OSA Webinar, online

Topological and Non-Hermitian Electromagnetism

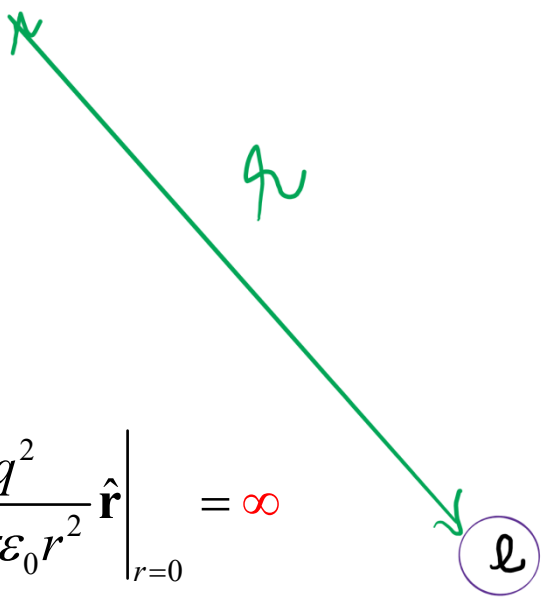
Mário G. Silveirinha

1st Part: Non-Hermitian Electromagnetism

Some old problems of classical electromagnetism

The electron self-field

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$



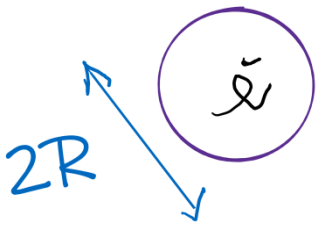
$$\mathbf{F}_{\text{self}} = \frac{q^2}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}\bigg|_{r=0} = \infty$$

"The theory of electron is not complete because it is not theoretically self-consistent… the charge produces an electric field around the electron… if we assume that the charge is concentrated at one point the electron force approaches infinity as one approaches this point…"

[P. Dirac, Lindau Lecture, 1956]

Self-field energy

$$U_{\text{elec}} = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 R} \xrightarrow{R=0} \infty$$

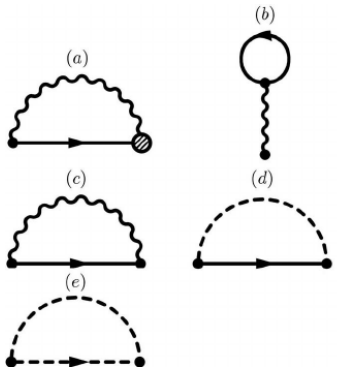


For a point particle, the self-energy and the self-field are infinitely large

Paths for a solution

- Quantum electrodynamics fixes the problem?

Not exactly: QED is plagued with the same problems as classical theory -> solved with renormalization



"... many infinities have been swept under the rug of a field-theorist's office..."

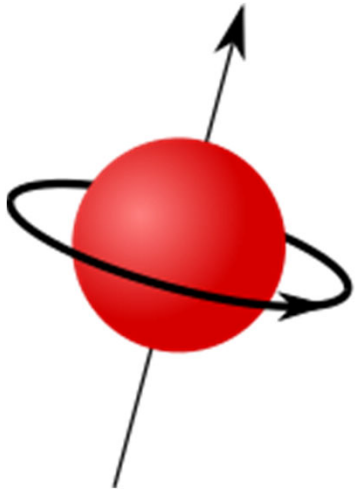
Paths for a solution (contd.)

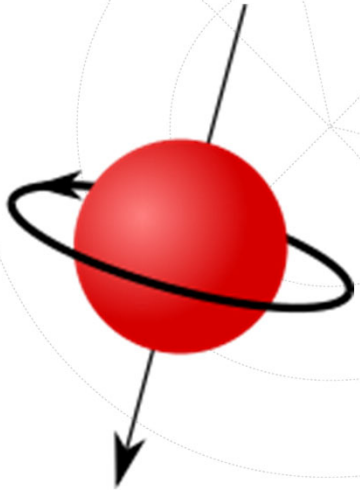
- Perhaps the Maxwell's equations (Coulomb's force) need to be modified to avoid the fields singularity? [e.g., Born-Infeld theory]

- Maybe the electron is not a point particle?

- Other idea?

Other failures of classical theory (only solved by quantum theory)





tinyurl.com/u72hbyta

Electron spin

Wave-particle duality

Quantization of energy, etc

New starting point:

Non-Hermitian Electromagnetism

M.G. Silveirinha, *Non-Hermitian Electromagnetism: a Nonlocal Time-Crystal Model for an Electron with Spin*

Hermitian physics

Hermitian physics = "energy" conserving dynamics

Hamilton, 1830

Non-Hermitian (without an Hamiltonian) approach to the theory of the electron

Why non-Hermitian?

The vacuum is not empty space

Bouncing droplets (by Yves Couder)

www.youtube.com/watch?v=W9yWv5dqSKk

Yves Couder and Emmanuel Fort, Single-Particle Diffraction and Interference at a Macroscopic Scale, Phys. Rev. Lett. 97, 154101, 2006

