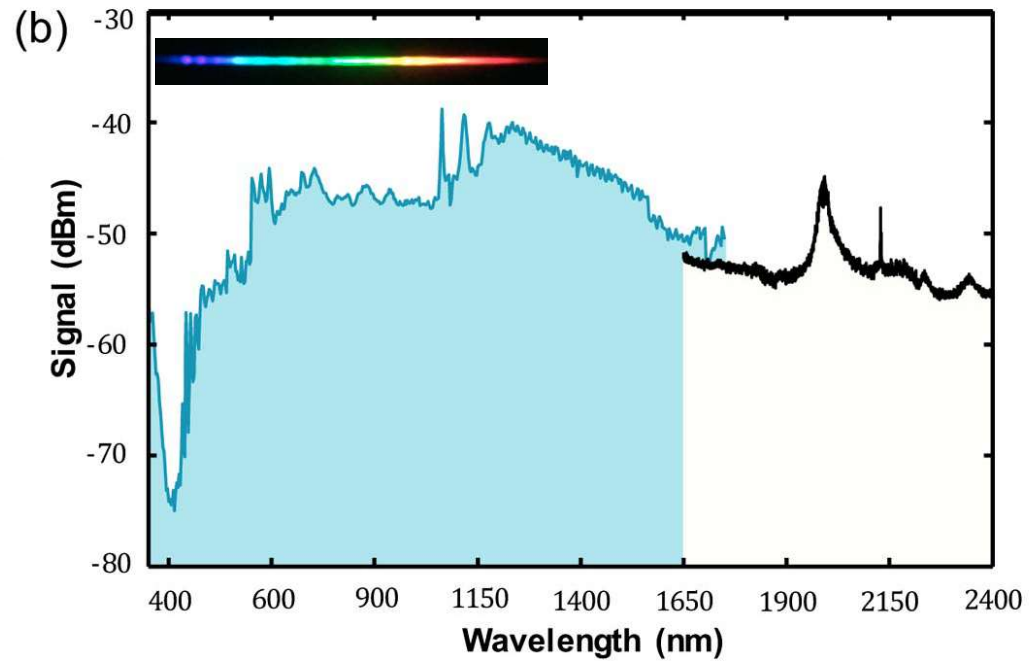


# High-power continuum

GRIN fiber  
50  $\mu\text{m}$  core  
28 m



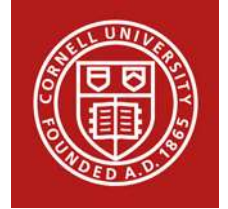
400 ps  
100  $\mu\text{J}$   
1064 nm



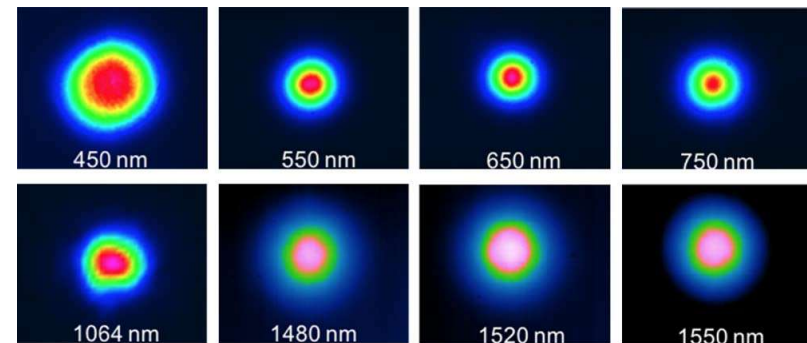


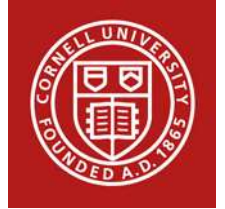


## High-power continuum

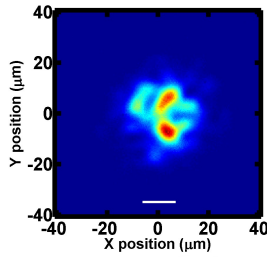


- Continuum from spatiotemporal MI, geometric parametric instability, Raman, and other 4-wave mixing processes
- Self-cleaning confirmed
- Speckle-free output with moderate  $M^2$
- 80  $\mu\text{J}$  pulse energy
- Route to compact, bright, multi-octave continuum

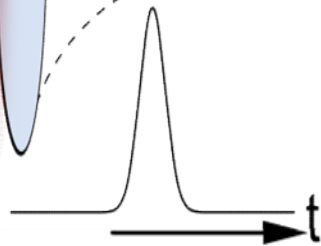
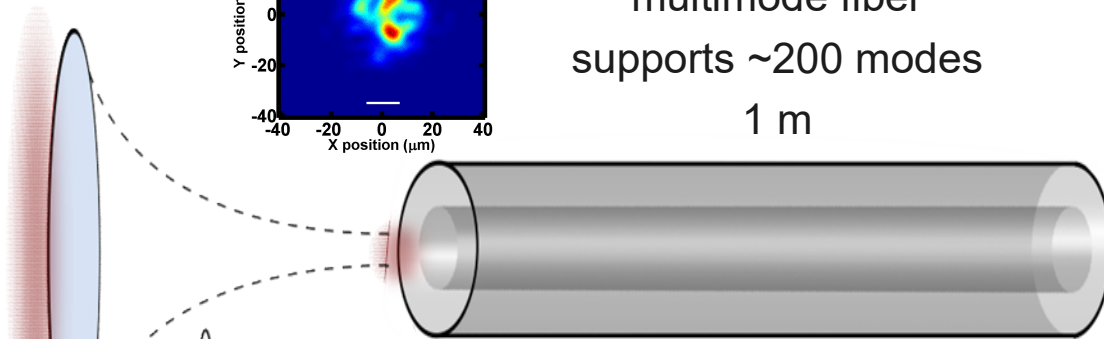




