Ultrafast Measurements and Extreme Events in Nonlinear Fibre Optics



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UNIVERSITÉ **BOURGOGNE FRANCHE-COMTÉ**



Ultrafast Measurements and Extreme Events in Nonlinear Fibre Optics



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2021 – Sixty years of nonlinear optics !

The high power and spatial coherence of laser light enabled the study of the nonlinear response of light to optical fields





(the experimental evidence was removed as a speck of dirt on the photographic plate)

Luckily they were republished elsewhere !





Nonlinear optics was actually considered earlier: 1926, 1930, 1950

More on Vavilov's Contributions to 20th-century Physics

120

What Yuri Nikolaievitch and his colleague omitted to say, but may be of interest to your readers, is that Sergei Vavilov was probably the first scientist to observe a nonlinear optic effect. In 1926, with Vadim L. Levshin, he found a reduction in the absorption of light by uranium glass with an increase of intensity of 454 nm light from a high-intensity spark source.¹ And it was Vavilov who introduced the term "nonlinear optics" into the literature, in a passage in his 1950 book *Mikrostruktura sveta* ("The Microstructure of Light").

References

SEPTEMBER 1996 PHYSICS TODAY

- S. I. Vavilov, V. L. Levshin, Z. Phys. 35, 920 (1926).
- R. G. W. Brown, E. R. Pike, in *Twentieth Century Physics*, B. Pippard, A. Pais, L. Brown, eds., Bristol (England)/Boston, Institute of Physics/American Institute of Physics (1995), p. 1385.
- S. I. Vavilov, Dok. Akad. Nauk SSR 11, 457 (1934).
- P. A. Čerenkov, Dok. Akad. Nauk SSR 11, 451 (1934).
- Usp. fiz. nauk 111, 702 (1973); 114, 533 (1974).
- Sov. Phys. Usp. 16, 702 (1974); 17, 950 (1975).

ROY PIKE

University of London London, England **ROBERT G. W. BROWN** University of Nottingham Nottingham, England Sharp Laboratories of Europe Ltd Oxford, England

Über Elementarakte mit zwei Quantensprüngen

Von Maria Göppert-Mayer

(Göttinger Dissertation)

(Mit 5 Figuren)

Einleitung

Der erste Teil dieser Arbeit beschäftigt sich mit dem Zusammenwirken zweier Lichtquanten in einem Elementarakt. Mit Hilfe der Diracschen Dispersionstheorie¹) wird die Wahrscheinlichkeit eines dem Ramaneffekt analogen Prozesses, nämlich der Simultanemission zweier Lichtquanten, berechnet.



(Eingegangen 7. Dezember 1930)

Motivation and Overview – Extreme Events

Extreme events are defined by rarity, unpredictability, and often highly destructive impact



Natural disasters

Motivation and Overview – Extreme Events

Extreme events are defined by rarity, unpredictability, and often highly destructive impact



This has created a new interdisciplinary field of science combining areas of specific expertise (geology, climate, hydrodynamics) with statistics, physics, simulations etc.

Rogue waves are a particular type of extreme event

Rogue waves are large and destructive waves that appear on the ocean's surface, outside the range of amplitudes expected from standard linear wave theory



Fig. I.2 Observation of the highest reported wave by the crew members of "Ramapo" (Dennis and Wolff 1996)











Rogue waves appear in the long tail of wave height distributions

The Draupner Wave of 1995 went beyond anecdote & provided **quantitative data**





Long term wave height measurements show asymmetric distributions. Rogue waves are "extreme events" in the tails, beyond the predicted Rayleigh distribution



M. S. Longuet-Higgins Journal of Marine Research 11, 245-266 (1952)

In 2007, "optical rogue waves" were reported in the supercontinuum

Optical rogue waves appeared in noisy **fibre supercontinuum generation** that showed a long-tailed distribution in intensity fluctuations at particular wavelengths



It is important to appreciate the context here

Supercontinuum generation was only possible because of the photonic crystal fiber



The photonic crystal fiber concept celebrated its 30th birthday in 2021 !!

Google now appreciates fibre optics!