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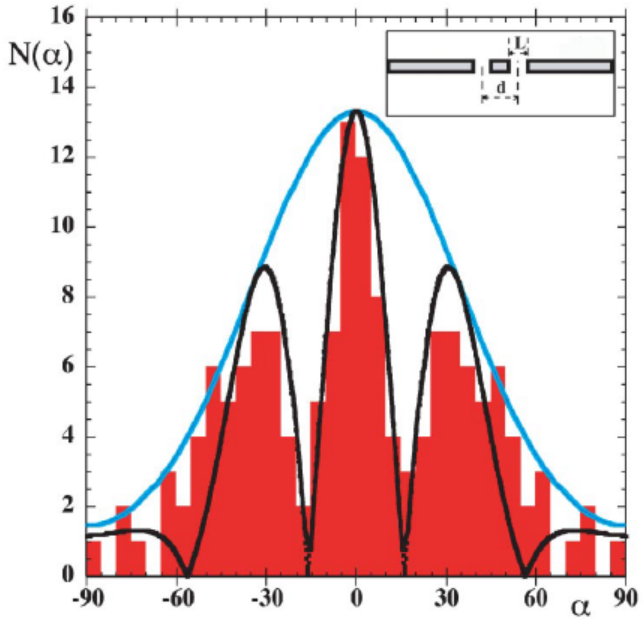


Yves Couder and Emmanuel Fort, Single-Particle Diffraction and Interference at a Macroscopic Scale, Phys. Rev. Lett. 97, 154101, 2006

How to do physics without an Hamiltonian?

A little inspiration from Feynman:

"There is one lucky break, however—electrons behave just like light. The quantum behavior of atomic objects… is the same for all, they are all particle-waves…".

“Electrons behave just like light”

A seemingly absurd proposal:

How far can one go with these postulates?

Some insight: planar trajectories

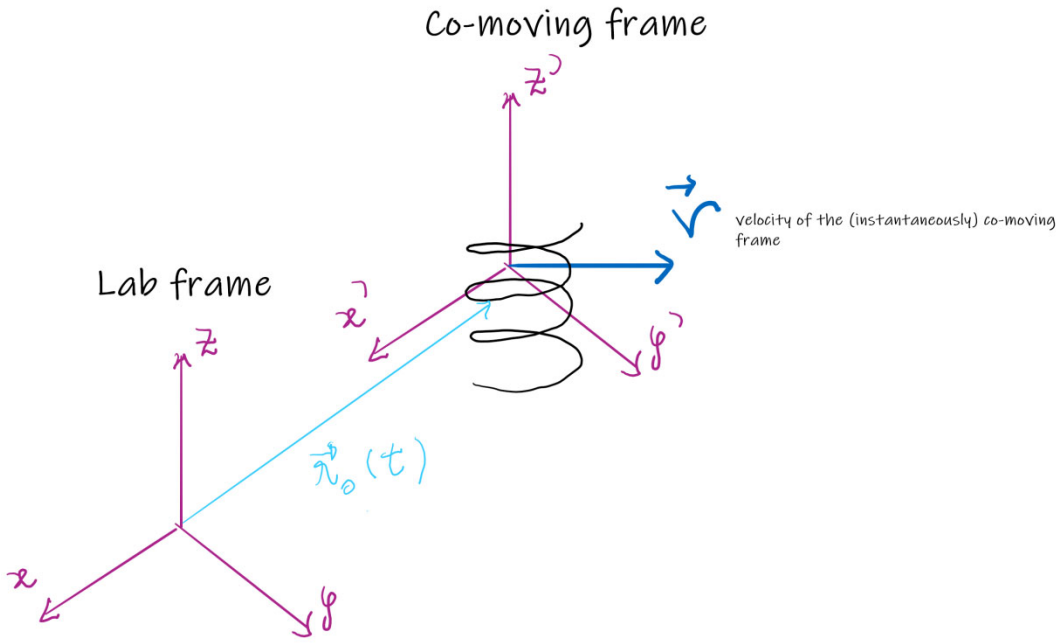
Where things start getting interesting

$(\mathbf{S}\cdot\boldsymbol{\beta},\mathbf{S})$ transforms as a 4-vector under a Lorentz boost

Energy-momentum 4-vector

The elephant in the zoom

Co-moving frame



Co-moving frame (properties)

- *For each time instant there is a unique inertial frame that satisfies the definition.*

- *The velocity of the co-moving frame depends only on kinematic parameters and on the energy of the electron*

- *The co-moving frame transforms in a Lorentz co-variant manner.*

- *It is possible to derive a (purely kinematic) dynamical law for the spin vector in terms of the velocity of the co-moving frame*

