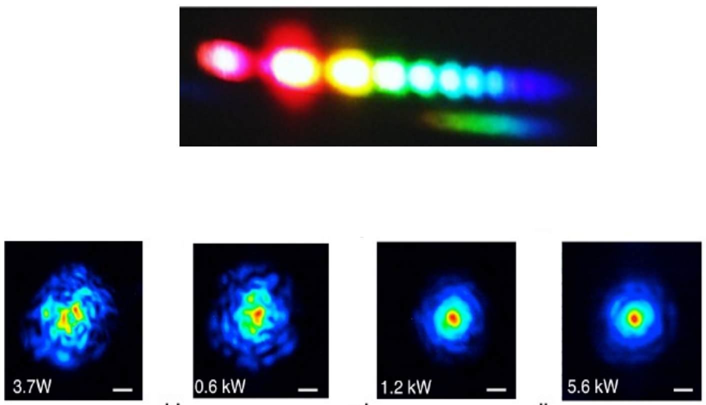
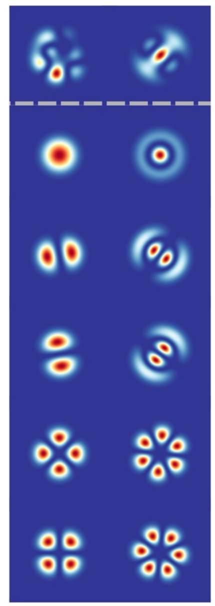
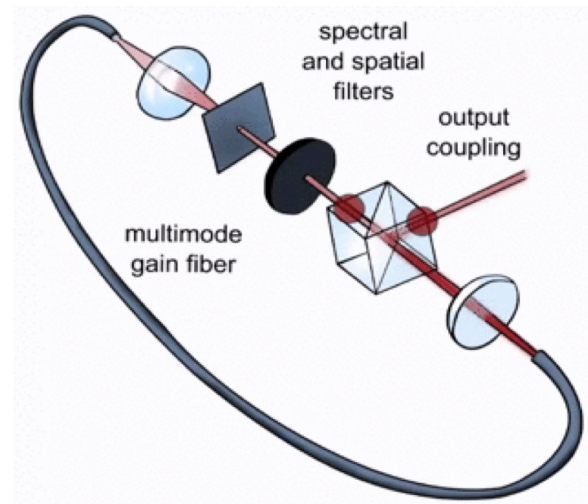
Spatiotemporal Dynamics of Optical Pulse Propagation in Multimode Fibers







F. W. Wise

Department of Applied Physics

Cornell University

Spatiotemporal Dynamics…

- Introduction to nonlinear wave propagation

- Beam self-cleaning
  Optical thermodynamics
  Measurement of 3D electric fields
- Multimode solitons
- Instabilities
- Spatiotemporal mode-locking

- Current / future directions

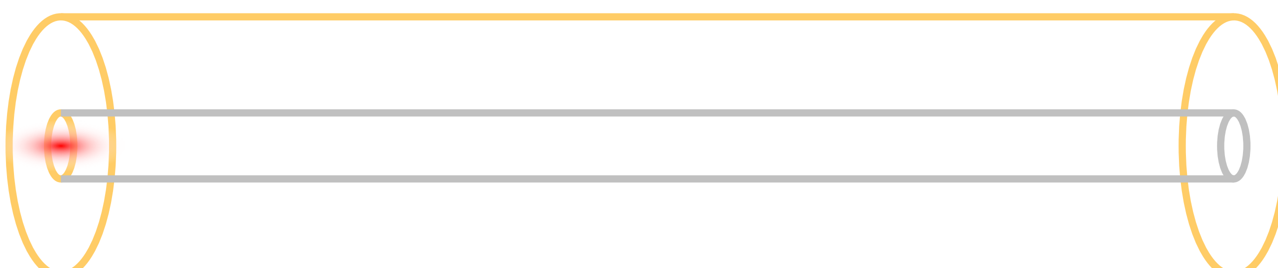
  - Emphasis on physics and science
  - Many questions remain
  - References will be provided

Introduction to Nonlinear Wave Propagation

Short pulses: dispersion

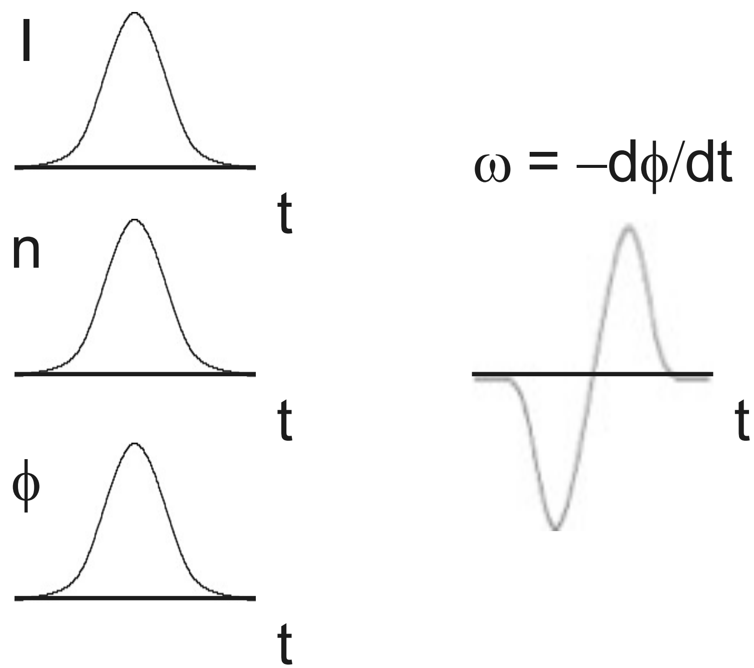
$$n = n(\omega)$$

$$v(\omega) = c/n(\omega)$$



Nonlinear propagation ($\chi^{(3)}$)

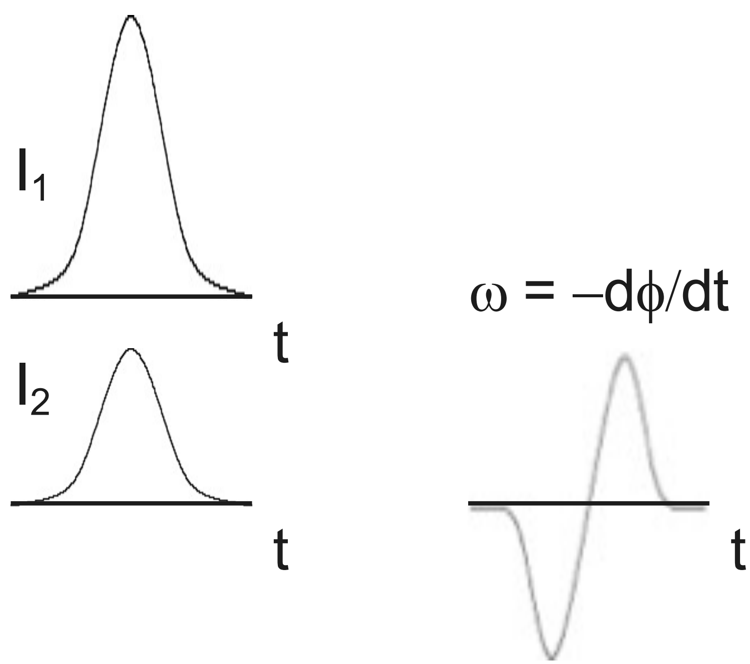
$$P = \chi^{(1)}E + \chi^{(2)}EE + \chi^{(3)}EEE + \ldots \qquad n_2 \sim \mathrm{Re}\,\chi^{(3)} \qquad n = n_0 + n_2 I$$



self-phase modulation produces new frequencies

Kerr nonlinearity from bound electrons

Nonlinear propagation ($\chi^{(3)}$)



cross-phase modulation

- produces new frequencies
- couples waves

Nonlinear propagation ($\chi^{(3)}$)

4-wave mixing

$$\frac{\partial A_p}{\partial z} \propto \sum_{(l,m,n)} S^k_{plmn}\, A_l A_m A_n^*\, e^{i(\beta_l+\beta_m-\beta_n-\beta_p)z}$$

$$\Delta\beta = \beta_l + \beta_m - \beta_n - \beta_p$$

- allows modes to exchange energy

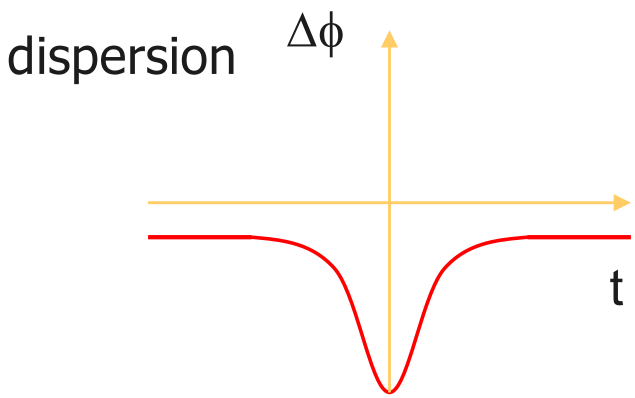
Dispersion and nonlinearity

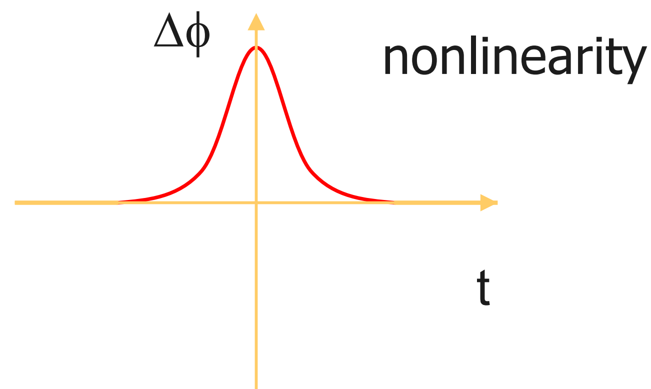
$$\frac{\partial A(z,t)}{\partial z} + i\,\frac{\beta^{(2)}}{2}\,\frac{\partial^2 A(z,t)}{\partial t^2} = i\gamma\,|A(z,t)|^2\,A(z,t)$$

- Wave usually decays

Soliton formation

$$\frac{\partial A(z,t)}{\partial z} + i\,\frac{\beta^{(2)}}{2}\,\frac{\partial^2 A(z,t)}{\partial t^2} = i\gamma\,|A(z,t)|^2\,A(z,t)$$



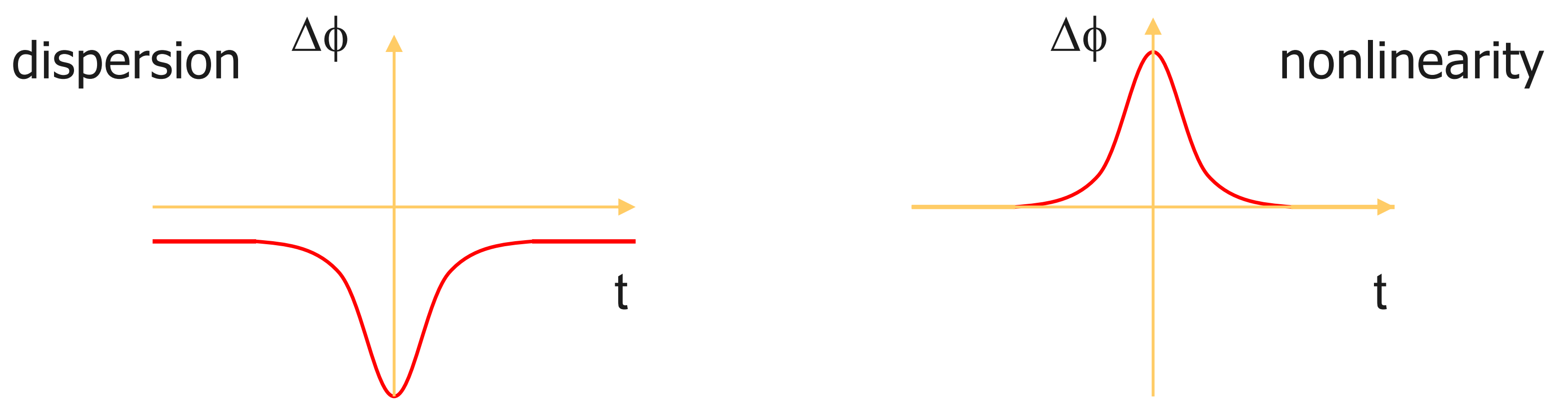


(anomalous) dispersion cancels nonlinearity for

$$A(t) = A_0\,\mathrm{sech}(t/\tau_p)\exp(iz/z_{sol})$$

Soliton formation

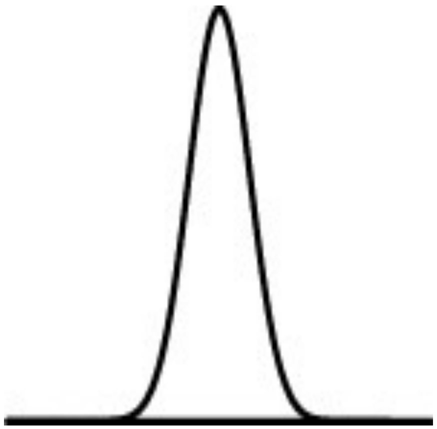
$$\frac{\partial A(z,t)}{\partial z} + i\frac{\beta^{(2)}}{2}\frac{\partial^2 A(z,t)}{\partial t^2} = i\gamma\,|A(z,t)|^2\,A(z,t)$$



(anomalous) dispersion cancels nonlinearity for

$$A_0\tau_p = \sqrt{\frac{\beta_2}{\gamma}}$$

Soliton formation



Why are solitons so important?