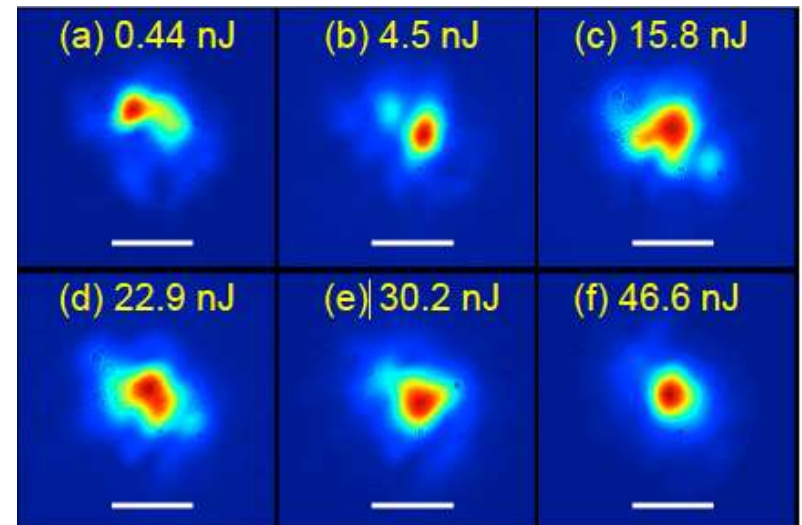
# Self-cleaning of femtosecond pulsed beams



multimode fiber  
supports ~200 modes  
1 m

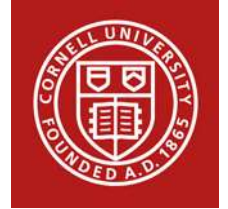


60 fs  
50 nJ  
1035 nm

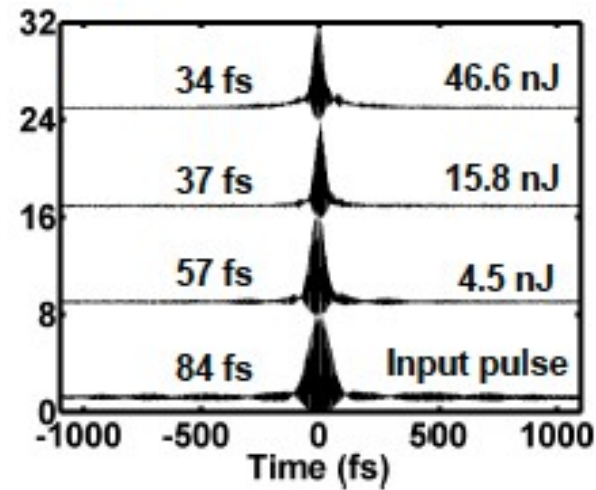
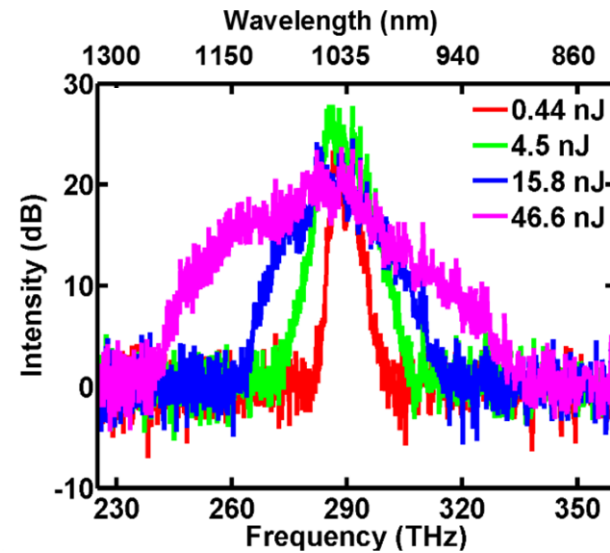




## Self-cleaning of femtosecond pulsed beams

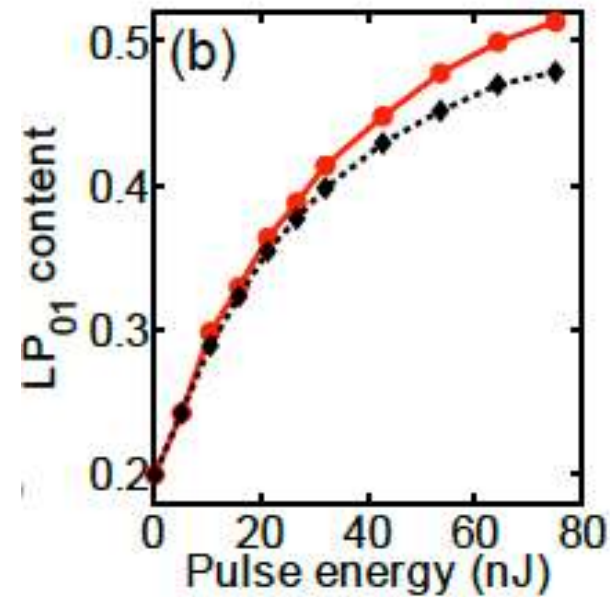
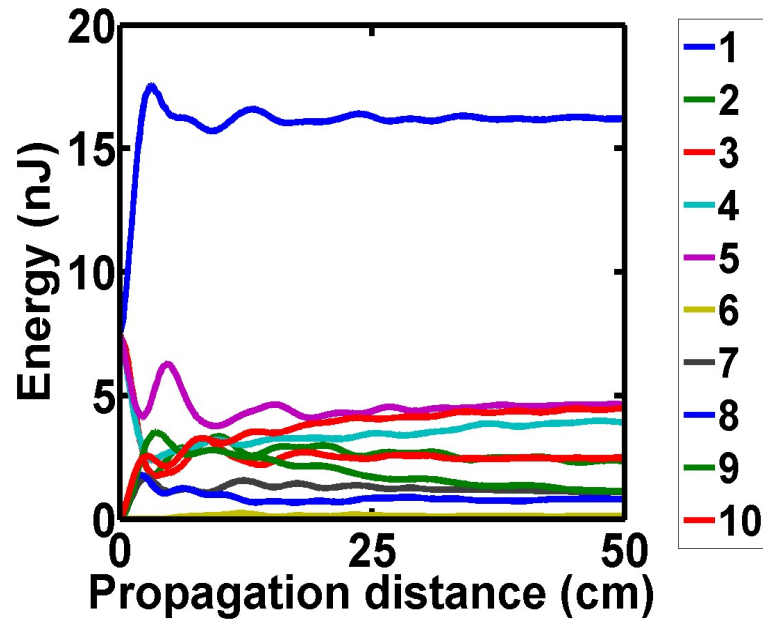
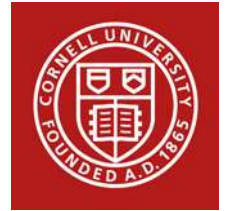