Reliable techniques for fabricating small-core waveguides yielded the birth of fibre optics

PROC. IEE, Vol. 113, No. 7, JULY 1966 Dielectric-fibre surface waveguides for optical frequencies







Details: (i) total internal reflection (ii) the binary sequences converted to ASCII spell K A O

The femtosecond Ti:Sapphire laser also celebrates its 30th birthday

It was the injection of femtosecond pulses from a Ti:Sapphire into the PCF that led to the supercontinuum



The discovery of self-modelocking led to the transfer of soliton concepts into ultrafast laser design, and the concept of the "dissipative soliton" laser

The femtosecond Ti:Sapphire laser also celebrates its 30th birthday

The Kerr lens modelocked Ti:Sapphire oscillates with a spatio-temporal balance between dispersion-managed temporal solitons and diffraction-managed spatial solitons



Fig. 1. Schematic of the cavity configuration for self-modelocked Ti:Al $_2O_3$ laser. The inset shows the intracavity prism sequence for dispersion compensation.





W Sibbett *et al.* The development and application of femtosecond laser systems Optics Express **20** 6989-7001 (2012)

1964 – nonlinear self-focusing and solitons in optics

OPN Optics & Photonics News November 2010

Jeff Hecht

How the Laser Launched Nonlinear Optics



The fact that the damaged zone was smaller than the focal spot-and that it did not increase as the beam passed through the glass-led Townes to suggest that optical nonlinearities were offsetting beam diffraction to cause self-trapping.



1964 – Townes theory & the nonlinear Schrödinger equation (NLSE)

VOLUME 13, NUMBER 15

PHYSICAL REVIEW LETTERS

12 October 1964

SELF-TRAPPING OF OPTICAL BEAMS*

R. Y. Chiao, E. Garmire, and C. H. Townes Massachusetts Institute of Technology, Cambridge, Massachusetts (Received 1 September 1964)

We shall discuss here conditions under which an electromagnetic beam can produce its own dielectric waveguide and propagate without spreading. This may occur in materials whose dielectric constant increases with field intensity, but which are quite homogeneous in the absence of the electromagnetic wave. Such self-trapping in dielectric waveguide modes appears to be possible in intense laser beams, and to produce marked optical and physical effects. In the case where E_t depends only on y, and under the assumption of linear polarization,

$$\frac{d^2}{dy^2}E_t(y) - \Gamma^2 E_t(y) + \frac{\epsilon_2}{2}k_0^2 E_t^3(y) = 0.$$
 (5)

If E_t represents a slab-shaped beam, confined in the y direction, the boundary conditions are $E(y) \rightarrow 0$ as $y \rightarrow \infty$ and dE/dy = 0 at y = 0. This excludes periodic solutions, so that $\Gamma^2 > 0$. A mechanical analog of (5) is a particle in a doublewell quartic potential-energy function. It is immediate from consideration of this analog that there is a unique solution which is not oscillatory, namely $E_t(y) = E_t(0)/\cosh\Gamma y$, where Γ must equal $\frac{1}{2}\epsilon_2^{1/2}k_0E_t(0)$. Note that, given a certain size of the beam $(\sim 1/\Gamma)$, the field inside the beam must attain a value $E_t(0)$ for trapping.

First statement of the cubic NLSE in optics & sech-soliton solution

Actually not the first statement of the NLSE

SOVIET PHYSICS JETP

VOLUME 23, NUMBER 6

DECEMBER 1966

SELF-FOCUSING AND SELF-TRAPPING OF INTENSE LIGHT BEAMS IN A NONLINEAR MEDIUM

S. A. AKHMANOV, A. P. SUKHORUKOV, and R. V. KHOKHLOV

Moscow State University

 $2ik\frac{\partial A}{\partial z} = \Delta_{\perp}A + \frac{n_2|A|^2}{n_0}k^2A + \frac{n_4|A|^4}{n_0}k^2A.$ (8)



¹G. A. Askar'yan, JETP **42**, 1567 (1962), Soviet Phys. JETP **15**, 1088 (1962).

² V. I. Talanov, Izv. Vuzov, Radiofizika 7, 564 (1934).

³ R. Y. Chiao, E. Garmire, and C. Townes, Phys. Rev. Lett. **13**, 479 (1964) (erratum, Phys. Rev. Lett. **14**, 1056 (1965)).

⁴ L. V. Keldysh, Report to the Session of the Department of General and Applied Physics of the U.S.S.R. Academy of Sciences, 1964. In 1967, Khokhlov & Akhmanov received the Lenin Prize, celebrated with a mural of them both riding a horse upon an SHG crystal converting red to green. It was on the wall for many years at Moscow State University. This photo is from @jeffhecht 's article. Where is the mural now?