- Localized wave packets that are stable

- Eigenmodes in linear systems ⇔ solitons in nonlinear systems

In 1D soliton dynamics help us understand

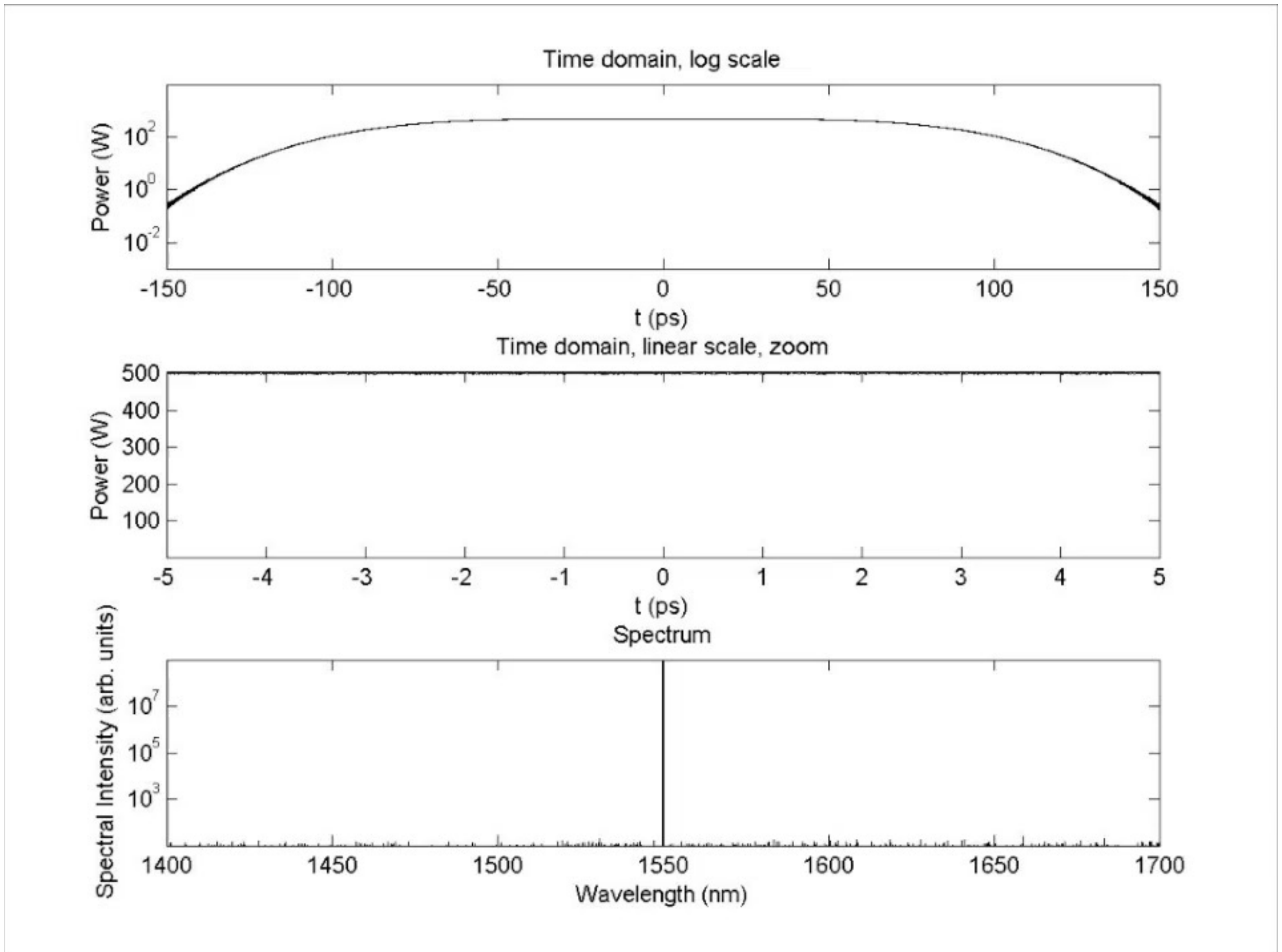
- modulation instability
- modelocked lasers
- continuum generation
- breathers, Peregrine soliton
- rogue waves
- …

2D and 3D: solitons are unstable

What will happen in multimode fiber ??

Modulation instability (temporal)

- Anomalous dispersion: a continuous wave breaks into temporal components



Modulation instability (temporal)

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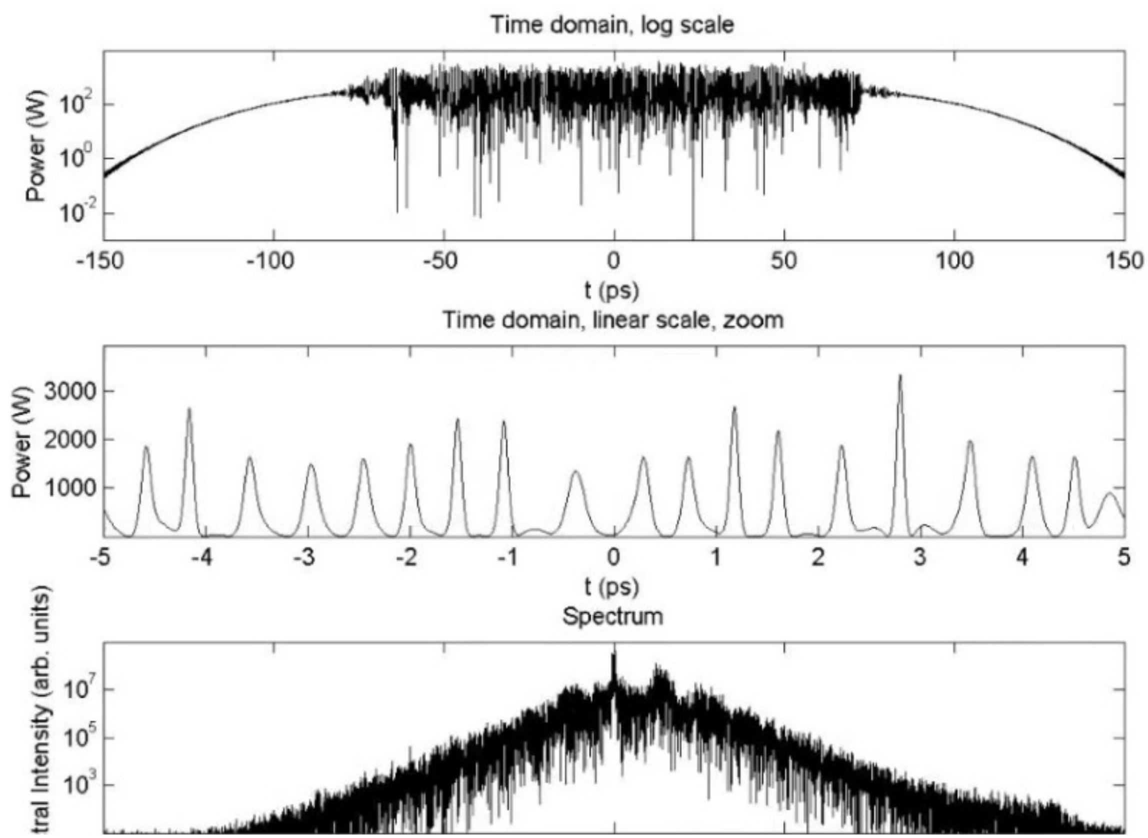
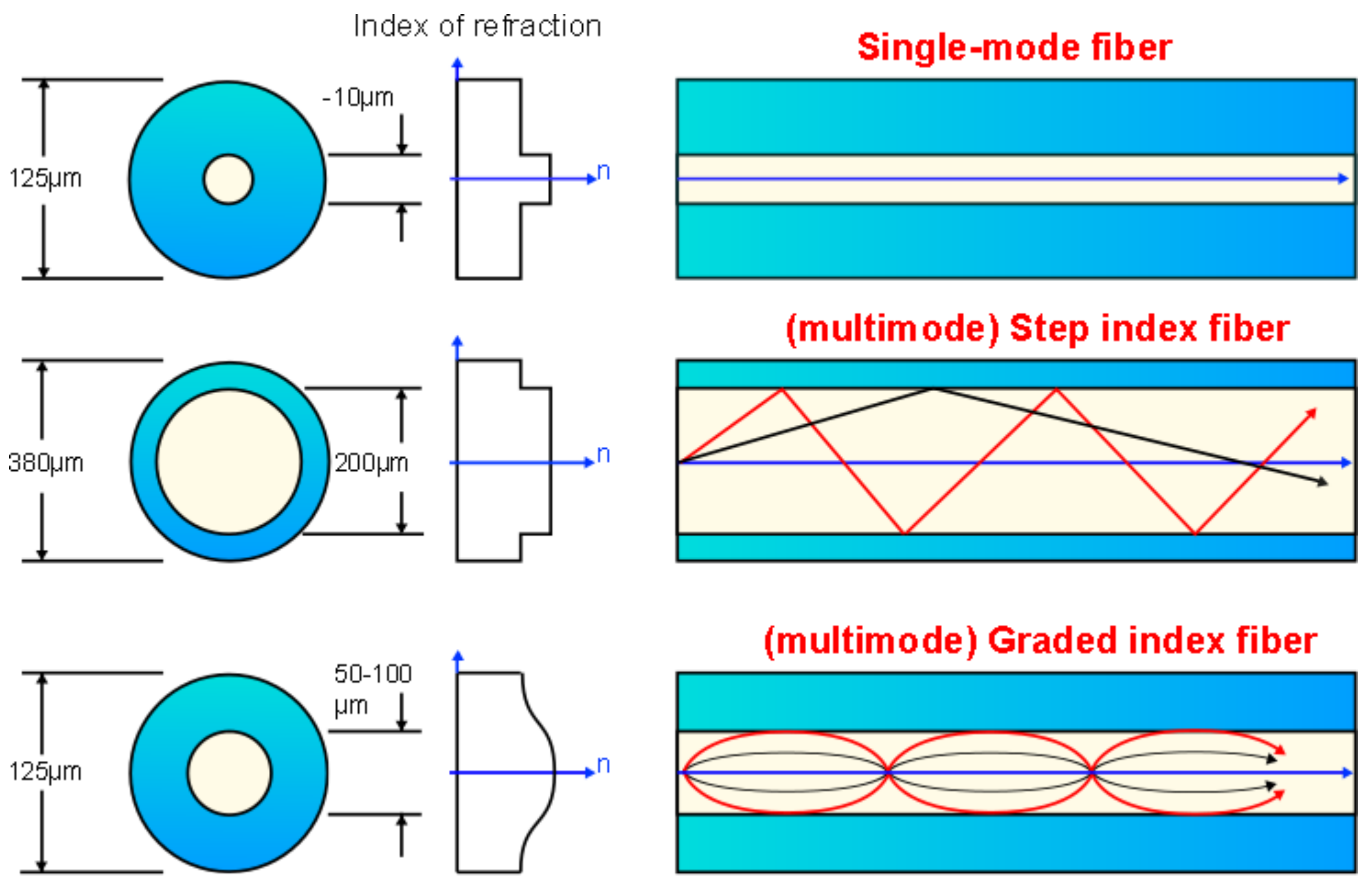
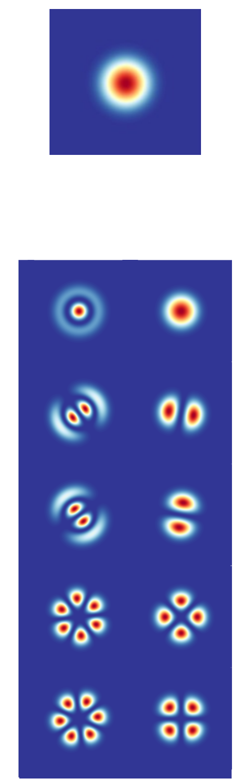


$A_0 \tau_p \sim$ constant

Multimode waveguides: between 1D and 3D





https://commons.wikimedia.org/wiki/File:Optical_fiber_types.svg

Why study propagation in multimode fiber now?

- Little work on *nonlinear* multimode pulse propagation before 2013

Why study propagation in multimode fiber now?