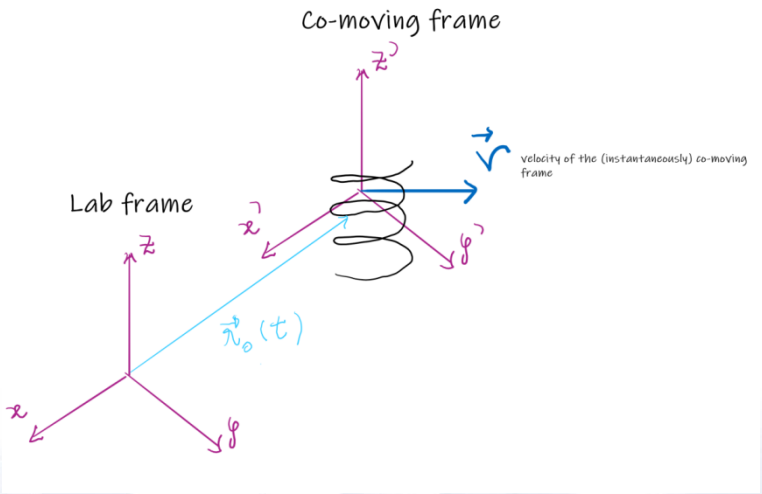
The flow of ideas

Massive energy-momentum 4-vector

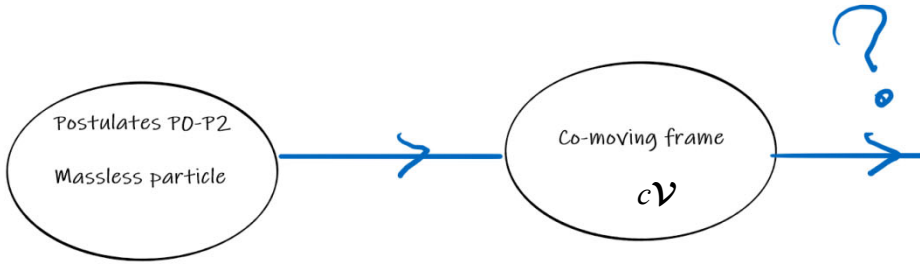
$$E = \gamma m_e c^2 \qquad \boldsymbol{\pi} = m_e \gamma c \boldsymbol{\mathcal{V}} \qquad \gamma = 1/\sqrt{1 - \boldsymbol{\mathcal{V}} \cdot \boldsymbol{\mathcal{V}}}$$

$(E, \boldsymbol{\pi})$ **transforms as 4-vector because** $\boldsymbol{\mathcal{V}}$ **transforms as a velocity under a Lorentz boost!**

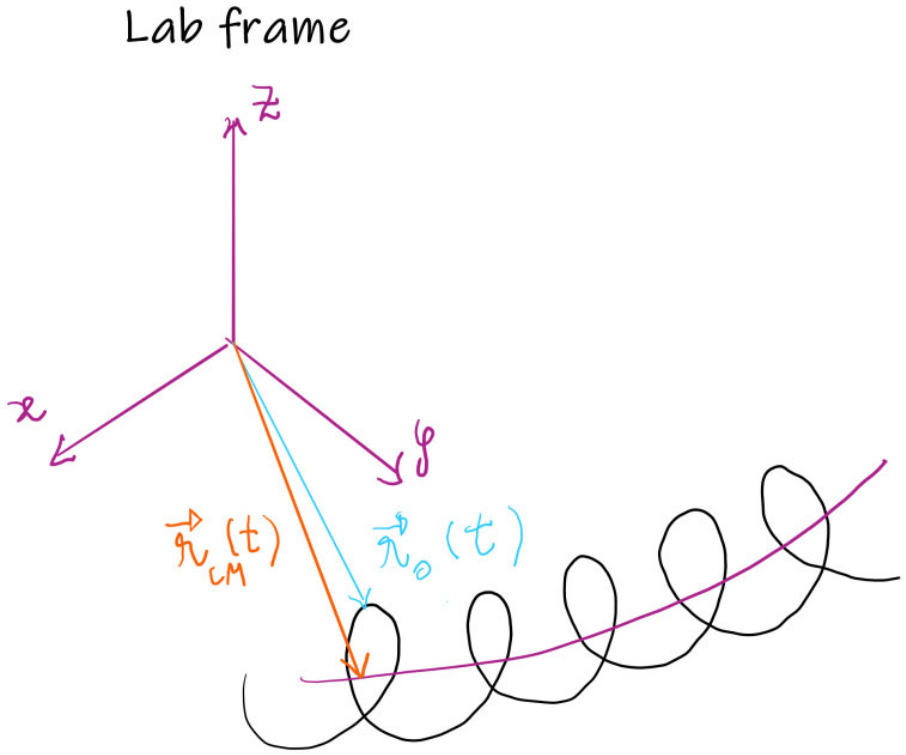
The full picture: 3 energy-momentum 4-vectors associated with the non-Hermitian electron



Coming full circle: from massless to massive



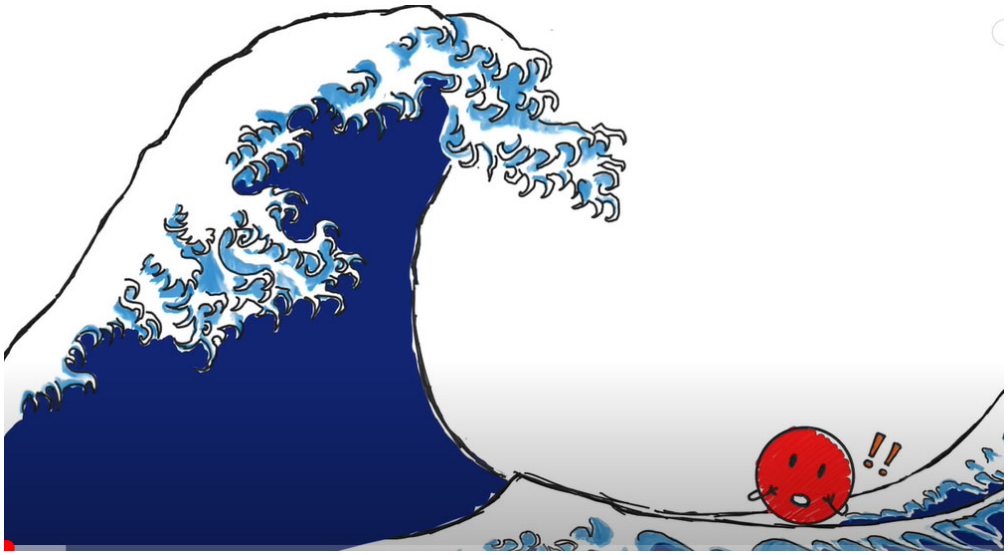
The non-Hermitian electron is a <u>two-component point particle</u>



Mass is an emergent property!