Now we return to "rogue waves" in the supercontinuum

Optical extreme events appeared in the statistics of noisy supercontinuum generation



An analogy is suggested by the same governing equation

Light pulse propagation in optical fibre and wave group propagation on deep water are both described by the nonlinear Schrödinger equation (NLSE)





Why are nonlinear Schrödinger equation-like systems so interesting?

Nonlinear optical waveguides provide a platform to study a wide range of other systems

www.sciencemag.org SCIENCE VOL 319 7 MARCH 2008 **Fiber-Optical Analog of the** nature NATURE REVIEWS | PHYSICS VOLUME 1 | NOVEMBER 2019 | 675 COMMUNICATIONS **Event Horizon** REVIEWS Received: 30 April 2019; Accepted: 20 January 2020; Thomas G. Philbin,^{1,2} Chris Kuklewicz,¹ Scott Robertson,¹ Stephen Hill,¹ Published online: 10 February 2020 Friedrich König.¹ Ulf Leonhardt¹* ARTICLE https://doi.org/10.1038/s41467-020-14634-0 The physics at the event horizon resembles the behavior of waves in moving media. Horizons are OPEN formed where the local speed of the medium exceeds the wave velocity. We used ultrashort pulses Supersymmetry in the time domain and its in microstructured optical fibers to demonstrate the formation of an artificial event horizon in Rogue waves and analogies in optics optics. We observed a classical optical effect: the blue-shifting of light at a white-hole horizon. We applications in optics also showed by theoretical calculations that such a system is capable of probing the quantum effects of horizons, in particular Hawking radiation. and oceanography Carlos García-Meca (^{1,2}*, Andrés Macho Ortiz (^{1,2}* & Roberto Llorente Sáez (¹ John M. Dudley^{1*}, Goëry Genty², Arnaud Mussot³, Amin Chabchoub⁴ and Frédéric Dias nature ARTICLES PHYSICAL REVIEW LETTERS 122, 010404 (2019) physics Editors' Suggestion PUBLISHED ONLINE: 31 AUGUST 2015 | DOI: 10.1038/NPHYS3451 Observation of Stimulated Hawking Radiation in an Optical Analogue **Optical simulations of gravitational effects in the** nature Newton-Schrödinger system Jonathan Drori,¹ Yuval Rosenberg,¹ David Bermudez,² Yaron Silberberg,¹ and Ulf Leonhardt¹ COMMUNICATIONS ¹Weizmann Institute of Science, Rehovot 7610001, Israel ²Departamento de Física, Cinvestav, A.P. 14-740, 07000 Ciudad de México, Mexico Rivka Bekenstein*, Ran Schley, Maor Mutzafi, Carmel Rotschild and Mordechai Segev ARTICIE (Received 28 August 2018; revised manuscript received 12 November 2018; published 9 January 2019) Some predictions of Einstein's theory of general relativity (GR) still elude observation, hence analogous systems, such as Received 14 Mar 2014 | Accepted 12 Aug 2014 | Published 17 Sep 2014 The theory of Hawking radiation can be tested in laboratory analogues of black holes. We use light optical set-ups, have been suggested as platforms for emulating GR phenomena. GR is inherently nonlinear: for example, the DOI: 10.1038/ncomms5969 curvature of space is induced by masses whose dynamics is also affected by the curved space they themselves induce. But, pulses in nonlinear fiber optics to establish artificial event horizons. Each pulse generates a moving thus far all GR emulation experiments with optical systems have reproduced only linear dynamics. Here, we study gravitational Nonlinear optics of fibre event horizons perturbation of the refractive index via the Kerr effect. Probe light perceives this as an event horizon when effects with optical wavepackets under a long-range nonlocal thermal nonlinearity. This system is mathematically equivalent to its group velocity, slowed down by the perturbation, matches the speed of the pulse. We have observed in the Newton-Schrödinger model proposed to describe the gravitational self-interaction of quantum wavepackets. We emulate gravitational phenomena by creating interactions between a wavepacket and the gravitational potential of a massive star, our experiment that the probe stimulates Hawking radiation, which occurs in a regime of extreme nonlinear observing gravitational lensing, tidal forces and gravitational redshift and blueshift. These wavepackets interact in the fiber optics where positive and negative frequencies mix. Karen E. Webb¹, Miro Erkintalo¹, Yiqing Xu^{1,2}, Neil G.R. Broderick¹, John M. Dudley³, curved space they themselves induce, exhibiting complex nonlinear dynamics arising from the interplay between diffraction, interference and the emulated gravitational effects. Goëry Genty⁴ & Stuart G. Murdoch¹

The validity or otherwise of some of these analogies is still an open question

What we set out to explain in 2007 ...