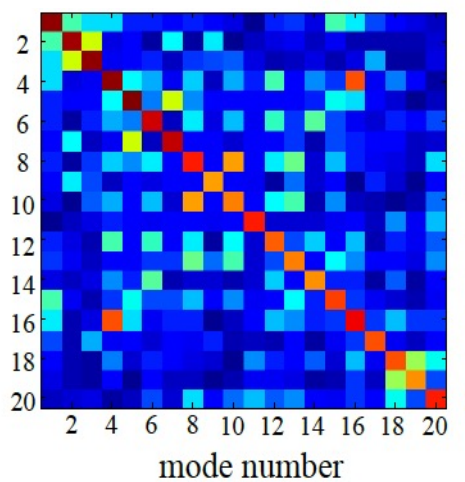
- Little work on *nonlinear* multimode pulse propagation before 2013

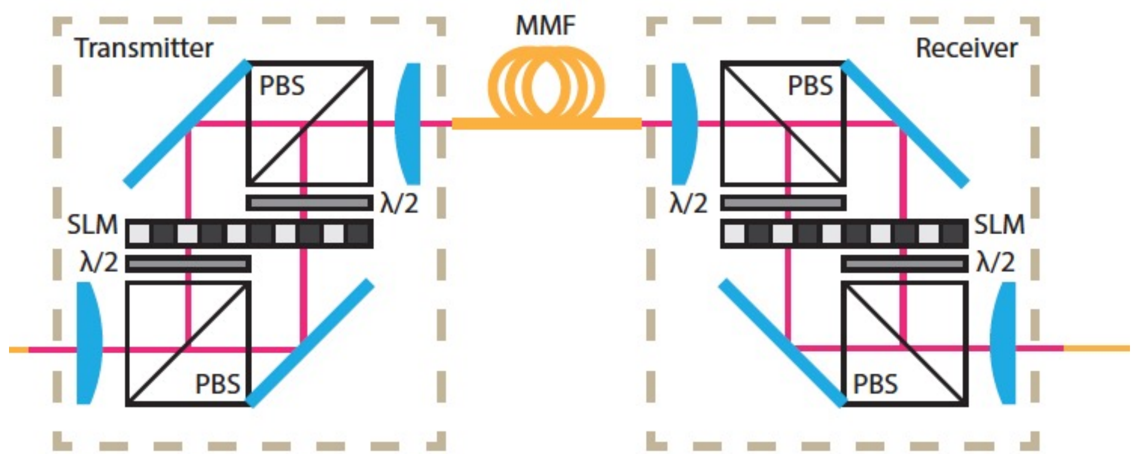
- Problem has
  Dispersion
  Linear and nonlinear mode coupling
  Disorder
  Dissipation

  With M modes there are M dispersion curves
  M^2 cross-phase modulation terms
  M^4 4-wave mixing interactions

Why study propagation in multimode fiber now?

- Recent theoretical, computational, experimental advances
  e.g., transfer matrix, principal modes, mode-resolved measurements,…





Carpenter et al.

Why study propagation in multimode fiber now?

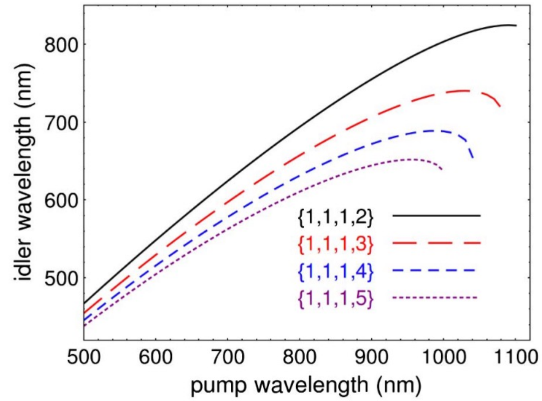
- Recent theoretical, computational, experimental advances
  e.g., transfer matrix, principal modes, mode-resolved measurements,…

- Relevance to imaging / complex media

Ploschner et al., Nature Photon 2015

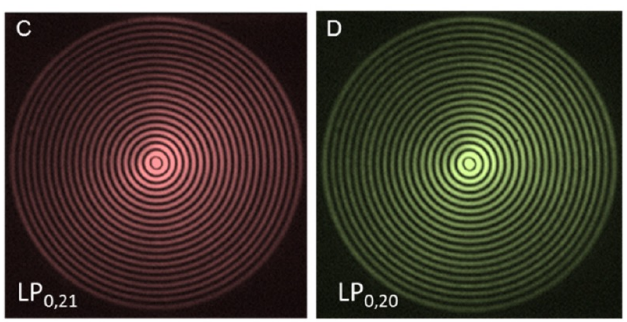
Why study propagation in multimode fiber now?

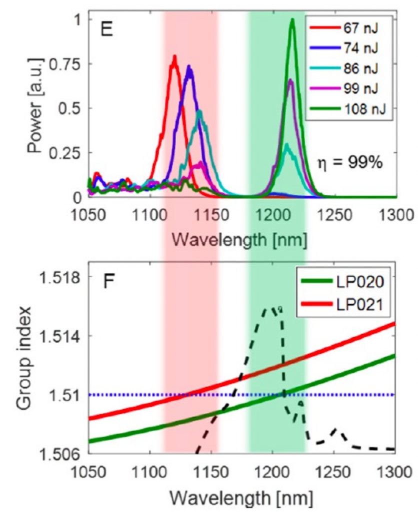
- Recent work on
  Intermodal nonlinear processes



Nazemosadat J Opt Soc Am B 2016

Propagation in higher-order modes of multimode fiber

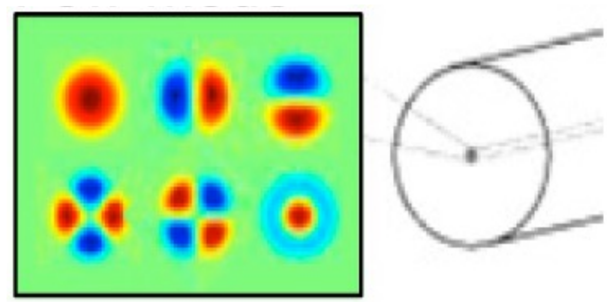




Rishøj et al., Optica 2019

Why study propagation in multimode fiber now?

- Space division multiplexing in telecom



Agrell et al., J Opt 2016

- Laser / amplifier / transmission applications

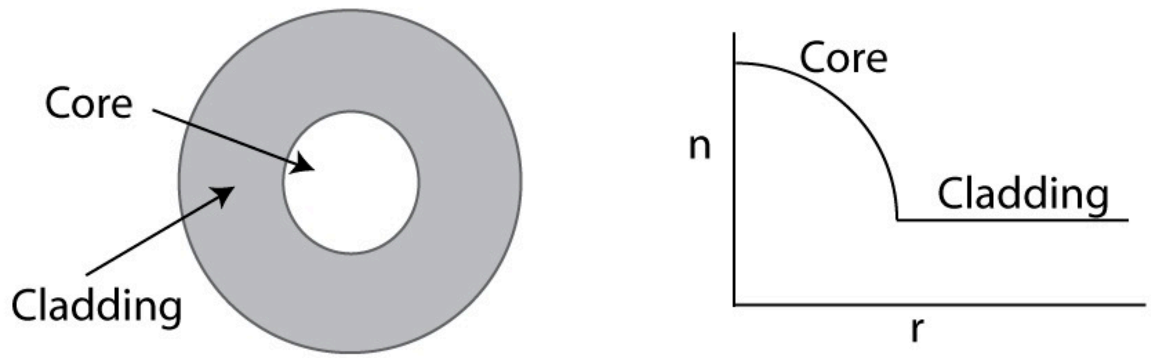
Impact of GRIN fiber

- Nonlinear optics always requires wave vector (phase) matching
- Short-pulse NLO requires matching of group velocities
- GRIN fiber has very small modal dispersion compared to step-index fibers

Ultrashort pulses in different modes interact strongly

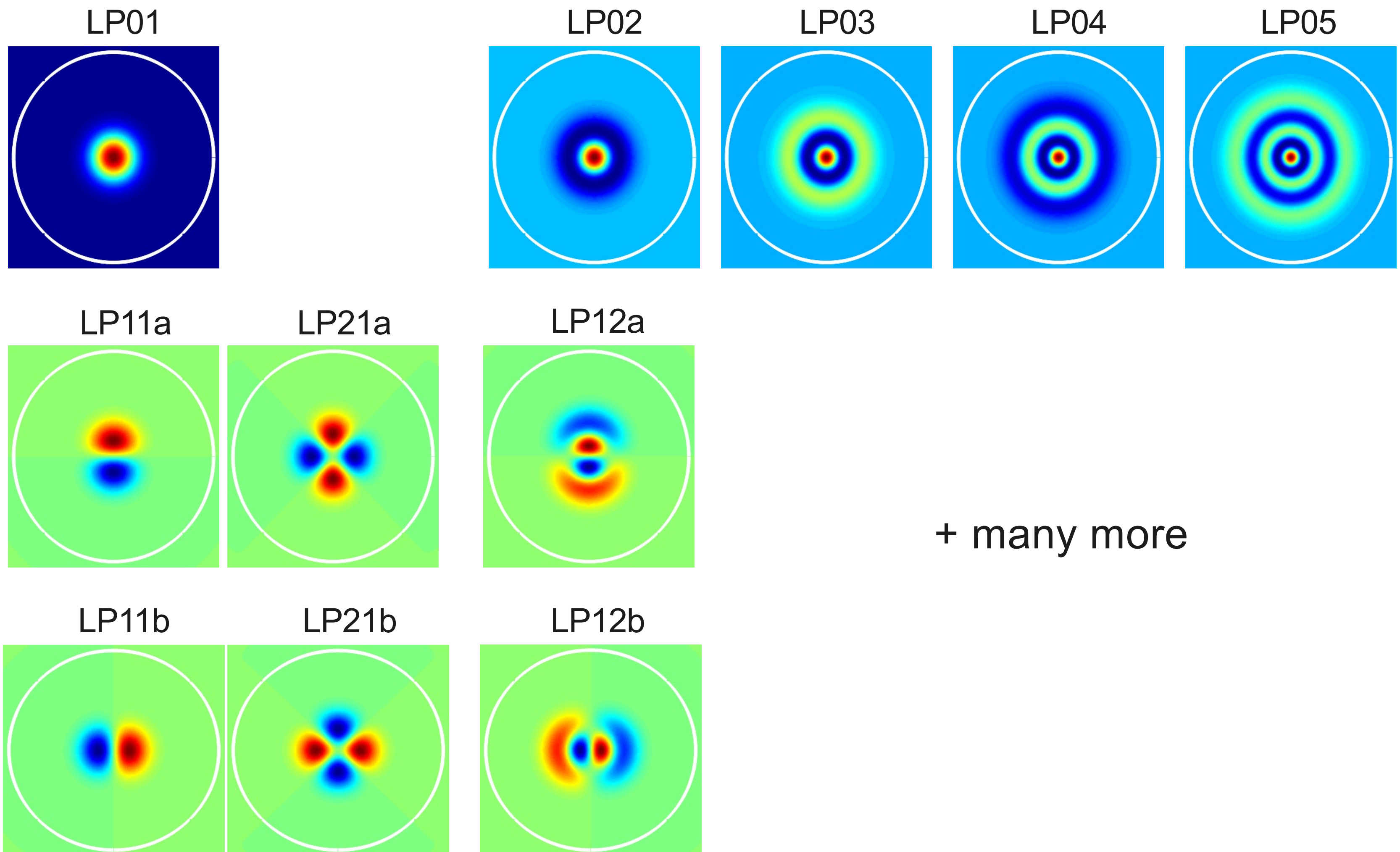
Result is quasi-3D pulse propagation

Graded-index (GRIN) multimode fiber



$$n^2(\rho) = n_0^2\left[1 - 2\Delta\left(\frac{\rho}{R}\right)^{\alpha}\right], \quad \rho \le R$$
$$= n_0^2(1 - 2\Delta), \quad \rho > R$$