Pilot-wave model

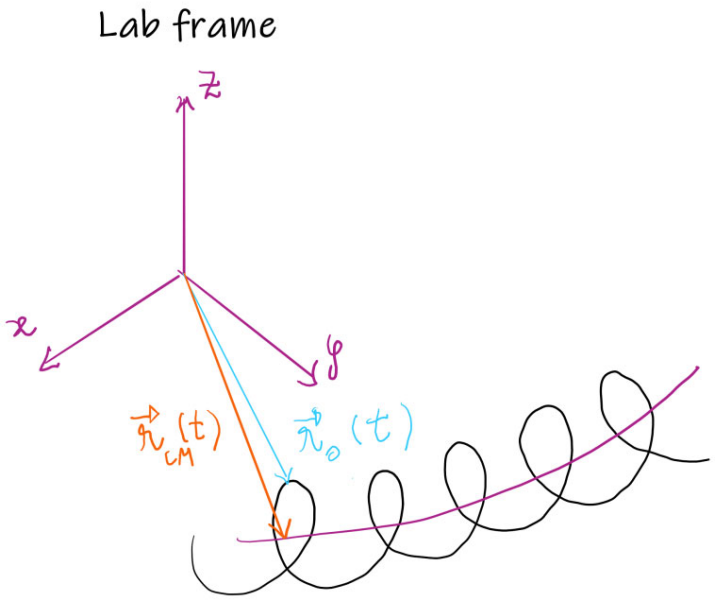
De Broglie – Bohm interpretation of quantum mechanics: realism and determinism



youtube.com/watch?v=r0plv_nIzsQ

An electron is (literally) a particle and a wave: the particle rides the wave

Pilot-wave mechanical model



Self-field free of infinities!

Free-particle states

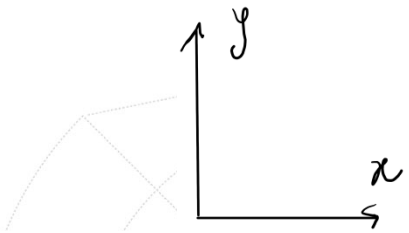
Free-particle

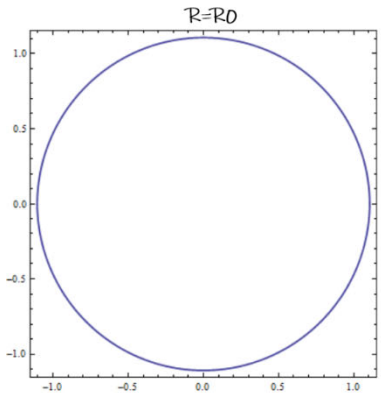
- *No external fields; self-force is neglected (but self-energy is taken into account)*

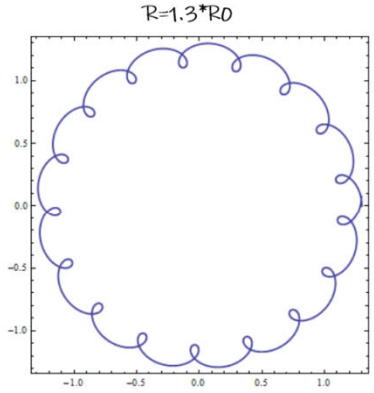
- *The center of mass of a free-particle moves with constant velocity.*

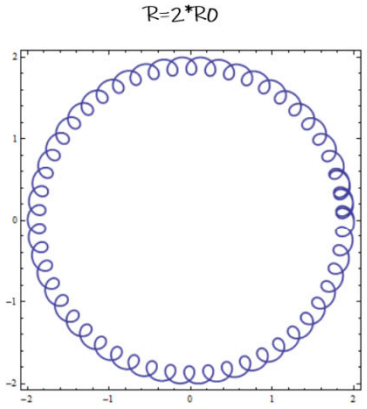
- *In the co-moving frame, the trajectory of the electron "wave"-component is planar with the spin vector perpendicular to the plane of motion. The center of mass (particle-component) is motionless*

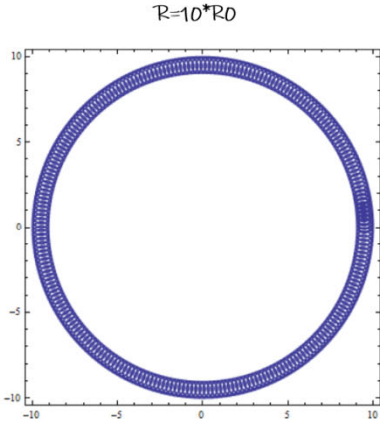
Time-crystal type states*







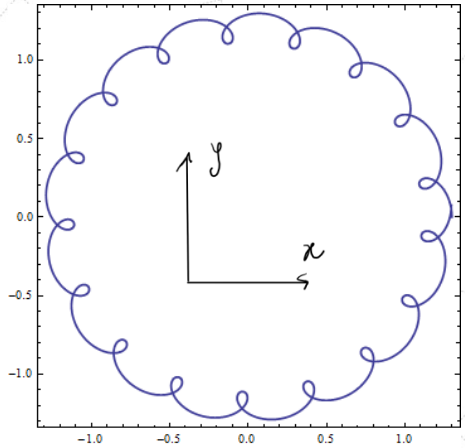




* Coined in A. Shapere and F. Wilczek, “Classical Time Crystals”, Phys. Rev. Lett. 109, 160402, (2012).