- $P < P_{cr}$
- Negligible dissipation
- Temporal coherence maintained

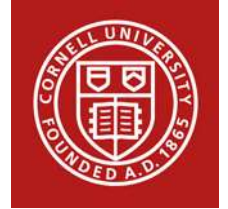




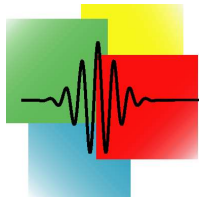
# Self-cleaning of femtosecond pulsed beams



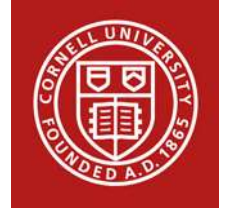
- Kerr nonlinearity underlies self-cleaning
- Process independent of pulse duration



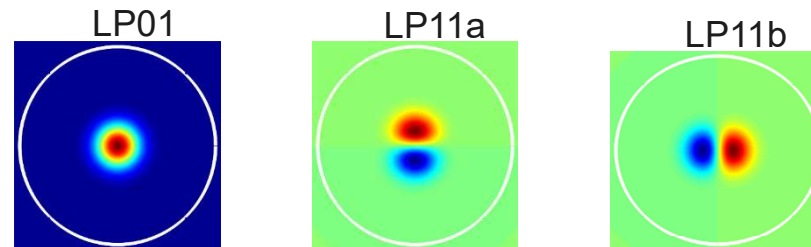
## *Implications / Future Directions*



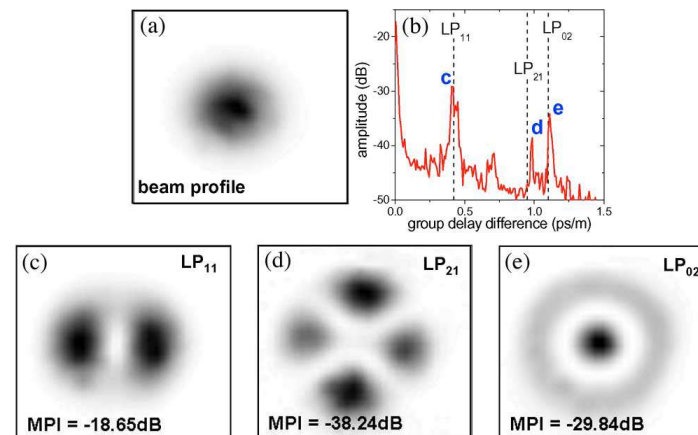
# Multimode solitons



- Solitons in few-mode fibers



- Mode-resolved studies



*Nicholson et al., JSTQE 2009*

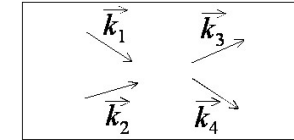
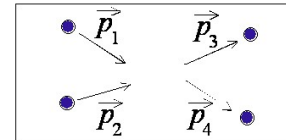


# Classical wave condensation



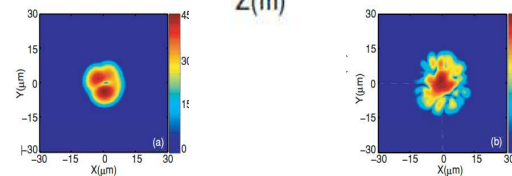
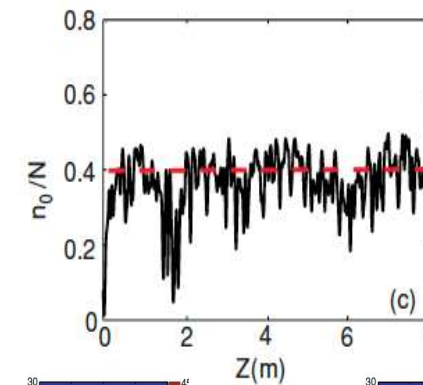
## Wave turbulence theory

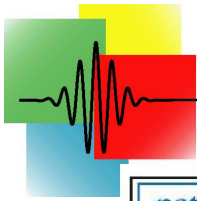
- random optical waves can “thermalize”



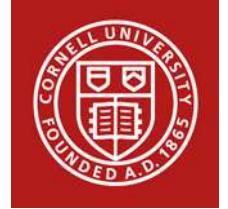
- initial *incoherent* field self-organizes to form large coherent structure
- equipartition of energy in higher-order modes

- 2D + parabolic waveguide:  
condensation predicted theoretically





# Optical turbulence



nature  
photonics

LETTERS

PUBLISHED ONLINE: 22 SEPTEMBER 2013 | DOI: 10.1038/NPHOTON.2013.246

## The laminar-turbulent transition in a fibre laser


E. G. Turitsyna<sup>1</sup>, S. V. Smirnov<sup>2</sup>, S. Sugavanam<sup>1</sup>, N. Tarasov<sup>1</sup>, X. Shu<sup>1</sup>, S. A. Babin<sup>2,3</sup>, E. V. Podivilov<sup>2,3</sup>,  
D. V. Churkin<sup>1,2,3</sup>, G. Falkovich<sup>4,5</sup> and S. K. Turitsyn<sup>1,2,\*</sup>

1362 OPTICS LETTERS / Vol. 39, No. 6 / March 15, 2014

## Optical turbulence in fiber lasers


Stefan Wabnitz

*Dipartimento di Ingegneria dell'Informazione, Universita' di Brescia, Via Branze 38,  
Brescia 25123, Italy (stefan.wabnitz@unibs.it)*

 **Physics Reports**

Volume 542, Issue 1, 1 September 2014, Pages 1–132

Optical wave turbulence: Towards a unified nonequilibrium thermodynamic formulation of statistical nonlinear optics



### Optical wave turbulence: Towards a unified nonequilibrium thermodynamic formulation of statistical nonlinear optics

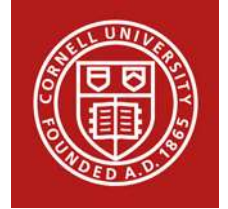
A. Picozzi<sup>a</sup>, J. Garnier<sup>b</sup>, T. Hansson<sup>c</sup>, P. Suret<sup>d</sup>, S. Randoux<sup>d</sup>, G. Millot<sup>a</sup>, D.N. Christodoulides<sup>e</sup>