- 1. Is the study of ocean rogue waves **using an analogy with optics** really valid? In any case, are optical rogue waves perhaps interesting in their own right?
- 2. Are ocean rogue waves generated from linear or nonlinear effects or both?



First we examine rogue waves in supercontinuum generation

The NLSE describes the evolution of an ultrashort pulse envelope in space and time

$$i\frac{\partial A(z,T)}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A(z,T)}{\partial T^2} - \gamma |A(z,T)|^2 A(z,T) \qquad \text{Kerr nonlinearity} \quad \gamma = n_2 \omega_0 / cA_{eff}$$

instantaneous power (W) $|A(z,T)|^2$

Linear dispersion (GVD) changes temporal pulse shape but does not alter the spectrum





First we examine rogue waves in supercontinuum generation

The NLSE describes the evolution of an ultrashort pulse envelope in space and time

$$i\frac{\partial A(z,T)}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A(z,T)}{\partial T^2} - \gamma |A(z,T)|^2 A(z,T)$$

$$\begin{aligned} & \text{co-moving time} \quad T = t - z/v_g = t - \beta_1 z \\ & \text{Kerr nonlinearity} \quad \gamma = n_2 \omega_0 / cA_{eff} \\ & \text{instantaneous power (W)} \quad |A(z,T)|^2 \end{aligned}$$





Supercontinuum physics and soliton dynamics

With fs pulses injected in the anomalous dispersion regime the supercontinuum develops from perturbed higher-order soliton propagation (soliton fission)

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k>2} \frac{\mathrm{i}^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT'\right)$$

Linear dispersion

Self-steepening

SPM, FWM, Raman



- 1. SPM & GVD on their own yield ideal periodic evolution
- 2. Perturbations (Raman, high-order dispersion...) induce <u>fission</u> into fundamental solitons
- 3. Solitons generate blue-shifted dispersive waves
- 4. Raman soliton self-frequency shift to longer wavelengths

Golovchenko, Dianov, Karasik, Prokhorov, Serkin., JETP Lett. **42** 87-91 (1985) Blow & Wood, IEEE J. Quant. Electron. **25** 2665-2673 (1989) Dudley, Genty, Coen, Rev. Mod. Phys. **78** 1135-1184 (2006) Agrawal, Nonlinear Fibre Optics 6th Ed (2019)

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More complex supercontinuum dynamics





5

Input pulses: FWHM 22 fs, 810 nm, 0.9 nJ, 15 cm PCF



- 1. SPM & GVD on their own yield ideal periodic evolution
- 2. Perturbations (Raman, high-order dispersion...) induce fission into fundamental solitons
- 3. Solitons generate blue-shifted **dispersive waves**

Pump

1000

4. Raman soliton self-frequency shift to longer wavelengths

More complex supercontinuum dynamics



FWHM 50 fs, 835 nm, 0.5 nJ, 15 cm PCF, N ~ 9

5



Input pulses: FWHM 22 fs, 810 nm, 0.9 nJ, 15 cm PCF

- 1. SPM & GVD on their own yield ideal periodic evolution
- 2. Perturbations (Raman, high-order dispersion...) induce <u>fission</u> into fundamental solitons
- 3. Solitons generate blue-shifted dispersive waves
- 4. <u>Raman soliton self-frequency shift to longer wavelengths</u>

But supercontinuum instabilities were seen in experiments

After the 1999 results, octave-spanning supercontinuum spectra were readily generated but why did some experiments show **highly-structured**, and others **smooth** spectra?



The smooth spectra were unsuitable for frequency combs - an underlying instability?

Modelling gave the answer, reproducing the instabilities

With long pulses, soliton-driven supercontinuum generation can be highly unstable.