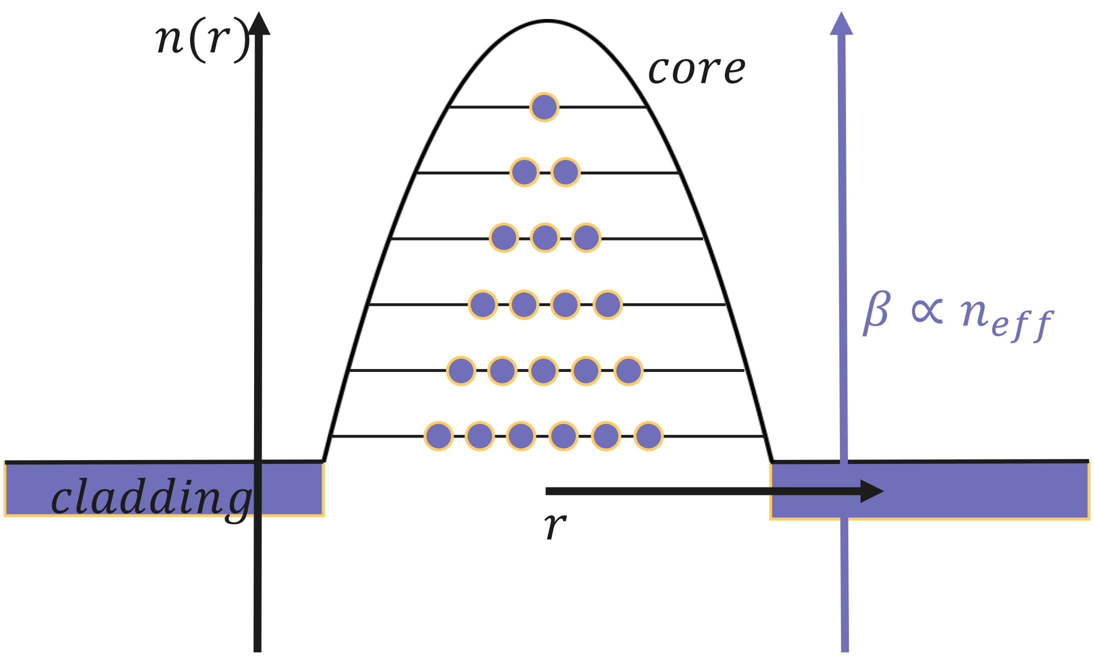
Modes of GRIN fiber

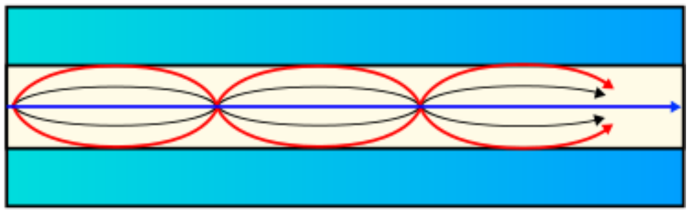


+ many more

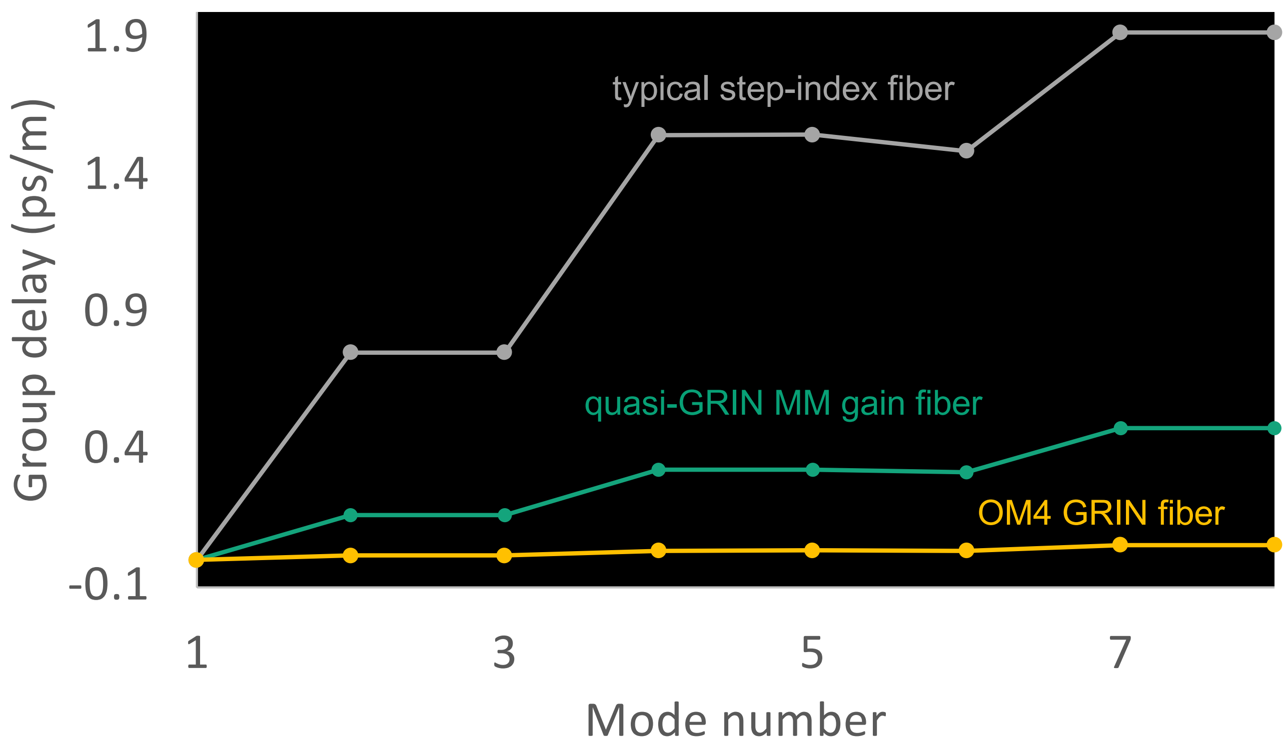
Modes of GRIN fiber

- Propagation constants equally-spaced





Multimode fibers



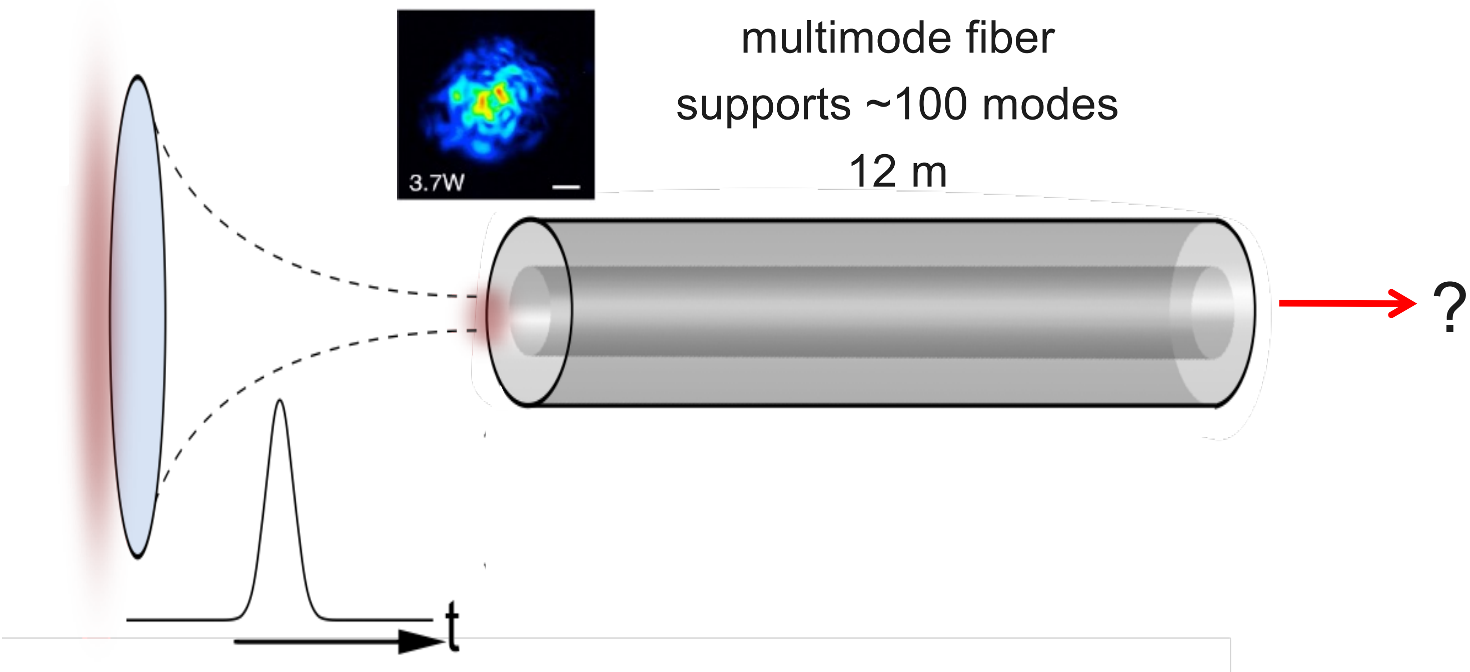
GRIN fiber

- modes have similar velocities
- allows stronger nonlinear interactions among modes

Beam Self-Cleaning

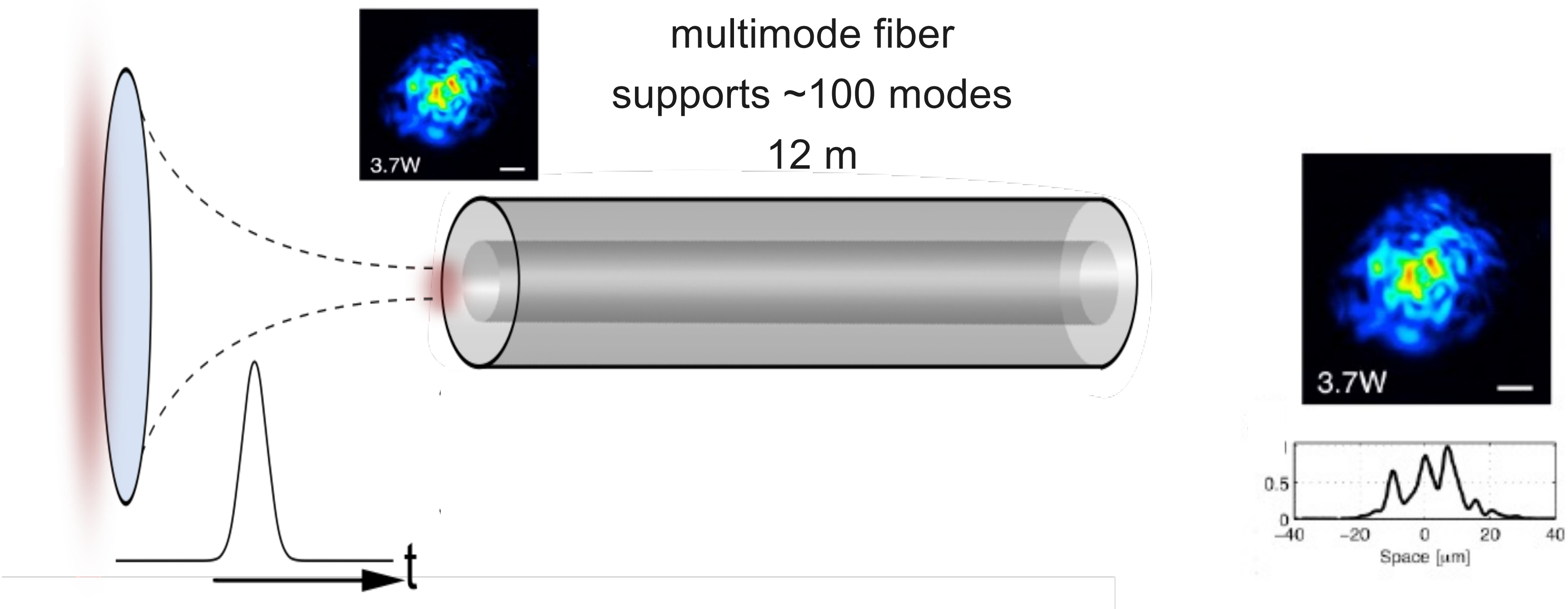
(spatial organization)

Beam self-cleaning in GRIN fiber



~1 ns
5 μJ
1064 nm

Beam self-cleaning in GRIN fiber



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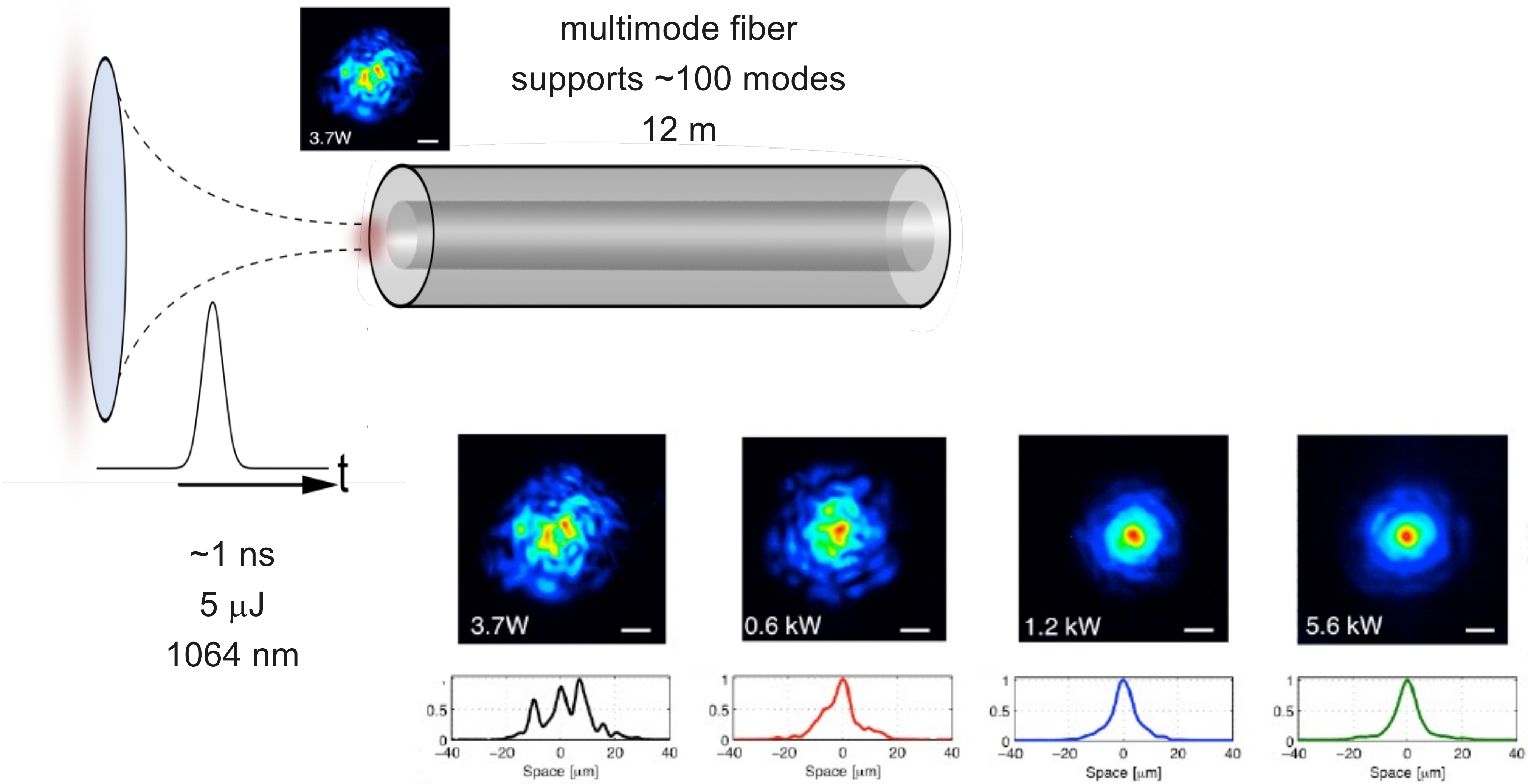
Beam self-cleaning in GRIN fiber