- Optical wave turbulence studied in 1D systems
- True turbulence requires 3D

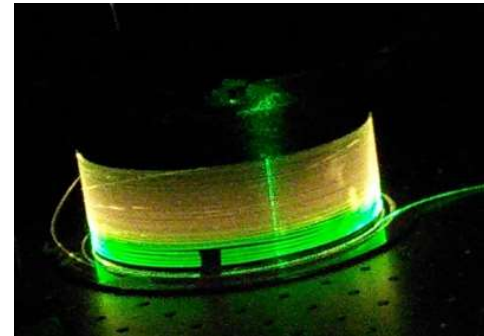




## *Effects of disorder and dissipation*



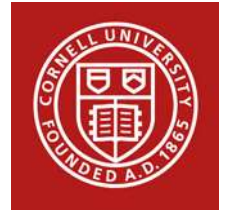
- Introduce  
random mode coupling  
gain, loss
- Complex system
- Controllable and measurable
- Testbed for  
cooperative phenomena  
self-organized critical behavior



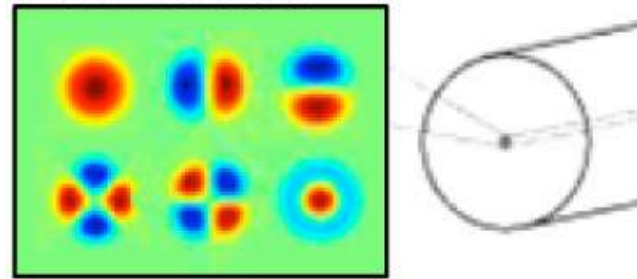
*Wright et al., arXiv 2016*



## Relevance to telecommunications

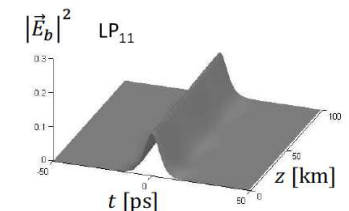
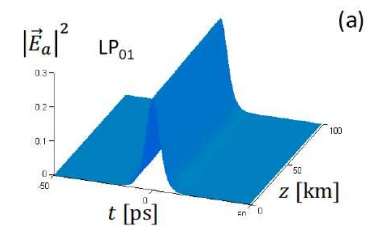


- N modes  $\rightarrow$  N channels



- Multimode solitons *versus* independent channels

- Strongly-coupled mode groups: Manakov solitons

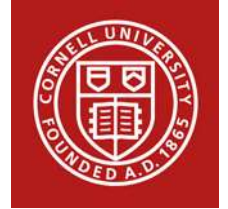


Mecozzi et al., *Opt Exp* 2012

- Instabilities may limit transmission

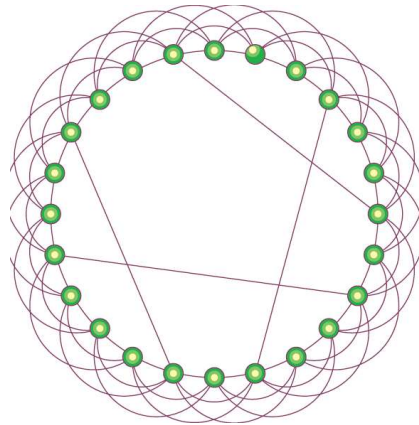


## Relevance to telecommunications



### Multimode fibers are small-world networks

- Coupling is primarily between nearest neighbors
- “Shortcut” links can lead to a strong-coupling transition, many-mode self-organization

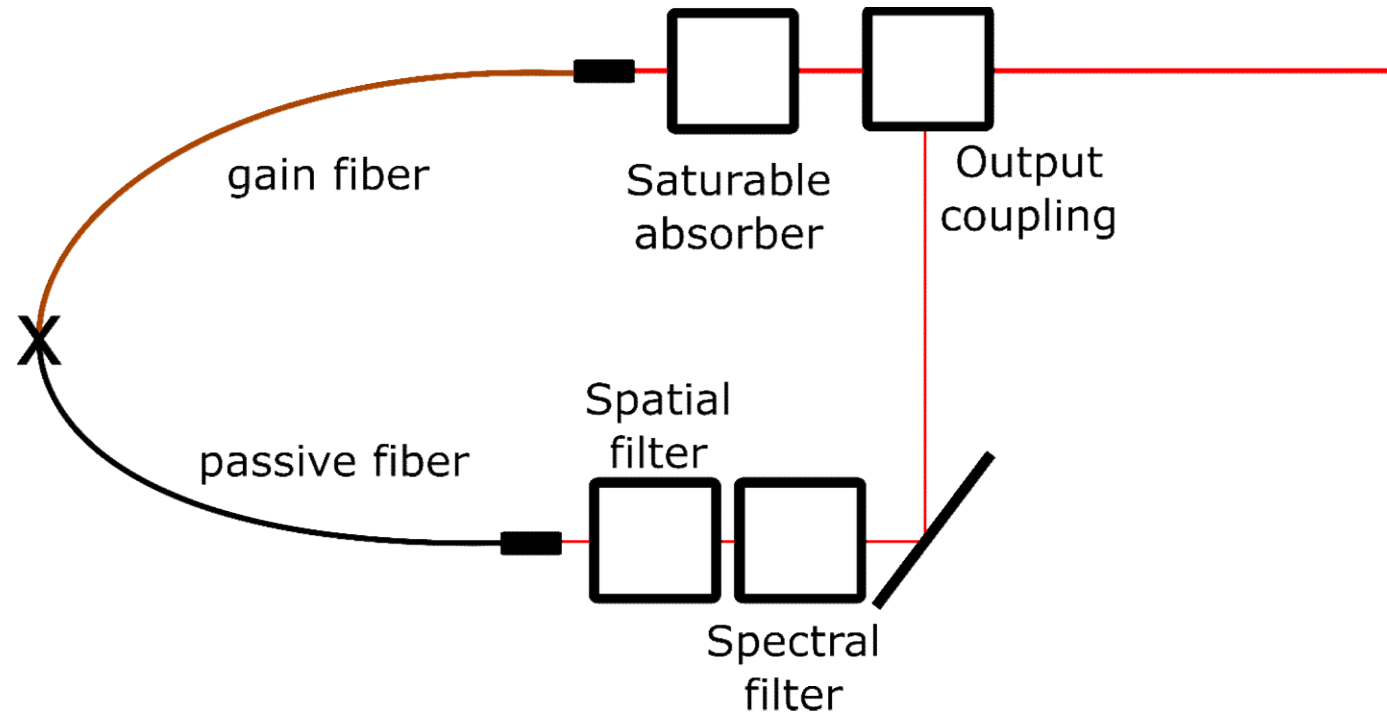
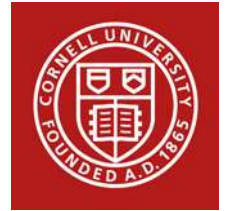