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k\geq 2} \frac{\mathrm{i}^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,t) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T-T')|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) \int_{-\infty}^{+\infty} R(T') |A(z,T)|^2 dT' + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \left(A(z,T) + \mathrm{i}\tau_{\mathrm{shock}} \frac{\partial}{\partial T}\right) \right)$$





The physics

- sensitivity of soliton fission to noise
- incoherent growth of spectral components outside the initial pulse bandwidth

J. M. Dudley, G. Genty, S. Coen, Rev. Mod. Phys. 78 1135 (2006)

The source of supercontinuum noise – modulation instability

Modulation instability (MI) is a fundamental property of nonlinear systems where modulation on a continuous wave grows exponentially





Lake, Yuen et. al. J. Fluid Mech. 83, 49 (1977)





	Physica D 238 (2009) 540-548
	Modulation instability: The beginning
PHYSICA D Nonlikkar phenomena	V.E. Zakharov ^{a,b} , L.A. Ostrovsky ^{c,d,*}
Conceptor	^a Department of Mathematics, University of Arizona, Tucson, AZ, USA ^b Lebedev Institute of Physics, Russian Acad. Sci., Russia ^c Zel Technologies/NOAA Earth Science Research Laboratory, USA ^d Institute of Applied Physics, Russian Acad. Sci., Russia

When stimulated by a coherent modulation, MI leads to **coherent solitons & breathers** When stimulated by noise, MI leads to chaos & **"rogue wave" statistics**

An extensive area of earlier work in mathematical physics



fore, E(f) receives complex values. As a consequence, b_f , b_f^* will involve a real exponential increasing with time, whence it follows that the states with small $N_f = b_f^* b_f$ are unstable.

In order to be sure in the stability of the excited states, let us restrict the class of possible types of interaction forces, supposing inequality (7) to be satisfied for all types we shall consider. It is interesting to note Bogolubov recognized the unstable branch but seems not to have studied it further!

N. Bogolubov On the Theory of Superfluidity *Journal of Physics* 11 23–32 (1947)



V. I. Bespalov and V. I. Talanov Filamentary structure of light beams in nonlinear liquids JETP Lett. 3, 307-310, (1966)



J. Lighthill Contributions to the theory of waves in nonlinear dispersive system J. Inst. Math. Appl. 1 269 (1965)



G.B. Whitham A general approach to linear and nonlinear dispersive waves using a Lagrangian J. Fluid Mech. 22 273 (1965)



T. B. Benjamin, J. E. Feir The disintegration of wave trains on deep water. Part I. Theory J. Fluid Mech. 27 417 (1967)

Coherent solitons and breathers

Before studying random nonlinear structures from modulation instability, we first see if we can excite the expected mathematical breather structures from a coherent modulation

$$i\psi_{\xi} + \frac{1}{2}\psi_{\tau\tau} + |\psi|^2\psi = 0$$



Figure 3 | Experimental set-up. ECL: external-cavity laser; OSA: optical spectrum analyser; FROG: frequency-resolved optical gating. HNLF: highly nonlinear fibre. EDFA: erbium-doped fibre amplifier.

We create frequency-domain initial conditions based on the analytic mathematical form for a particular soliton structure



Solitons and breathers in modulation instability

With coherent initial modulation, modulation instability evolves towards stable breather or soliton structures. This has been confirmed in experiments.



Transferring results into hydrodynamics

The generation of coherent structures from induced modulation instability has also been performed in hydrodynamic wave tanks, confirming the NLSE analogy between the systems



week ending 20 MAY 201

Rogue Wave Observation in a Water Wave Tank

A. Chabchoub,^{1,*} N. P. Hoffmann,¹ and N. Akhmediev² ¹Mechanics and Ocean Engineering, Hamburg University of Technology, Eißendorfer Straße 42, 21073 Hamburg, Germany ²Optical Sciences Group, Research School of Physics and Engineering, The Australian National University, Canberra ACT 0200, Australia (Received 28 February 2011; published 16 May 2011)



OPEN O ACCESS Freely available online

PLOS ONE

Rogue Waves: From Nonlinear Schrödinger Breather Solutions to Sea-Keeping Test

Miguel Onorato^{1,2}, Davide Proment³*, Günther Clauss⁴, Marco Klein⁴

1 Dipartimento di Fisica, Università degli Studi di Torino, Torino, Italy, 2 INFN, Sezione di Torino, Torino, Italy, 3 School of Mathematics, University of East Anglia, Norwich, United Kingdom, 4 Ocean Engineering Division, Technical University of Berlin, Berlin, Germany