Spin angular momentum

$$\mathcal{L}_{\text{spin}} = \mathbf{R} \times \mathcal{P}$$

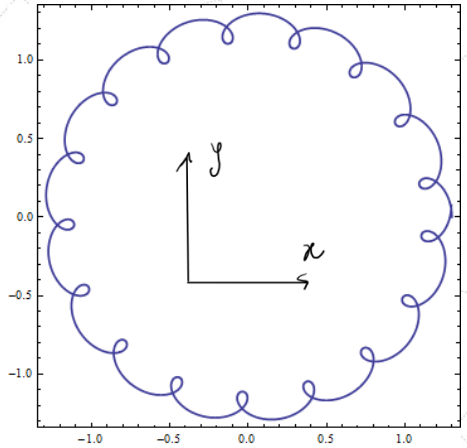


Dispersion of time-crystal states:

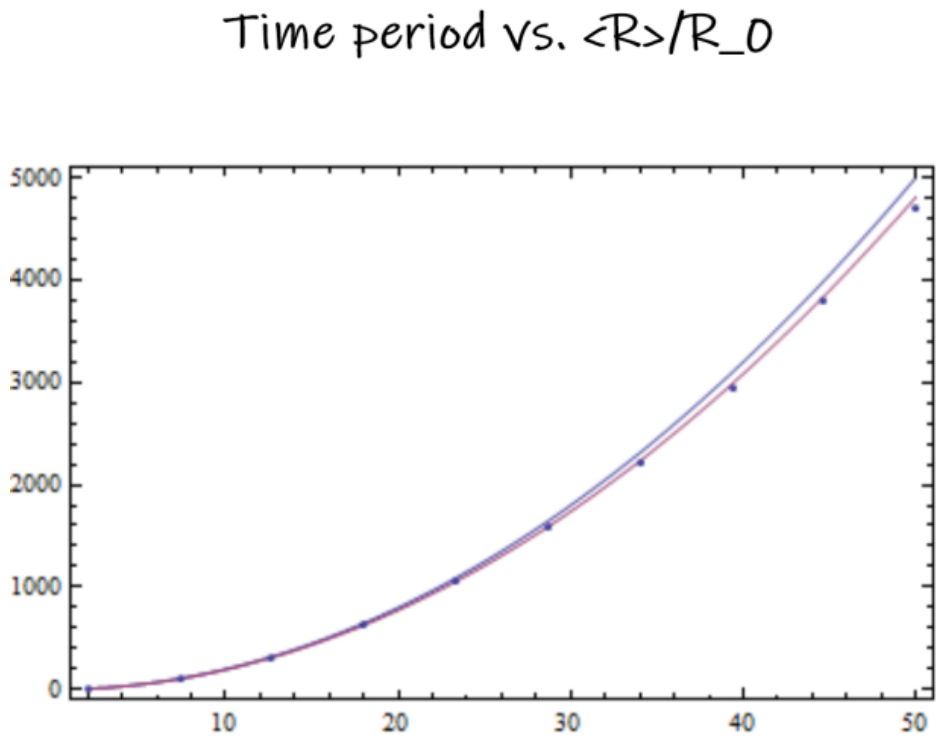
T_{super} = period of the super-orbit

$\langle R \rangle$ = average radius of the super-orbit

$$\omega_{\text{super}} = \frac{2\pi}{T_{\text{super}}}$$



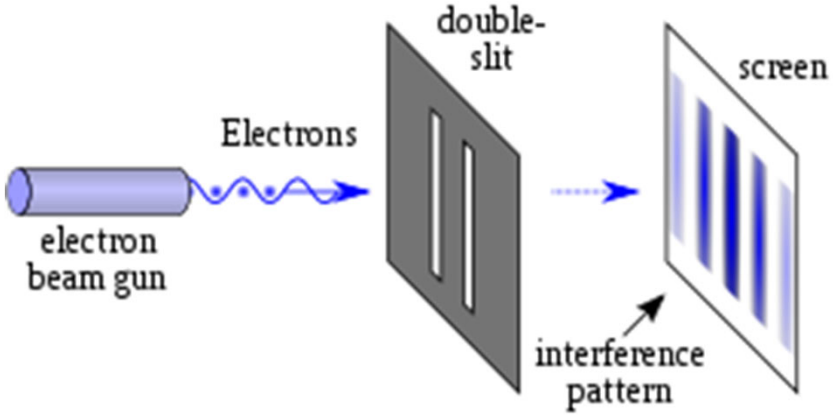
Dispersion of time-crystal states:

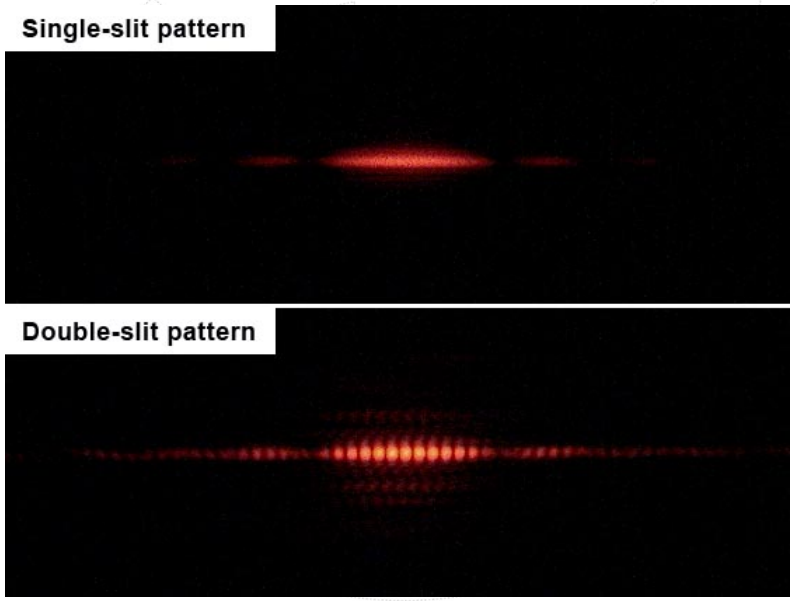


Double-slit experiment

Double-slit experiment

https://tinyurl.com/v7mcwr3f



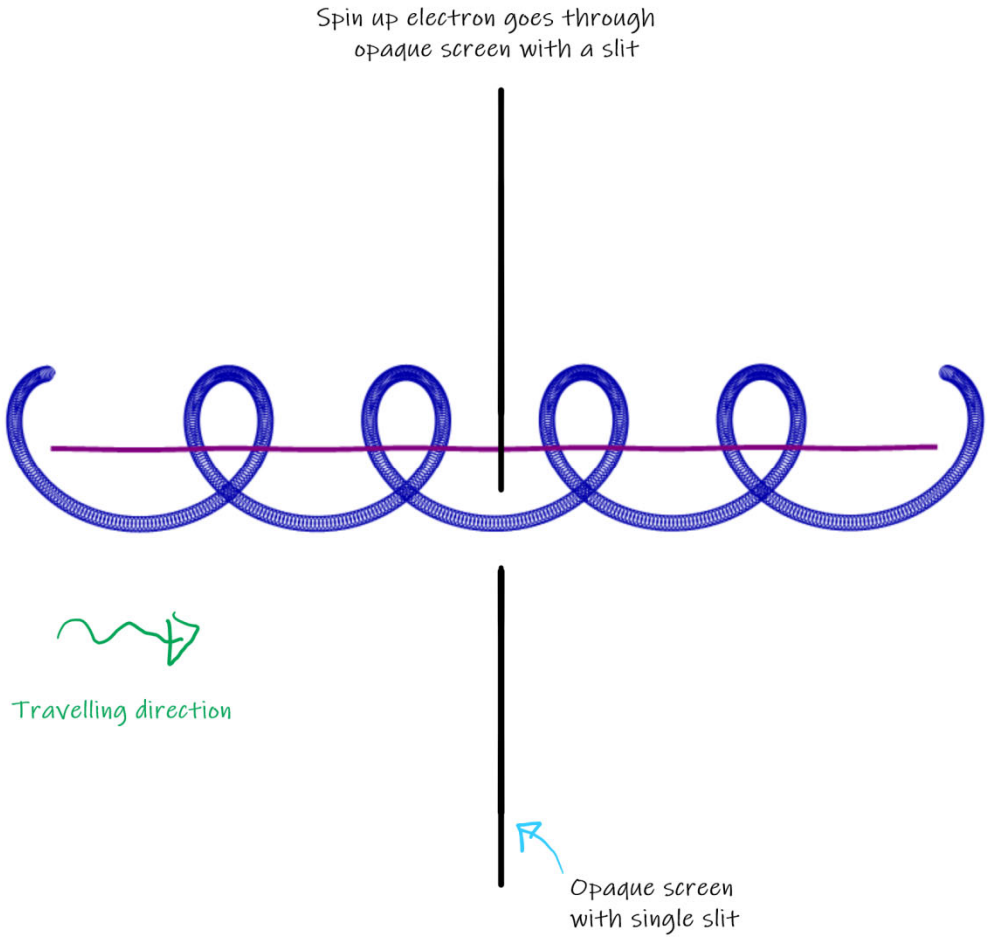


Feynman, Vol. III:

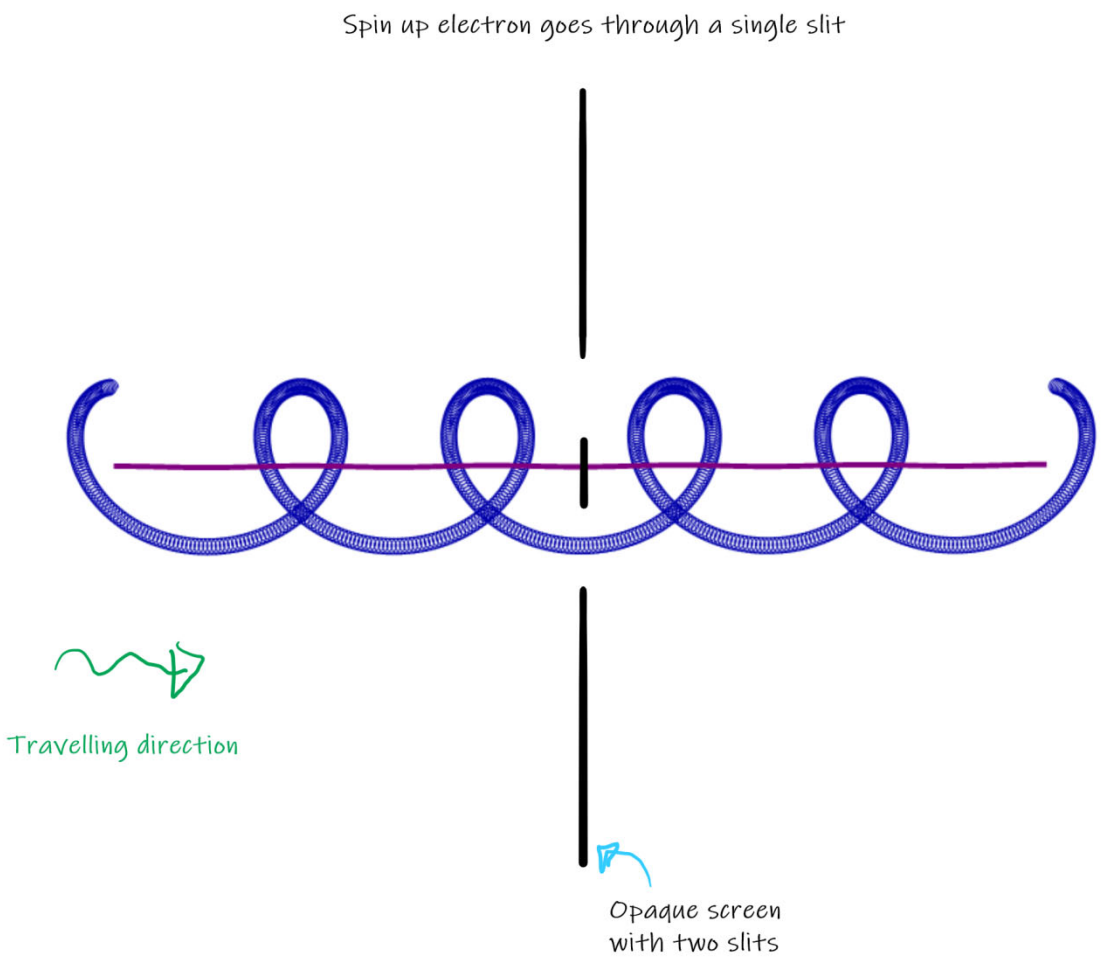
We shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery…