*A small-world network  
Strogatz, Nature 2001*

- Need to understand many-mode nonlinear interactions
  - Mode-dependent gain and loss
  - Mode-dependent, longitudinally-varying disorder

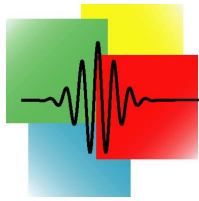


## Multimode soliton lasers



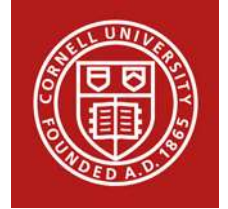
A multimode fiber laser is a new environment for nonlinear waves. It adds

- spatially-dependent gain, saturable absorption
- spatial and spectral filtering



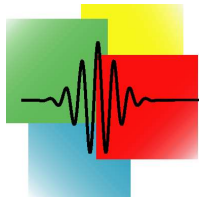
## *Multimode soliton lasers*

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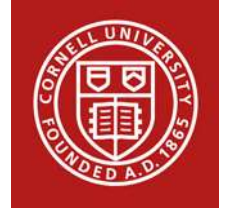


Multimode fiber lasers can have much higher energy than single-mode fiber lasers

- Larger mode area



## Multimode soliton lasers



Multimode fiber lasers can have much higher energy than single-mode fiber lasers

- Larger mode area

$$E \sim A_{eff}$$

single mode fiber

$$A_{eff} = 50-100 \mu\text{m}^2$$

large-mode-area microstructure fiber

$$A_{eff} \sim 5,000 \mu\text{m}^2$$

single higher-order mode

$$A_{eff} \sim 3,000 \mu\text{m}^2$$

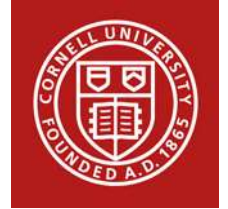
multimode fiber

$$A_{eff} > 30,000 \mu\text{m}^2$$

(1550 nm)



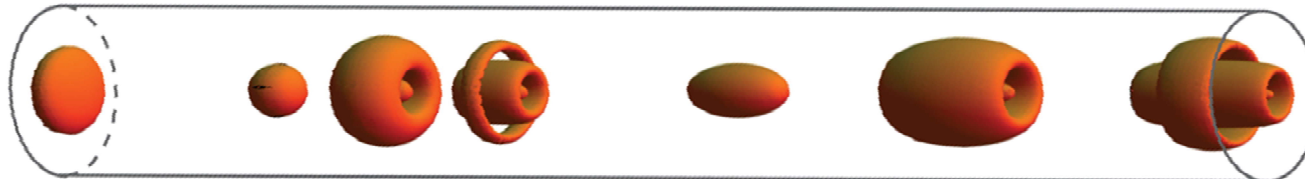
## Multimode soliton lasers



Multimode fiber lasers can have much higher energy than single-mode fiber lasers

- Larger mode area
- Modal dispersion

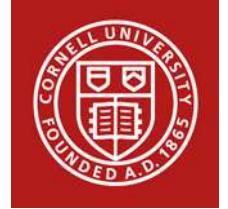
$$E \sim \sum \text{dispersion}$$





## *Multimode soliton lasers*

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Multimode fiber lasers can have much higher energy than single-mode fiber lasers

- Larger mode area
- Modal dispersion
- New (spatiotemporal) pulse evolutions

Role of spatiotemporal instabilities?

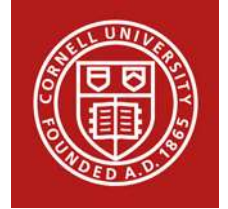
Ultimate limit from self-focusing



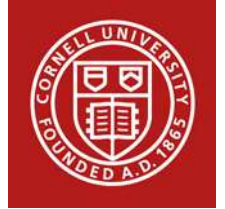


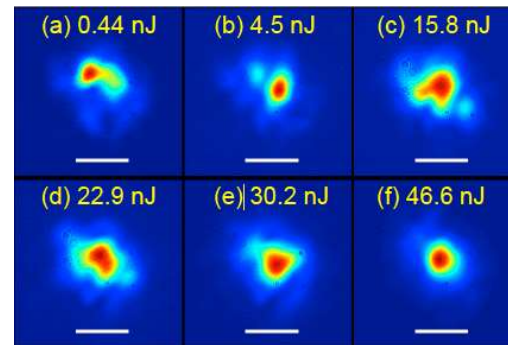
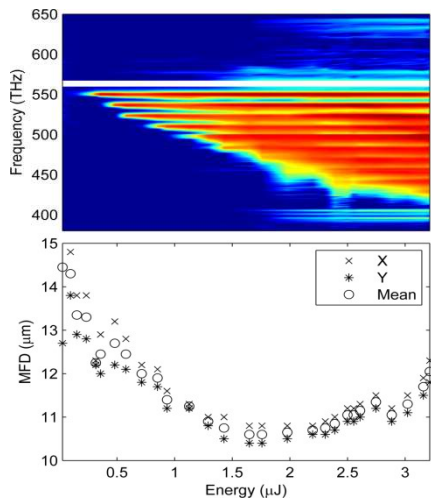
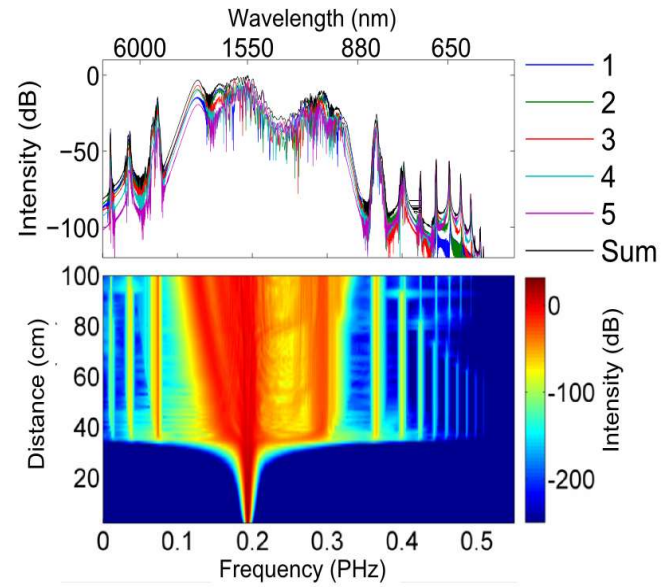
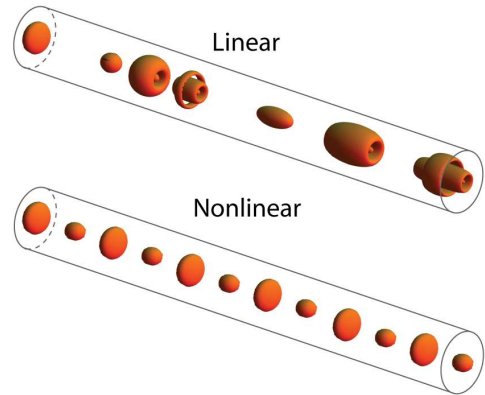
## Overall Summary