Measuring spectral instabilities in real-time

What was significant about the 2007 measurements in supercontinuum generation was the ability to measure the spectral instabilities in real time





Measuring spectral instabilities in real-time

Dispersive Fourier Transform

(measures spectral intensity, sub-nm resolution)

Principle: The temporal intensity of a pulse stretched by large linear dispersion is proportional to its spectral intensity





Shot-to-shot spectra are stretched to 5-10 nanoseconds and measured using a fast oscilloscope

This was in fact a much earlier idea – the "spectron"

Self-action of wave packets in a nonlinear medium and femtosecond laser pulse generation

S. A. Akhmanov, V. A. Vysloukh, and A. S. Chirkin

M. V. Lomonosov Moscow State University

Usp. Fiz. Nauk 149, 449-509 (July 1986)

1.4.1. "Spectron"; puise shape in the far-zone

Let us analyze the propagation of PM pulses in a dispersive medium for arbitrary initial shape of the profile $\rho_0(t)$.

At the output of a frequency-modulating device a pulse has the form

$$A_0(t) = \rho_0(t) e^{-i\alpha_0 t^2/2}.$$
 (1.37)

Evolution of this pulse in a dispersive medium in the secondorder approximation of dispersion theory is described by the expression (1.12). In this case, at a distance z = F $= (\alpha_0 k_2)^{-1}$ we obtain

$$A (\eta, z) = (i \cdot 2\pi k_2 z)^{-1/2} \widetilde{\rho}_0 (\alpha_0 \tau_0 \eta) e^{i\alpha_0 \eta^2/2}, \qquad (1.38)$$

$$\widetilde{\rho}_0(\alpha_0\tau_0\eta) = \int_{-\infty}^{+\infty} \rho_0\left(\frac{t}{\tau_0}\right) e^{-i\alpha_0\eta t} dt.$$
(1.39)

From the obtained result it is possible to draw the following conclusions about the pulse in the "focal" plane of the "time" lens. The pulse shape is exactly the same as the Fourier-spectrum of the initial pulse.^{34,39} Such pulses are called "spectrons."^{20,40} The profile of pulses turns out to be sym232 OPTICS LETTERS / Vol. 8, No. 4 / April 1983

Real-time Fourier transformation in dispersive optical fibers

Tomasz Jannson

Research Division, National Technical Systems, Inc., Los Angeles, California, 90066

Received November 22, 1982

The general concept of temporal Fourier transformation in dispersive media is analyzed. The real-time optical Fourier transformer is shown to be realizable by using dispersive single-mode fibers and chirping lasers.

²⁰Yu. E. D'yakov and S. Yu. Nikitin, Zadachi po statisticheskoĭ radiofizike i optike, Moscow State University Press, M., 1985 (Problems on statistical radiophysics and optics).

Measuring spectral instabilities in real-time

When applied to supercontinuum generation we can directly measure noise-induced shotto-shot fluctuations in spectral structure, and the associated statistics





Around 830 nm

Around 1550 nm

What makes an optical rogue wave different?

To understand the physics of optical rogue waves in the supercontinuum, we use simulations to see the effect of random noise on the input pulse

Most simulations yield dynamics like this



A small number of simulations yield dynamics like this (RS = rogue soliton)



What makes an optical rogue wave different?

Random noise can lead to very different dynamics, and in a small number of cases, the emergence of extreme rogue solitons that undergo dramatic shifts to longer wavelengths

A small number of simulations yield dynamics like this (RS = rogue soliton)



The physics

Random noise Chaotic modulation instability Chaotic soliton dynamics Inelastic (Raman) energy transfer

One soliton can become larger than the others

The next step: looking at incoherent modulation instability

Can we measure noise-seeded random modulation instability dynamics in more detail?



In particular

Can we also measure statistics of the temporal fluctuations?

Can we measure the intensity profiles of the chaotic temporal peaks and compare with analytic soliton / breather structures?