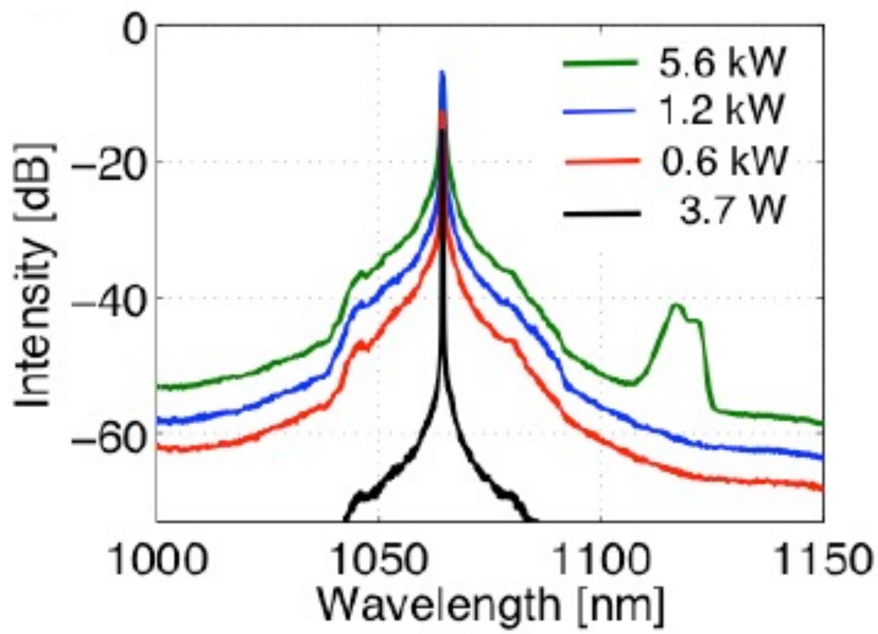


Krupa et al., Nature Photon 2017

Beam self-cleaning

- $P \ll P_{cr}$

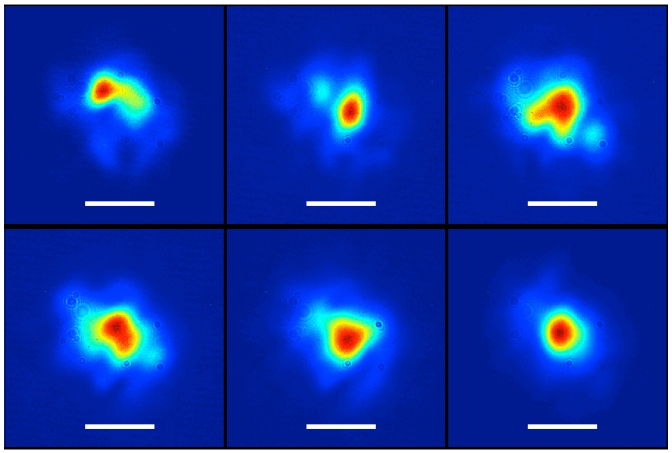
- negligible dissipation

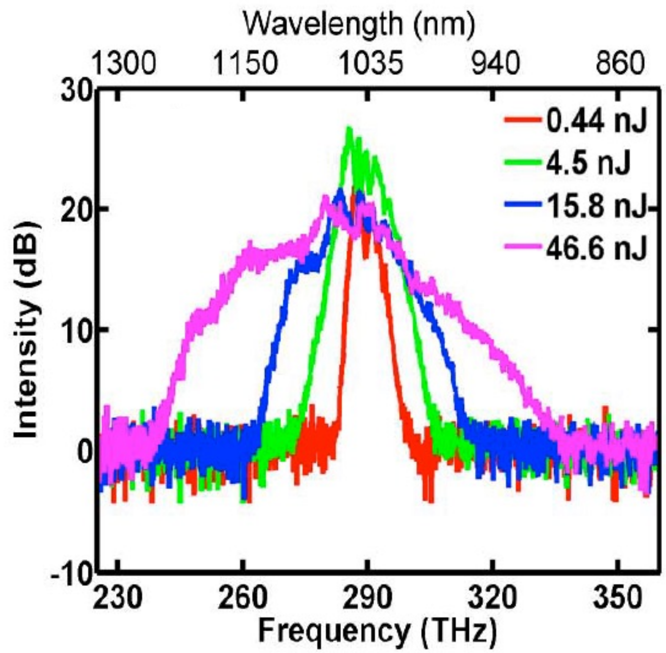


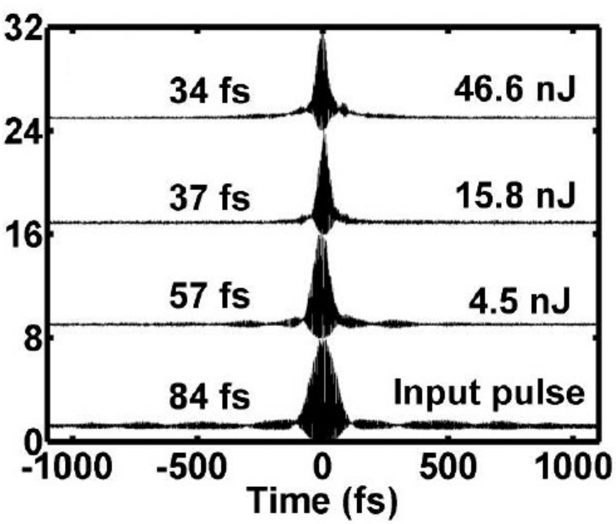
Krupa et al., Nature Photon 2017

Beam self-cleaning

- Femtosecond pulses



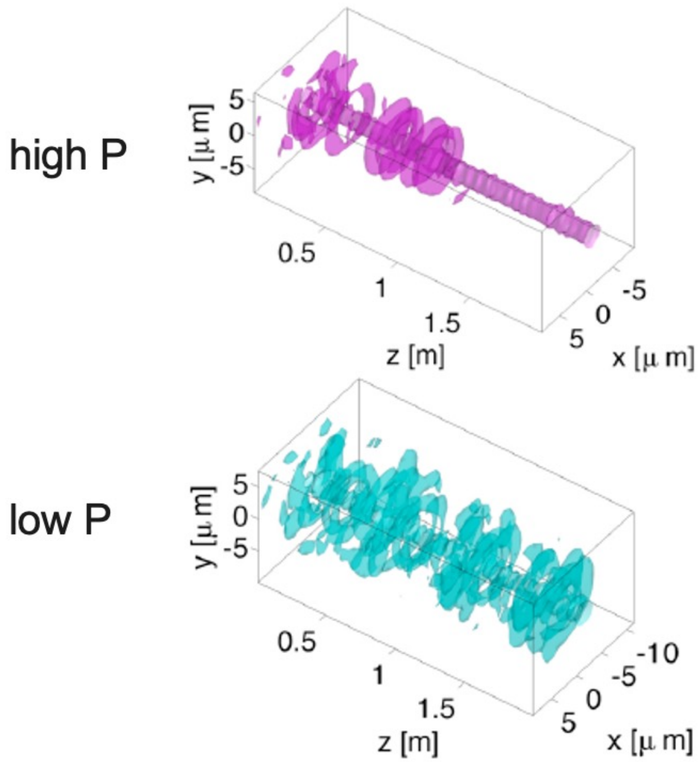




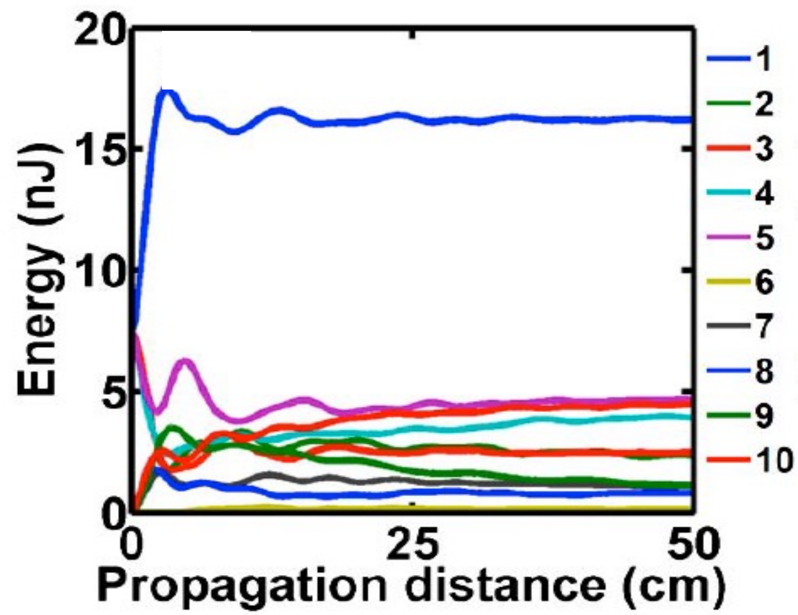
Liu et al., Opt Lett 2016

Beam self-cleaning: theory

nanosecond pulses
Krupa et al., Nature Photon 2017

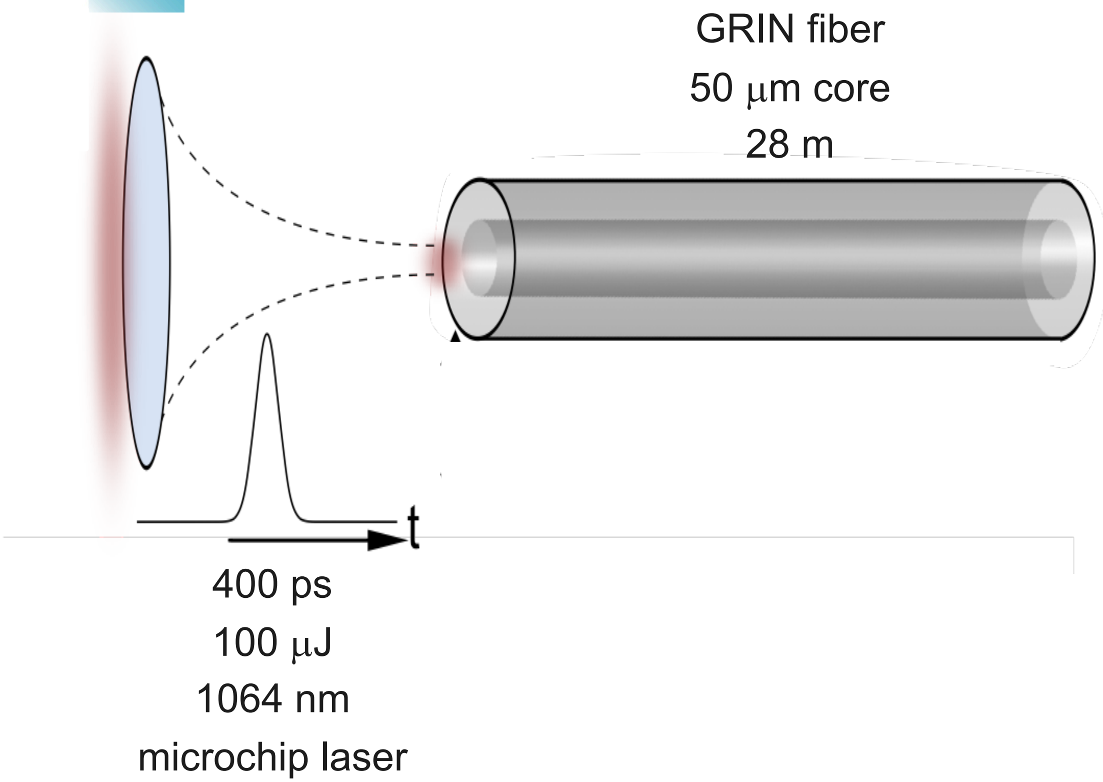


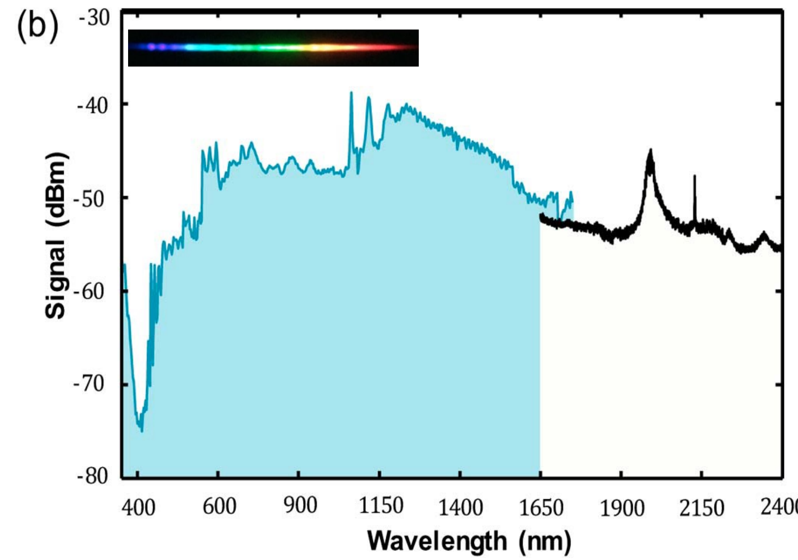
femtosecond pulses
Liu et al., Opt Lett 2016

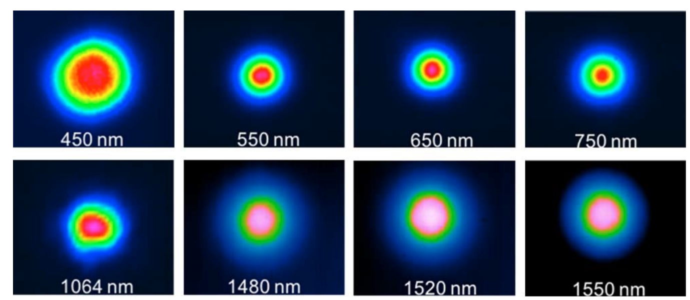


- Kerr nonlinearity underlies self-cleaning
- Why does it occur?
- Role of disorder?