Single slit screen

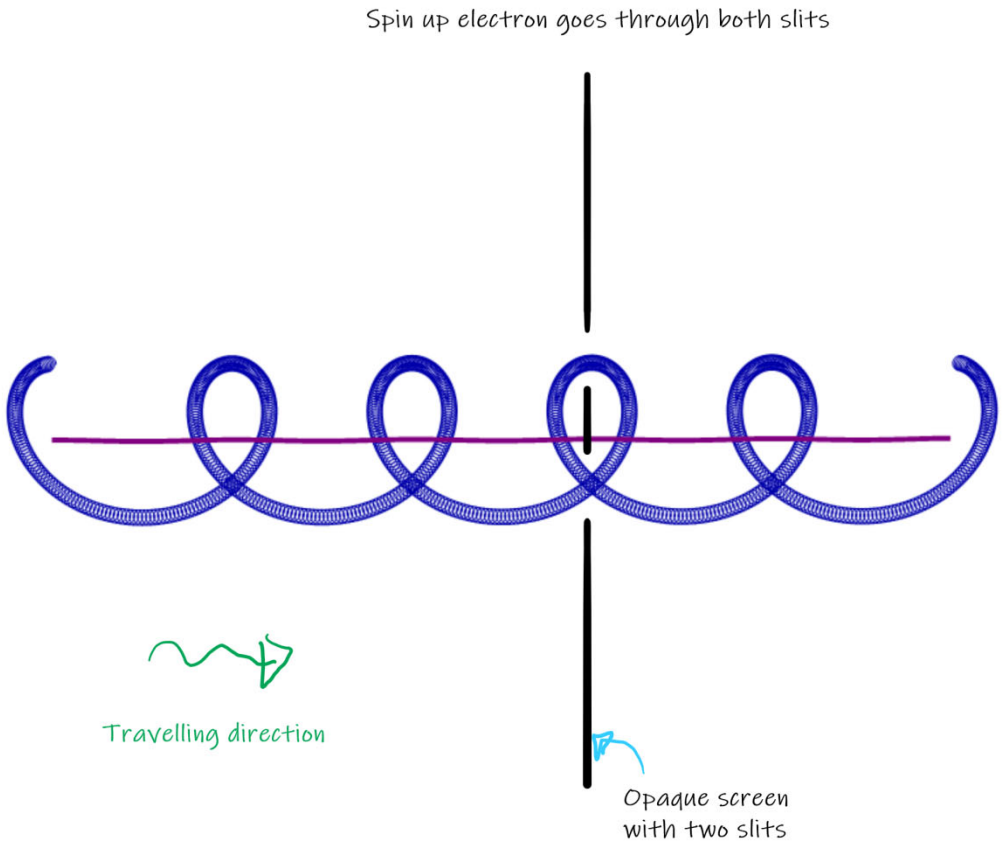
Trajectory is obtained by integration of equations of motion: not a sketch!



Screen with two slits (I)



Screen with two slits (II)