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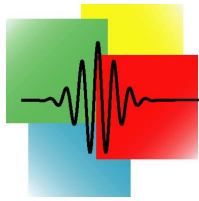
- Multimode fiber supports a variety of new spatiotemporal phenomena
- Initial results indicate that multimode solitons will help understand complex dynamics
- Relevance of nonlinear dynamics to applications
  - High-power, multi-octave continua
  - Connection to optics of complex media
  - Space-division multiplexing in telecommunications
  - Laser / amplifier / transmission applications



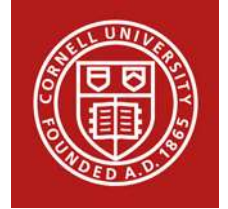




*Reserve slides*



## *Theory of pulse propagation in MM fiber*



F. Poletti and P. Horak, "Description of ultrashort pulse propagation in multimode optical fibers," *J. Opt. Soc. Am. B* 25, 1645 (2008).

P. Horak and F. Poletti, "Multimode Nonlinear Fiber Optics: Theory and Applications," in "Recent Progress in Optical Fiber Research," M. Yasin, ed. (2012), chap. 1, pp. 3–24.

A. Mafi, "Pulse Propagation in a Short Nonlinear Graded-Index Multimode Optical Fiber," *J. Lightwave Technol.* 30, 2803–2811 (2012).

F. Poletti and P. Horak, "Dynamics of femtosecond supercontinuum generation in multimode fibers," *Opt. Express* 17, 6134 (2009).

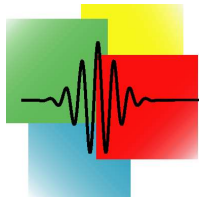
G. Hesketh, F. Poletti, and P. Horak, "Spatio-Temporal Self-Focusing in Femtosecond Pulse Transmission Through Multimode Optical Fibers," *J. Lightwave Technol.* 30, 2764–2769 (2012).

S. Mumtaz, R.J. Essiambre & G.P. Agrawal, Nonlinear propagation in multimode and multicore fibers: generalization of the Manakov equations. *Journal of Lightwave Technology*, 31(3), 398-406 (2013).

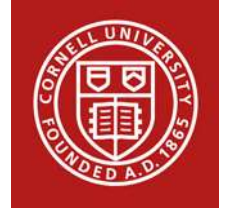
A. Mecozzi, C. Antonelli & M. Shtaif, Nonlinear propagation in multi-mode fibers in the strong coupling regime. *Optics express* 20.11, 11673-11678 (2012).

A. Mecozzi, C. Antonelli & M. Shtaif, Coupled Manakov equations in multimode fibers with strongly coupled groups of modes." *Optics express* 20.21, 23436-23441.(2012).

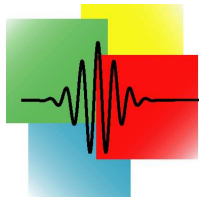
J. Andreasen and M. Kolesik, "Nonlinear propagation of light in structured media: Generalized unidirectional pulse propagation equations", *Phys. Rev. E* 86 (2012)



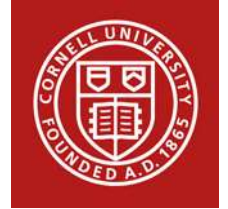
## *Theory: solitons in multimode fiber*



- A. Hasegawa, "Self-confinement of multimode optical pulse in a glass fiber," *Opt. Lett.* 5, 416 (1980).
- B. Crosignani and P. D. Porto, "Soliton propagation in multimode optical fibers," *Opt. Lett.* 6, 329 (1981).
- B. Crosignani, A. Cutolo, and P. D. Porto, "Coupled-mode theory of nonlinear propagation in multimode and single-mode fibers: envelope solitons and self-confinement," *J. Opt. Soc. Am.* 72, 1136 (1982).
- N. Akhmediev and A. Ankiewicz, "Multi-soliton complexes," *Chaos (Woodbury, N.Y.)* 10, 600–612 (2000).
- S. Buch and G. P. Agrawal, "Soliton stability and trapping in multimode fibers," *Opt. Lett.* 40, 225–228 (2015).



## Graded-index fiber



- Predicted 3D wave-packets from analytical models

Yu, et al., Spatio-temporal solitary pulses in graded-index materials with Kerr nonlinearity. *Optics Communications* 1995.

S Raghavan and Govind P Agrawal. Spatiotemporal solitons in inhomogeneous nonlinear media. *Optics Communications* 2000.

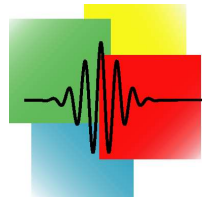
- Experiments

P. L. Baldeck, F. Raccach, and R. R. Alfano, "Observation of self-focusing in optical fibers with picosecond pulses," *Opt. Lett.* 12, 588 (1987).