Application: high-power continuum







- Speckle-free output with $M^2 \sim 2$
- 80 μJ pulse energy
- Compact, bright, multi-octave continuum

Lopez-Galmiche et al., Opt Lett 2016

How to understand beam-cleaning?

PHYSICAL REVIEW LETTERS **122,** 103902 (2019)

Hydrodynamic 2D Turbulence and Spatial Beam Condensation in Multimode Optical Fibers

E. V. Podivilov,[1,2] D. S. Kharenko,[1,2] V. A. Gonta,[1] K. Krupa,[3] O. S. Sidelnikov,[1,4] S. Turitsyn,[1,5] M. P. Fedoruk,[1,4] S. A. Babin,[1,2] and S. Wabnitz[1,6]

Optics Letters

Spatial beam cleanup by pure Kerr processes in multimode fibers

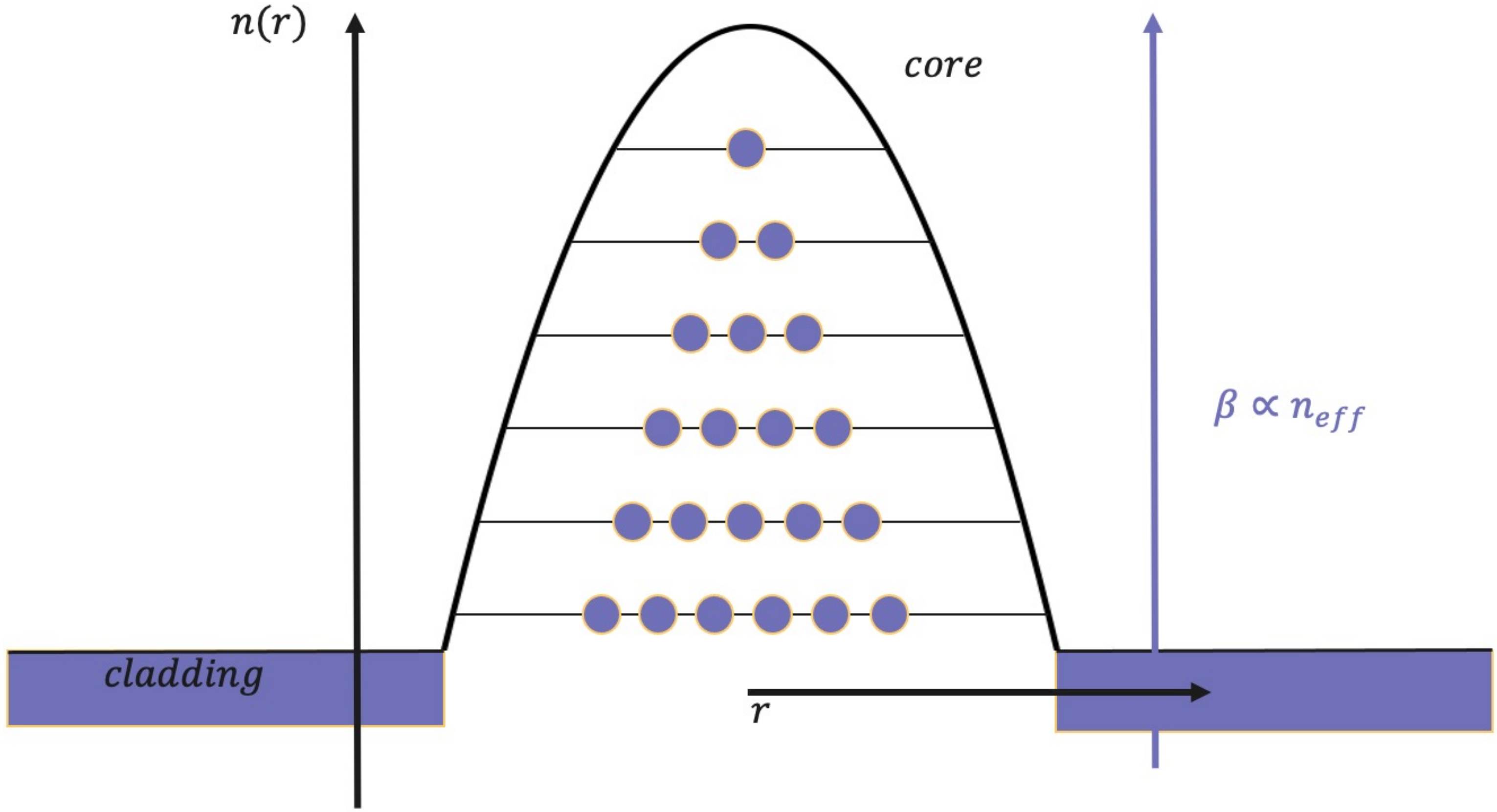
JESPER LÆGSGAARD
DTU Fotonik, Department of Photonics Engineering, Technical University of Denmark, Ørsteds Plads, 2800 Kongens Lyngby, Denmark (jlag@fotonik.dtu.dk)

PHYSICAL REVIEW LETTERS **122,** 123902 (2019)

Dramatic Acceleration of Wave Condensation Mediated by Disorder in Multimode Fibers

Adrien Fusaro,[1] Josselin Garnier,[2] Katarzyna Krupa,[3,1] Guy Millot,[1] and Antonio Picozzi[1]

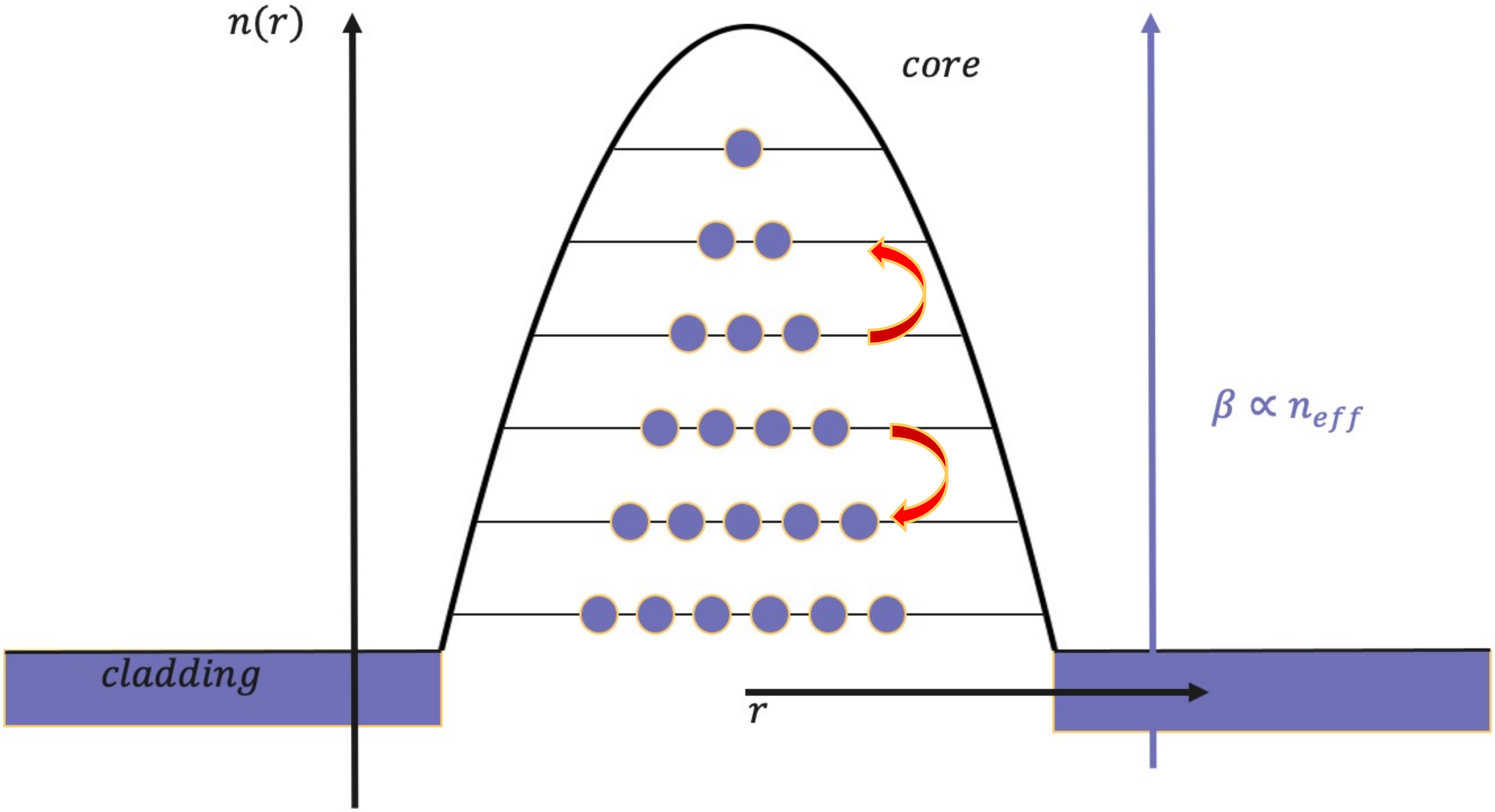
Nonlinear mode coupling



$$\frac{\partial A_p}{\partial z} \propto \sum_{(l,m,n)} f_{plmn} A_l A_m A_n^* \, e^{i(\beta_l+\beta_m-\beta_n-\beta_p)z}$$

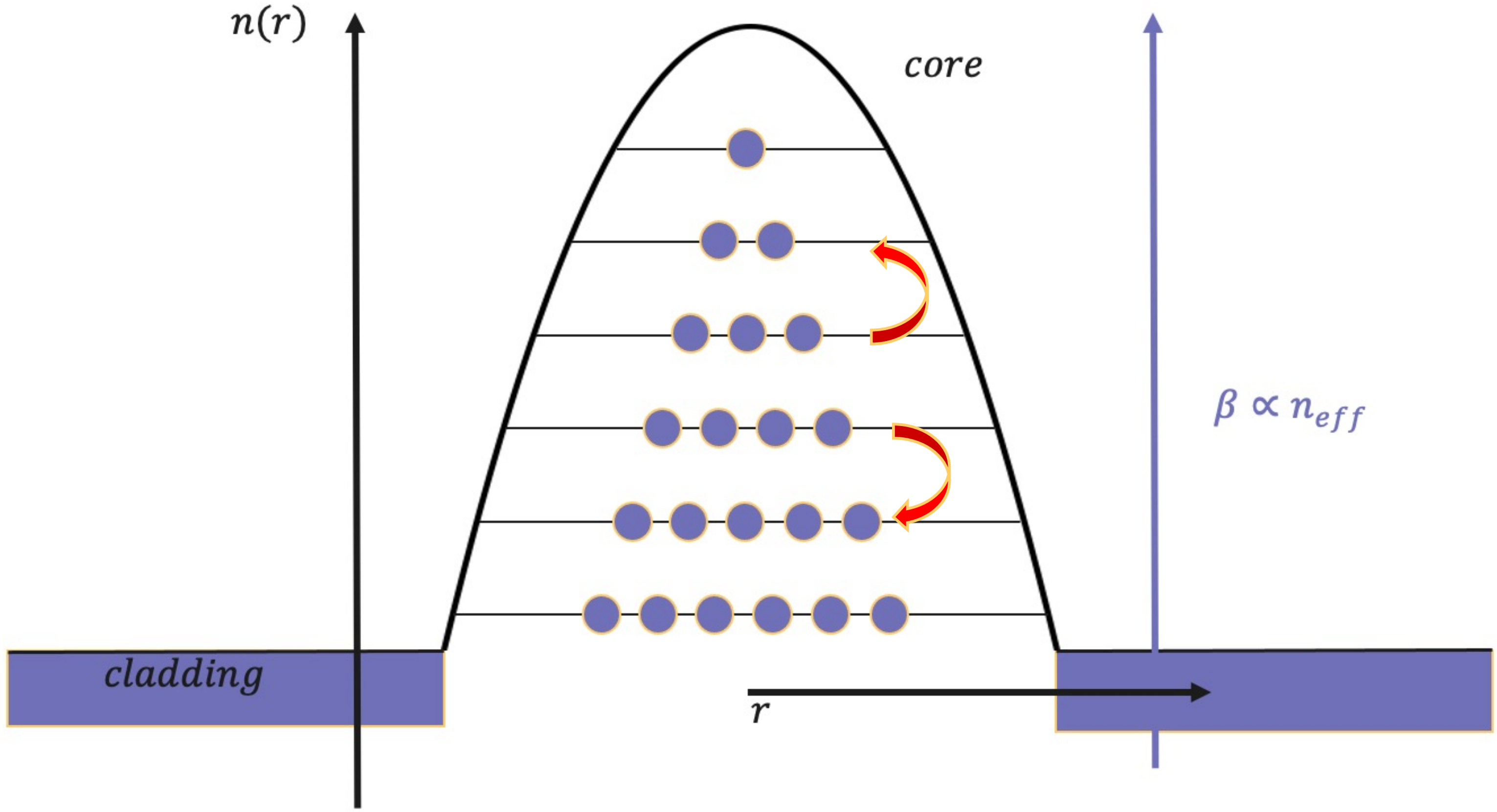
$$\Delta\beta = \beta_l + \beta_m - \beta_n - \beta_p$$