Measuring temporal instabilities in real-time

Time lens magnifier

(temporal intensity, sub-ps resolution)

Principle: A signal experiencing dispersion before and after quadratic temporal phase is temporally magnified



Analogous to temporal imaging



Picosecond structures are stretched to the nanosecond scale and measured using a fast oscilloscope

Time-lens measurements of modulation instability

NATURE COMMUNICATIONS | 7:13675 | DOI: 10.1038/ncomms13675 | www.nature.com/naturecommunications

Real-time measurements of spontaneous breathers and rogue wave events in optical fibre modulation instability

Mikko Närhi¹, Benjamin Wetzel^{2,3}, Cyril Billet⁴, Shanti Toenger^{1,4}, Thibaut Sylvestre⁴, Jean-Marc Merolla⁴, Roberto Morandotti^{2,5,6}, Frederic Dias⁷, Goëry Genty¹ & John M. Dudley⁴



Direct measurement of instability time series and histogramComparison with stochastic NLSE simulationsComparing intensity peaks with analytic soliton profiles



What if you do not have a time lens?

Even when time lens measurements are not possible, a **neural network** algorithm can be trained to determine key temporal characteristics based only on measurements of spectral intensity (i.e. no spectral phase information).



Can we configure a neural network to map complex shot to shot spectra to the peak intensity of the corresponding modulated temporal intensity profile (rogue waves)? Can $S_k(\lambda)$ predict P_k^{max} ?

Experimental setup

Modelling suggests that this requires a very high dynamic range real-time spectrometer, so experiments are performed using a reduced repetition rate laser and scanning bulk monochromator

NATURE COMMUNICATIONS | DOI: 10.1038/s41467-018-07355-y

Machine learning analysis of extreme events in optical fibre modulation instability

Mikko Närhi ¹, Lauri Salmela ¹, Juha Toivonen¹, Cyril Billet², John M. Dudley ² & Goëry Genty¹





Single shot spectra (red) compared to mean (black)

900

900

Training a neural network to analyse modulation instability

We use a standard neural network architecture and use simulations to train a network to correlate the full MI spectrum with the corresponding peak of the associated temporal intensity profile.



$\mathbf{X}_n = [x_1, x_2 \dots x_N]$

ArchitectureFeedforward2 hidden layers (30, 10 nodes)

Training using NLSE simulations

 (\mathbf{X}_n, Y_n) , n = 1....30,000

 $\mathbf{X}_n = [x_1, x_2 \dots x_N]$, N = 121

300 epochs of 30,000 simulations

Testing the trained neural network

We use a standard neural network architecture and use simulations to train a network to correlate the full MI spectrum with the corresponding peak of the associated temporal intensity profile.



Testing the trained network

Use 20,000 simulations not in the training set.

For each spectrum we predict the peak intensity.

We plot the predicted intensity against the known ground truth intensity from the corresponding simulation.

We obtain very high correlation.

Analysing experimental spectra to predict time-domain peaks

Based on 3000 <u>measured</u> real-time <u>spectra</u>, we determine the corresponding temporal peaks and compute the associated <u>temporal</u> probability density function.



* Training for experimental data also includes the wavelength response of the spectrometer

We compare the machine learning predictions with fully realistic simulations of our experiments.

Can we do the same in supercontinuum generation?

In other words, based on spectral data only, can we determine the properties of the most red-shifted solitons ("rogue solitons") in a temporal supercontinuum field?



Temporal peak power Temporal duration Temporal walk-off

Spectrum

SCIENTIFIC REPORTS (2020) 10:9596 [https://doi.org/10.1038/s41598-020-66308-y Machine learning analysis of rogue solitons in supercontinuum generation

Lauri Salmela^{1⊠}, Coraline Lapre², John M. Dudley² & Goëry Genty¹





Machine learning and rogue solitons

We train a neural network to correlate the full supercontinuum spectrum with the peak intensity of the the red-shifted "rogue" soliton



The Approach

- 1. Train using 20000 simulations
- 2. Test using 10000 independent simulations
- 3. Compare predicted peak intensity with simulation "ground truth"



Conclusion: a neural network can use only supercontinuum spectral data to infer maximum temporal peak power of red-shifted Raman solitons

Model free modelling of nonlinear fibre propagation

Numerical integration of generalized nonlinear propagation equations can be time-consuming. Can we train a neural network to model nonlinear propagation directly?



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- 4. G. Genty et al. Nat. Photon. 15, 91 (2021)
- 5. U. Teğin et al. Nat. Mach. Intell. **3**, 387 (2021)



Top: Ten consecutive numerically simulated spectral-intensity profiles are input into a recurrent neural network, the output of which is the predicted spectrum at the next step. Bottom left: Operation of a cell in the long short-term memory (LSTM) recurrent layer. Bottom right: The network's predicted spectral-intensity evolution along the fiber.

From waveguides to lasers ...