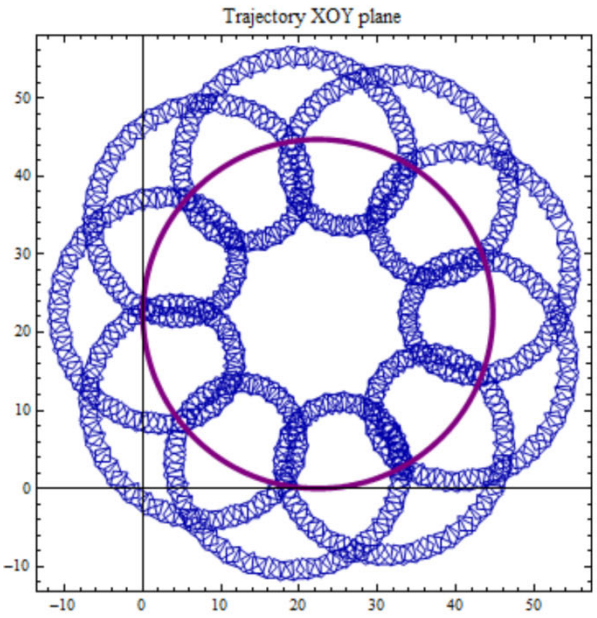


$$2 > 1 + 1$$

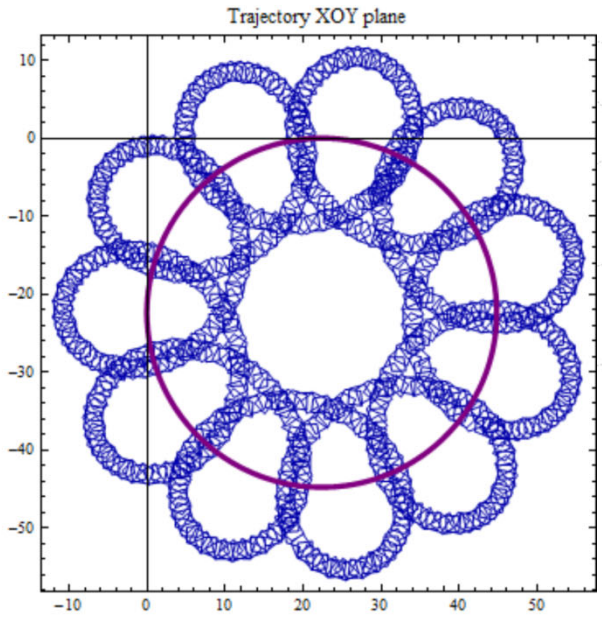
Excitation by an external field

Landau problem (electron under the influence of a static magnetic field)

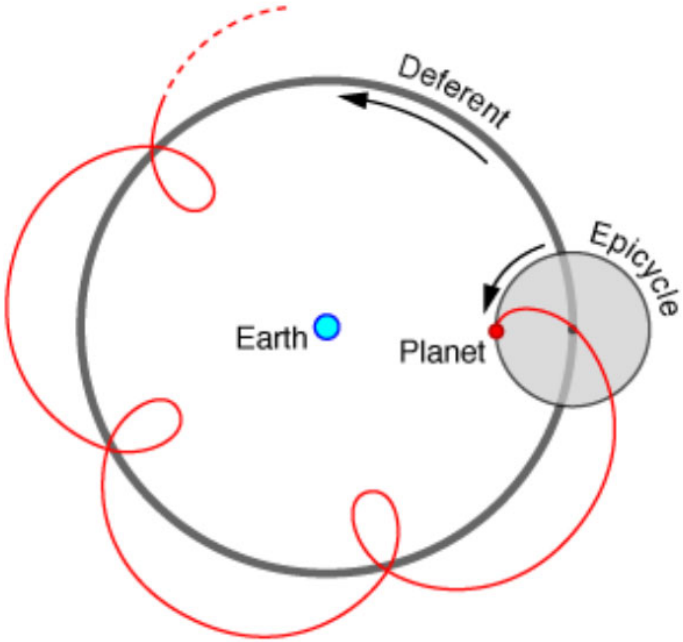
Spin up

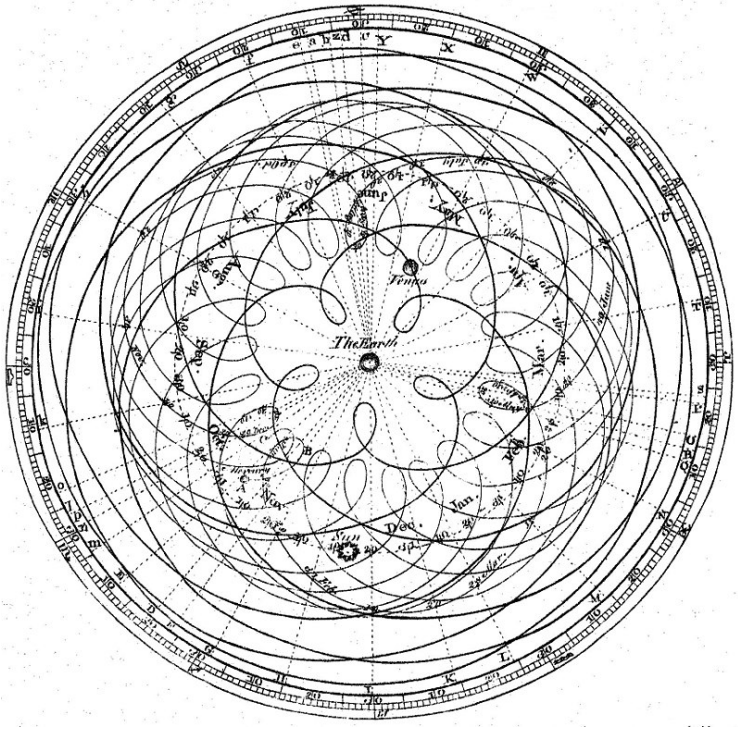


Spin down



Epicycles and Ptolemy (before Copernicus)





tinyurl.com/yepcwv46

Partial summary

- ***I introduced a Lorentz covariant non-Hermitian theory of the electron.***

- ***Relying on minimal hypotheses (postulates) the proposed theory leads naturally to the idea that a charged particle is formed by two (particle-like and wave-like) components.***

- ***The proposed model may be regarded as some sort of mechanical version of the pilot-wave model of de Broglie-Bohm.***

M.G. Silveirinha, *Non-Hermitian Electromagnetism: a Nonlocal Time-Crystal Model for an Electron with Spin*

Partial summary (contd.)

- ***Unlike the usual classical models, the self-field and self-energy of the non-Hermitian electron are finite.***

- ***The non-Hermitian electron has spin and a spin angular momentum that is precisely quantized.***

- ***The theory may shed some light on the inner workings of the "double slit experiment" and suggests that some of the weirdness of quantum physics may be captured by mechanical non-Hermitian models.***

M.G. Silveirinha, *Non-Hermitian Electromagnetism: a Nonlocal Time-Crystal Model for an Electron with Spin*

2nd Part: Link between topological photonics and geometry of surfaces