Nonlinear mode coupling



$$\Delta\beta = \beta_l + \beta_m - \beta_n - \beta_p$$

Nonlinear mode coupling



- 4WM phase-matched in GRIN fiber
- Power can transfer to lower and higher modes
- Transfer nonreciprocal owing to nonlinear phase

Optical thermodynamics

With M modes there are M dispersion curves

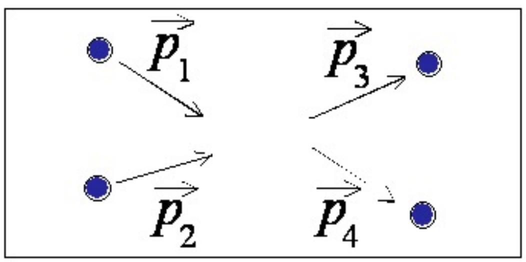
M^2 cross-phase modulation terms

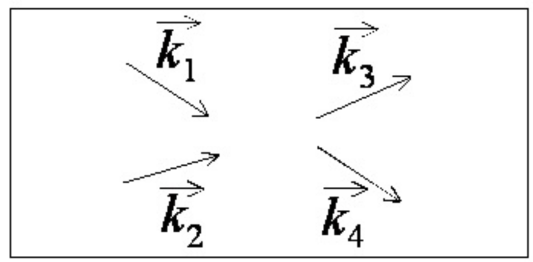
M^4 4-wave mixing interactions

$$
\begin{aligned}
\frac{\partial \tilde{A}_1(z,\Omega)}{\partial z} &= i\left[\tilde{\beta}^{(1)}(\omega) + i\alpha^{(1)}(\omega)/2\right]\tilde{A}_1(z,\Omega) + i\frac{n_2\omega_0}{c}(1+\frac{\Omega}{\omega_0})F(2(Q^{(1)}_{1111}R*(A_1A_1^*)+\cdots \\
&+Q^{(1)}_{1144}R*(A_4A_4^*))A_1 + 2(Q^{(1)}_{1313}R*(A_1A_3^*)+Q^{(1)}_{1331}R*(A_3A_1^*))A_3 + (1-f_R)(Q^{(2)}_{1111}A_1A_1+\cdots \\
&+Q^{(2)}_{1144}A_4A_4)A_1^* + (1-f_R)(Q^{(2)}_{1313}A_1A_3 + Q^{(2)}_{1331}A_3A_1)A_3^*) \\
\frac{\partial \tilde{A}_2(z,\Omega)}{\partial z} &= i\left[\tilde{\beta}^{(2)}(\omega) + i\alpha^{(2)}(\omega)/2\right](\omega)\tilde{A}_2(z,\Omega) + i\frac{n_2\omega_0}{c}(1+\frac{\Omega}{\omega_0})F(2(Q^{(1)}_{2211}R*(A_1A_1^*)+\cdots \\
&+Q^{(1)}_{2244}R*(A_4A_4^*))A_2 + 2(Q^{(1)}_{2424}R*(A_2A_4^*)+Q^{(1)}_{2442}R*(A_4A_2^*))A_4 + (1-f_R)(Q^{(2)}_{2222}A_2A_2+\cdots \\
&+Q^{(2)}_{2244}A_4A_4)A_2^* + (1-f_R)(Q^{(2)}_{2424}A_2A_4 + Q^{(2)}_{2442}A_4A_2)A_4^*) \\
\frac{\partial \tilde{A}_3(z,\Omega)}{\partial z} &= i\left[\tilde{\beta}^{(3)}(\omega) + i\alpha^{(3)}(\omega)/2\right]\tilde{A}_3(z,\Omega) + i\frac{n_2\omega_0}{c}(1+\frac{\Omega}{\omega_0})F(2(Q^{(1)}_{3311}R*(A_1A_1^*)+\cdots \\
&+Q^{(1)}_{3344}R*(A_4A_4^*))A_3 + 2(Q^{(1)}_{3131}R*(A_3A_1^*)+Q^{(1)}_{3113}R*(A_1A_3^*))A_1 + (1-f_R)(Q^{(2)}_{3333}A_3A_3+\cdots \\
&+Q^{(2)}_{3344}A_4A_4)A_3^* + (1-f_R)(Q^{(2)}_{3131}A_3A_1 + Q^{(2)}_{3113}A_1A_3)A_1^* \\
\frac{\partial \tilde{A}_4(z,\Omega)}{\partial z} &= i\left[\tilde{\beta}^{(4)}(\omega) + i\alpha^{(4)}(\omega)/2\right]\tilde{A}_4(z,\Omega) + i\frac{n_2\omega_0}{c}(1+\frac{\Omega}{\omega_0})F(2(Q^{(1)}_{4411}R*(A_1A_1^*)+\cdots \\
&+Q^{(1)}_{4444}R*(A_4A_4^*))A_4 + 2(Q^{(1)}_{4242}R*(A_4A_2^*)+Q^{(1)}_{4224}R*(A_2A_4^*))A_2 + (1-f_R)(Q^{(2)}_{4411}A_1A_1+\cdots \\
&+Q^{(2)}_{4444}A_4A_4)A_4^* + (1-f_R)(Q^{(2)}_{4242}A_4A_2 + Q^{(2)}_{4224}A_2A_4)A_2^*).
\end{aligned}
$$

Optical thermodynamics

With M modes there are M dispersion curves
M^2 cross-phase modulation terms
M^4 4-wave mixing interactions





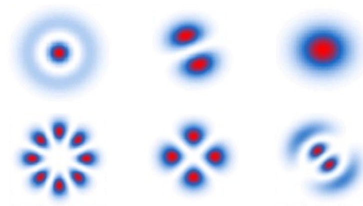
Picozzi et al., "Optical wave turbulence: Towards a unified nonequilibrium thermodynamic formulation of statistical nonlinear optics," *Phys. Rep.* 542, 1 (2014)

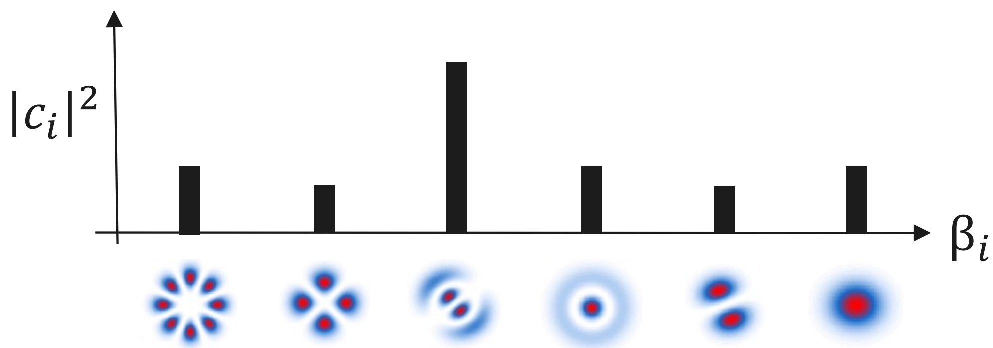
Optical thermodynamics

M total number of modes

$|c_i|^2$ mode occupancies

β_i propagation constant





Optical thermodynamics: finite # of modes

M modes

Power $P = \sum |c_i|^2$

Hamiltonian $H = \sum \beta_i \, |c_i|^2$

Internal energy $U = -H$

Entropy $S = \sum \ln |c_i|^2$

entropy maximization

⬇

equilibrium state
Rayleigh–Jeans distribution

$$|c_i|^2 = -\frac{T}{\beta_i + \mu}$$

T and μ
optical thermodynamic parameters

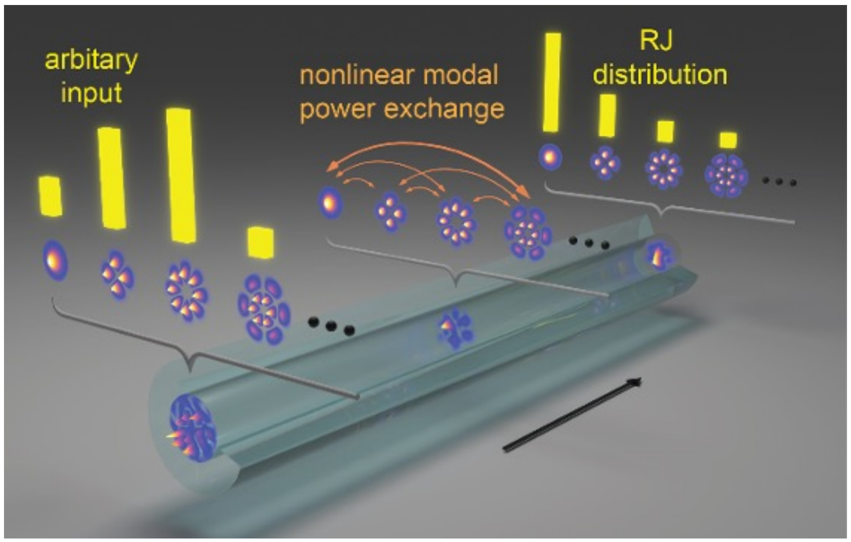
$$U - \mu P = MT$$

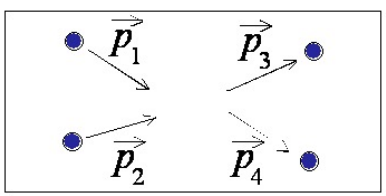
(*T has nothing to do with ordinary temperature)*

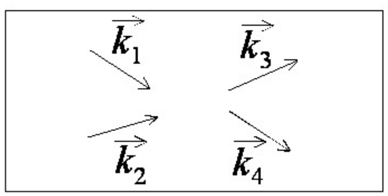
Wu and Christodoulides, *Nature Photonics* 13, 776 (2019)

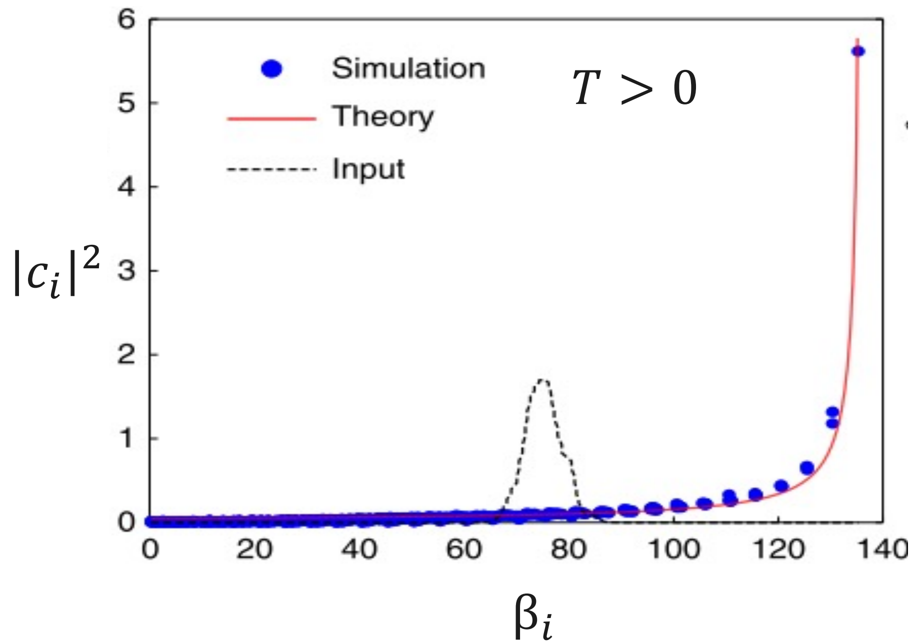
Optical thermodynamics

- Nonlinearity mediates thermalization





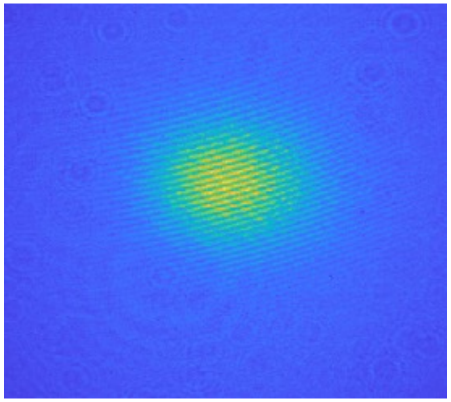




Wu and Christodoulides
Nature Photonics 13, 776 (2019)