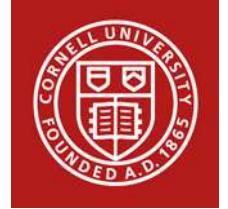
A. B. Grudinin, E. M. Dianov, D. V. Korbkin, A. M. Prokhorov, and D. V. Khaïdarov, "Nonlinear mode coupling in multimode optical fibers; excitation of femtosecond-range stimulated-Raman-scattering solitons," *J. Exp. Theor. Phys.* 47 297–300 (1988).

H. Poubeyram, G. P. Agrawal, and A. Mafi, "Stimulated Raman scattering cascade spanning the wavelength range of 523 to 1750 nm using a graded-index multimode optical fiber," *Appl. Phys. Lett.* 102, 201107 (2013).

K.O. Hill, D.C. Johnson & B.S. Kawasaki, Efficient conversion of light over a wide spectral range by four-photon mixing in a multimode graded-index fiber. *Appl. Opt.* **20**, 2769 (1981).



## *Hollow-core multimode nonlinear optics*

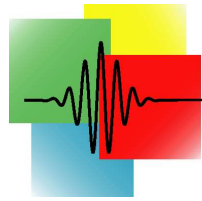


P. St. J. Russell, P. Hölzer, W. Chang, A. Abdolvand & J. C. Travers, Hollow-core photonic crystal fibres for gas-based nonlinear optics, *Nature Photonics* **8**, 278–286 (2014)

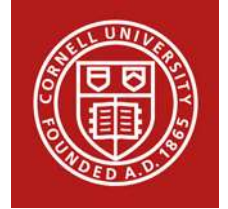
F. Tani, J.C. Travers, & P.St.J Russell, Multimode ultrafast nonlinear optics in optical waveguides: numerical modeling and experiments in kagomé photonic-crystal fiber. *JOSA B*, **31**(2), 311-320 (2014)

G. Fibich and A. L. Gaeta, “Critical power for self-focusing in bulk media and in hollow waveguides,” *Opt. Lett.* **25**, 335–337 (2000)





## *Recent work on nonlinear optics in other multimode fibers*



J. Ramsay et al. Generation of infrared supercontinuum radiation: spatial mode dispersion and higher-order mode propagation in ZBLAN step-index fibers. *Opt. Express* 21, 10764–71 (2013).

M. Guasoni, Generalized modulational instability in multimode fibers: Wideband multimode parametric amplification. *Phys. Rev. A* 92, 033849 (2015).

I. Kubat & O. Bang, Multimode supercontinuum generation in chalcogenide glass fibres. *Opt. Express* 24, 2513–26 (2016).

J. Demas, P. Steinvurzel, B. Tai, L. Rishøj, Y. Chen, and S. Ramachandran, "Intermodal nonlinear mixing with Bessel beams in optical fiber," *Optica* 2, 14-17 (2015)

J. Demas, T. He, and S. Ramachandran, "Generation of 10-kW Pulses at 880 nm in Commercial Fiber via Parametric Amplification in a Higher Order Mode," in Conference on Lasers and Electro-Optics, OSA Technical Digest (2016) (Optical Society of America, 2016), paper STh3P.6.

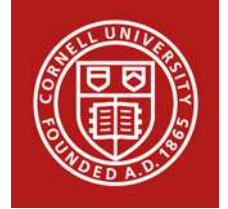
L. Rishoj, G. Prabhakar, J. Demas, and S. Ramachandran, "30 nJ, ~50 fs All-Fiber Source at 1300 nm Using Soliton Shifting in LMA HOM Fiber," in Conference on Lasers and Electro-Optics, OSA Technical Digest (2016) (Optical Society of America, 2016), paper STh3O.3.

J. Cheng, M.E. Pedersen, K. Charan, K. Wang, C. Xu, L. Grüner-Nielsen & D. Jakobsen, Intermodal four-wave mixing in a higher-order-mode fiber. *Applied Physics Letters*, 101(16), 161106 (2012)

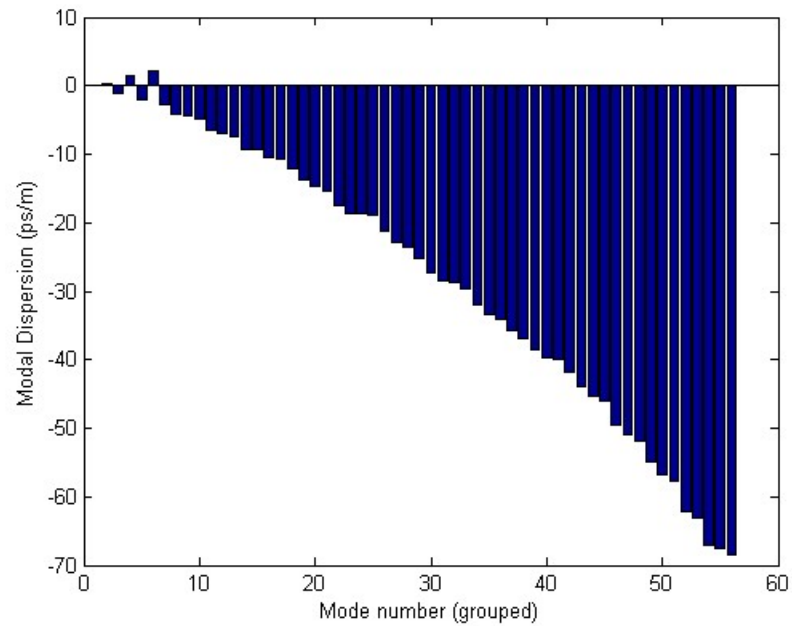
J. Cheng, M.E. Pedersen, K. Charan, K. Wang, C. Xu, L. Grüner-Nielsen & D. Jakobsen, Intermodal Čerenkov radiation in a higher-order-mode fiber, *Optics letters* 37 (21), 4410-4412 (2012).