Instabilities have been observed since the first development of lasers in the 1960s but we can now use ultrafast real-time characterization tools to uncover new details



Understanding instabilities in lasers

In addition to new measurement techniques, we now have the general framework of the **dissipative soliton** to describe lasers where nonlinear dynamics coexist with gain & loss





Spectral Instabilities in a Soliton-Similariton Laser

The soliton similariton laser is a novel fibre oscillator where two conceptually different nonlinear structures co-exist within a highly dissipative fibre laser cavity



Spectral Instabilities in a Soliton-Similariton Laser

A similariton or self-similar amplifier is one where all input pulses evolve asymptotically to the same output pulse (a nonlinear attractor)



G. I. Taylor: Proc. Royal Soc. London A 201 175-196 (1950)

Q. Can similariton evolution make fibre lasers more robust against nonlinear instability?

Characterizing more extreme intracavity laser dynamics

Experiments using real-time dispersive Fourier transform measurements reveal comparable rich dynamics as suggested by the modelling



fibre amplifier.

Startup

Chaos and Intermittence



Work by many other groups as well

What actually happens when you turn on a fibre laser?

The build-up of stable mode-locking in a fiber laser typically shows complex dynamics



Time (100 ns/div.)

0.2

Time (50 µs/div.)

Slow detection over long time window

Structures with different time scales

Pseudo-periodic Q-switched bursts with sub-ms period

Periodic quasi mode-locked pulses with roundtrip period (50 ns)

Q: Can we measure the sub-picosecond chaotic build-up dynamics?

Time (100 ns/div.)

Measuring instabilities in lasers

Simultaneous time lens & dispersive Fourier transform provides complete real time characterization of soliton buildup from noise in a SESAM-modelocked soliton fibre laser



In how much detail can we study startup dynamics in a dissipative soliton laser?



Multiscale dynamics where we need ps resolution over a measurement window of 50 μs

Build up of solitons from noise





The regime just before stability shows periodic soliton breathing





Build up of solitons from noise

We see complex dynamics and multiple pulses, with phases of growth and collapse



Ryczkowski et al. Nature Photonics 12, 221-227 (2018)

New results from a broadband noise-like pulse laser

Time lens measurements in a 1000 nm broadband **noise-like pulse laser** reveal soliton instability properties and statistics in excellent quantitative agreement with modelling

NATURE COMMUNICATIONS | (2021)12:5567 | https://doi.org/10.1038/s41467-021-25861-4 ARTICLE

 Oneck for updates

https://doi.org/10.1038/s41467-021-25861-4 OPEN

Intracavity incoherent supercontinuum dynamics and rogue waves in a broadband dissipative soliton laser

Fanchao Meng¹, Coraline Lapre¹, Cyril Billet¹, Thibaut Sylvestre⊙ ¹, Jean-Marc Merolla¹, Christophe Finot², Sergei K. Turitsyn⊙ ^{3,4}, Goëry Genty⊙ ⁵ & John M. Dudley⊙ ¹⁸³



Experiment Intensity (arb. units) mmmmmm Simulation Intensity (arb. units) -20 20 -10 10 0 Time (ps)



Conclusions

- 1. Optical rogue waves can emerge out of fibre nonlinear dynamics due to the noise sensitivity of modulation instability and soliton propagation
- 2. But other experiments & modelling in both optics and hydrodynamics suggest that nonlinearity is not the only way in which extreme wave events can occur





	REVIEWS
and oceanography	Mussot ³ , Amin Chabchoub ⁴ and
ohn M. Dudley of *, Coèry Genty?, Arnaud M rédéric Dias of s bstract (Over a decade ago, an analogy was draw nd the propagation of light fields in optical fibres tudies in both systems, which we review here. In c eal-time measurement techniques, whereas in oc	Aussot ¹ , Amin Chabchoub ⁴ and vn between the generation of large ocean waves . This analogy drove numerous experimental ptics, we focus on results arising from the use of eanography we consider insights obtained from lide experiments in wave tanks. This Review of

Conclusions

- 3. Real-time measurement techniques provide new windows into studying instabilities in both fibre propagation and in ultrafast lasers
- 4. Techniques such as the dispersive Fourier transform and the time lens are now becoming necessary elements in experimental setups
- 5. Machine learning is an extremely promising addition to both experiment and analysis in nonlinear fibre optics, and maybe the future of ultrafast laser development