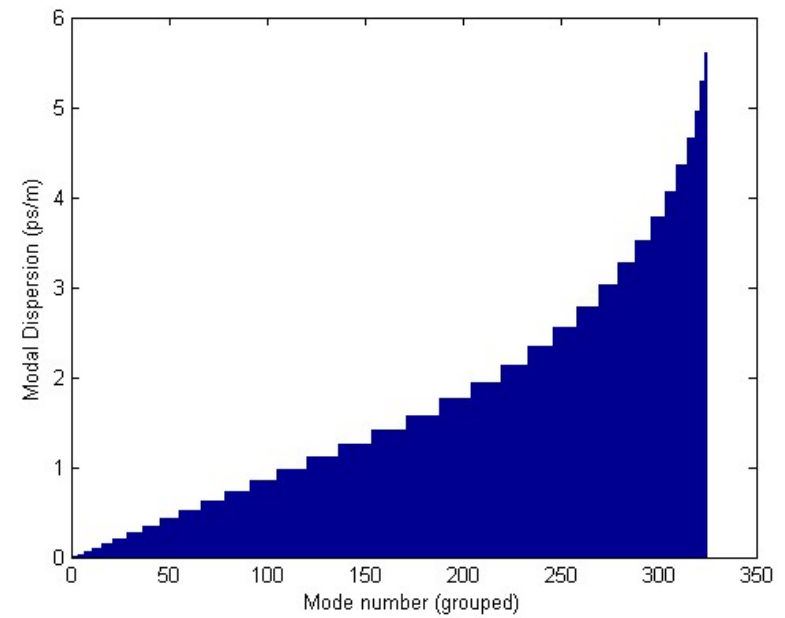
# Multimode fibers



Step-Index



GRIN



- In GRIN fiber, modes have similar group velocities



## Single-field model for GRIN fiber



$$\frac{\partial A}{\partial z} = \frac{i}{2k_0} \left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) - i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - i \frac{k_0 \Delta}{R^2} (x^2 + y^2) A + i \gamma |A|^2 A$$

diffraction

dispersion

index profile

Kerr

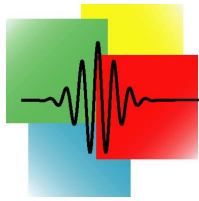


## Single-field model for GRIN fiber

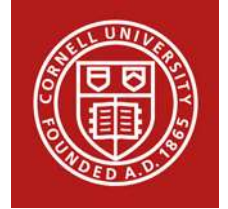


$$\frac{\partial A}{\partial z} = \frac{i}{2k_0} \left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) - i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - i \frac{k_0 \Delta}{R^2} (x^2 + y^2) A + i \gamma |A|^2 A$$

- Gross-Pitaevskii equation



# Coupled mode analysis



“GMMNLSE”

$$\partial_z A_p(z, t) = i \left( \beta_0^{(p)} - \Re [\beta_0^{(0)}] \right) A_p - \left( \beta_1^{(p)} - \Re [\beta_1^{(0)}] \right) \frac{\partial A_p}{\partial t} + \sum_{m=2}^3 i^{m+1} \frac{\beta_m}{m!} \partial_t^m A_p$$

modal wavenumber mismatch   modal velocity mismatch   group velocity dispersion

$$+i \frac{n_2 \omega_0}{c} \left( 1 + \frac{i}{\omega_0} \partial_t \right) \sum_{l,m,n} \{ (1 - f_R) S_{plmn}^k A_l A_m A_n^* + f_R A_l S_{plmn}^R \int_{-\infty}^t d\tau A_m(z, t - \tau) A_n^*(z, t - \tau) h_R(\tau) \}$$

**shock**
**Kerr**
**Raman**

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# Relation between modes

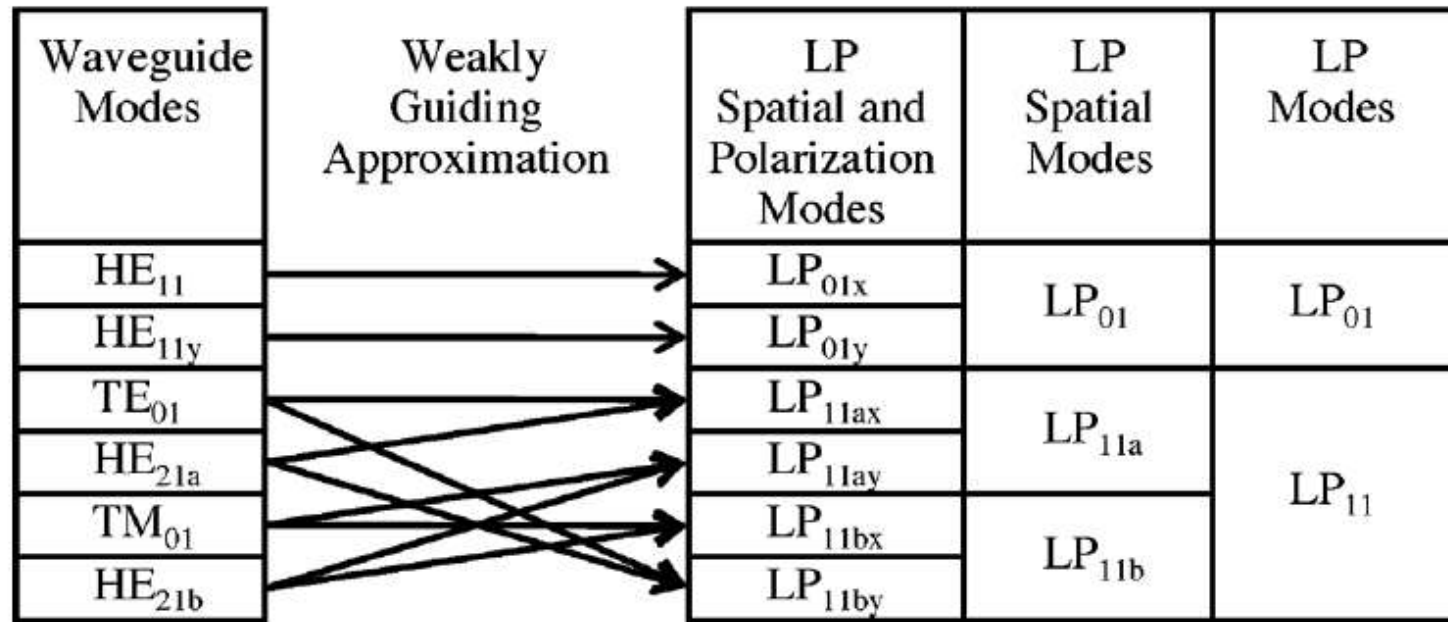


Fig. 1. Relation between the LP modes and the real waveguide modes HE<sub>11x</sub>, HE<sub>11y</sub>, TE<sub>01</sub>, TM<sub>01</sub>, HE<sub>21a</sub>, and HE<sub>21b</sub> of the six-mode FMF.