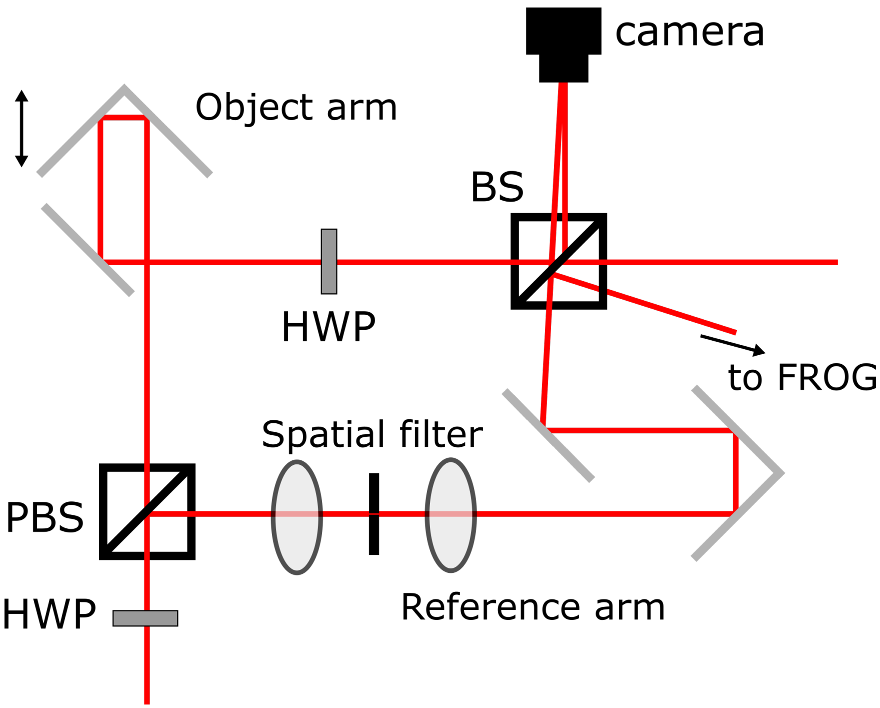
- 4WM phase-matched in GRIN fiber
- Measurement of distribution requires measurement of $\vec{E}$

3D field measurement



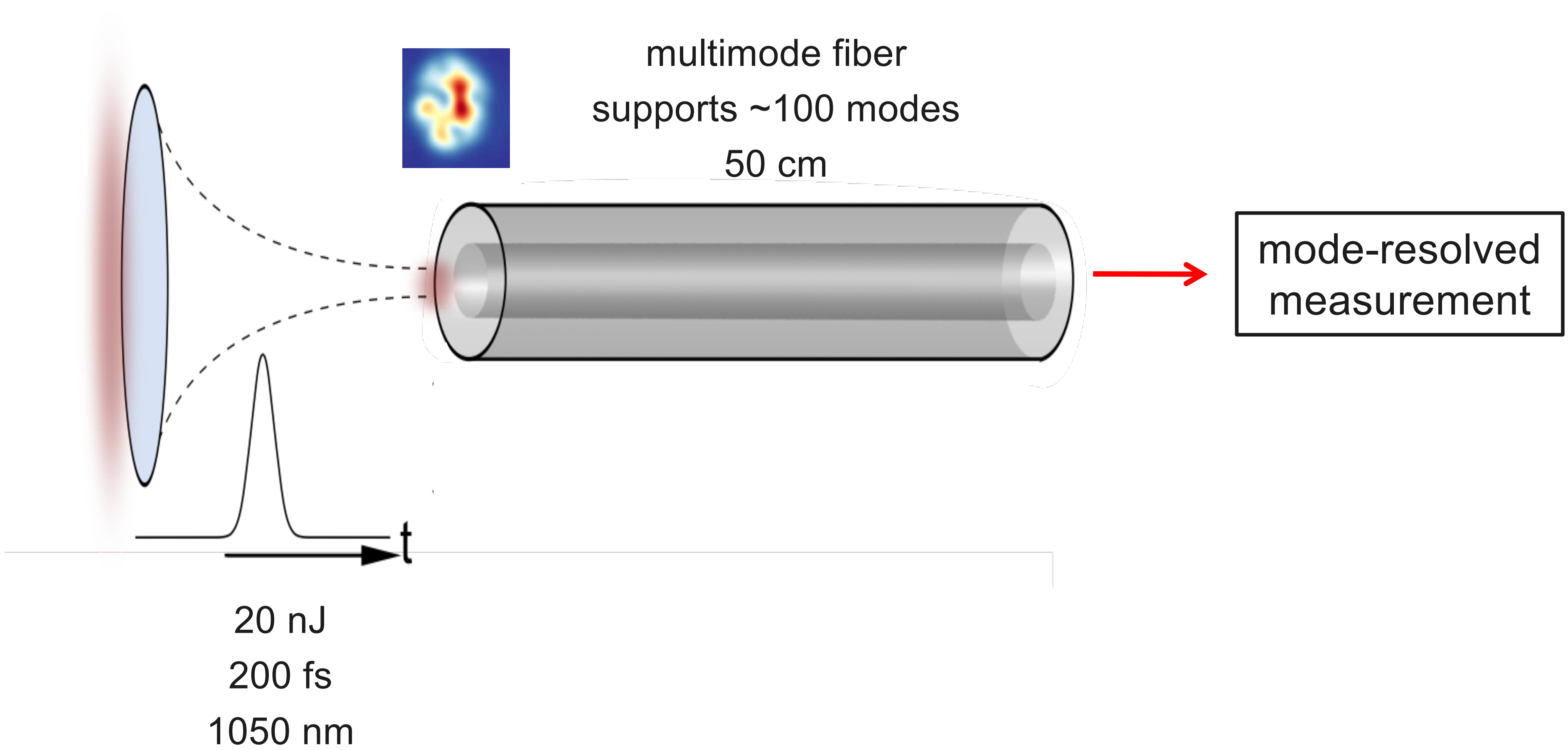
$$E(x,y) = |\mathcal{R}(x,y)|^2 + |\mathcal{O}(x,y)|^2 + \mathcal{O}^*(x,y)\mathcal{R}(x,y) + \mathcal{O}(x,y)\mathcal{R}^*(x,y)$$

,

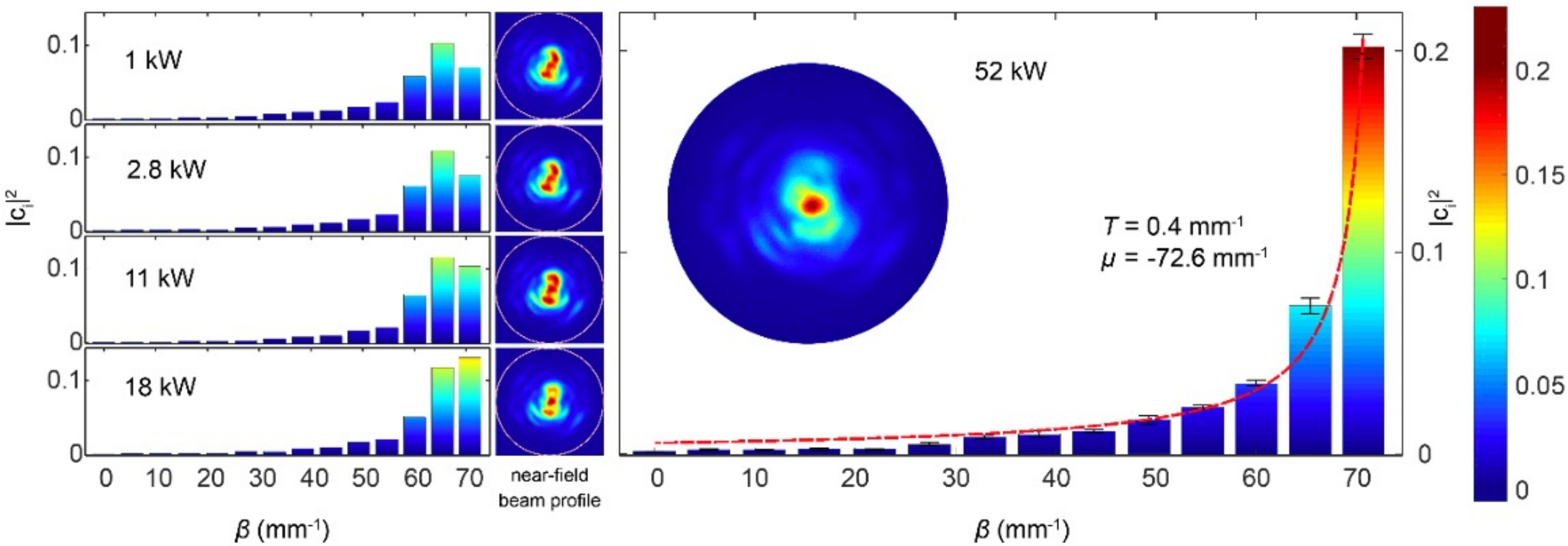


- Spatial phase from fringes
- Spectral phase from FROG
  (Pariente et al., *Nature Photon* 2016)
- Field decomposed into eigenmode basis

Thermalization experiment



Thermalization



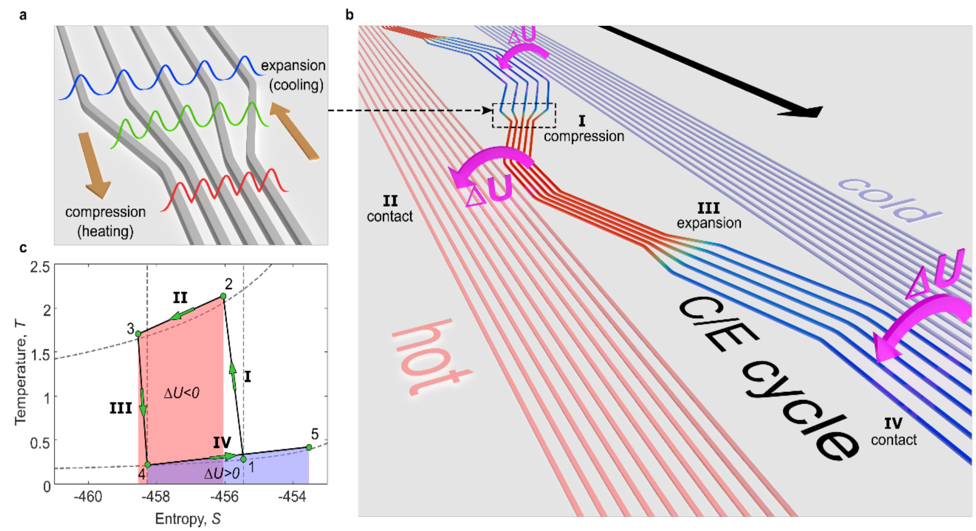
- Direct observation of Rayleigh-Jeans distribution
- Optical entropy is maximized in Kerr beam-cleaning

Pourbeyram et al., arXiv

Optical thermodynamics

Other processes

- Carnot cycles
- Optical cooling
- Isentropic processes
- …
- …



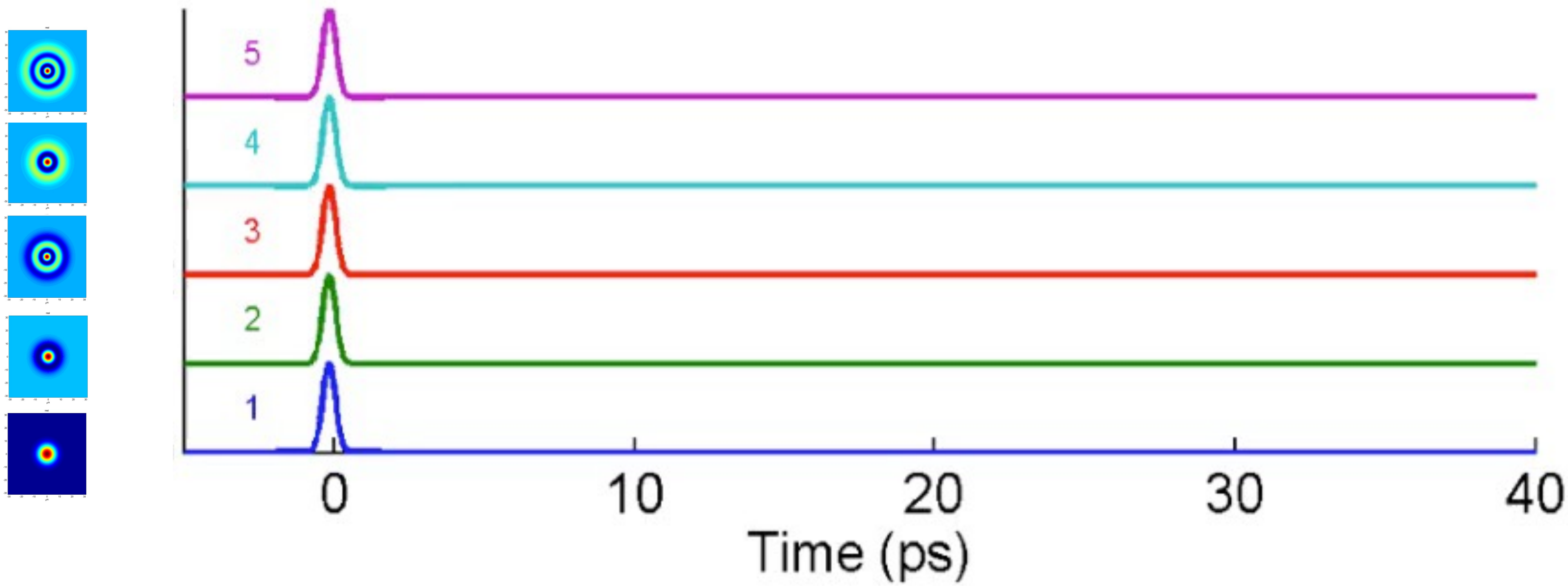
Future: design of high-performance light sources

Wu and Christodoulides, *Nature Photonics* 13, 776 (2019)

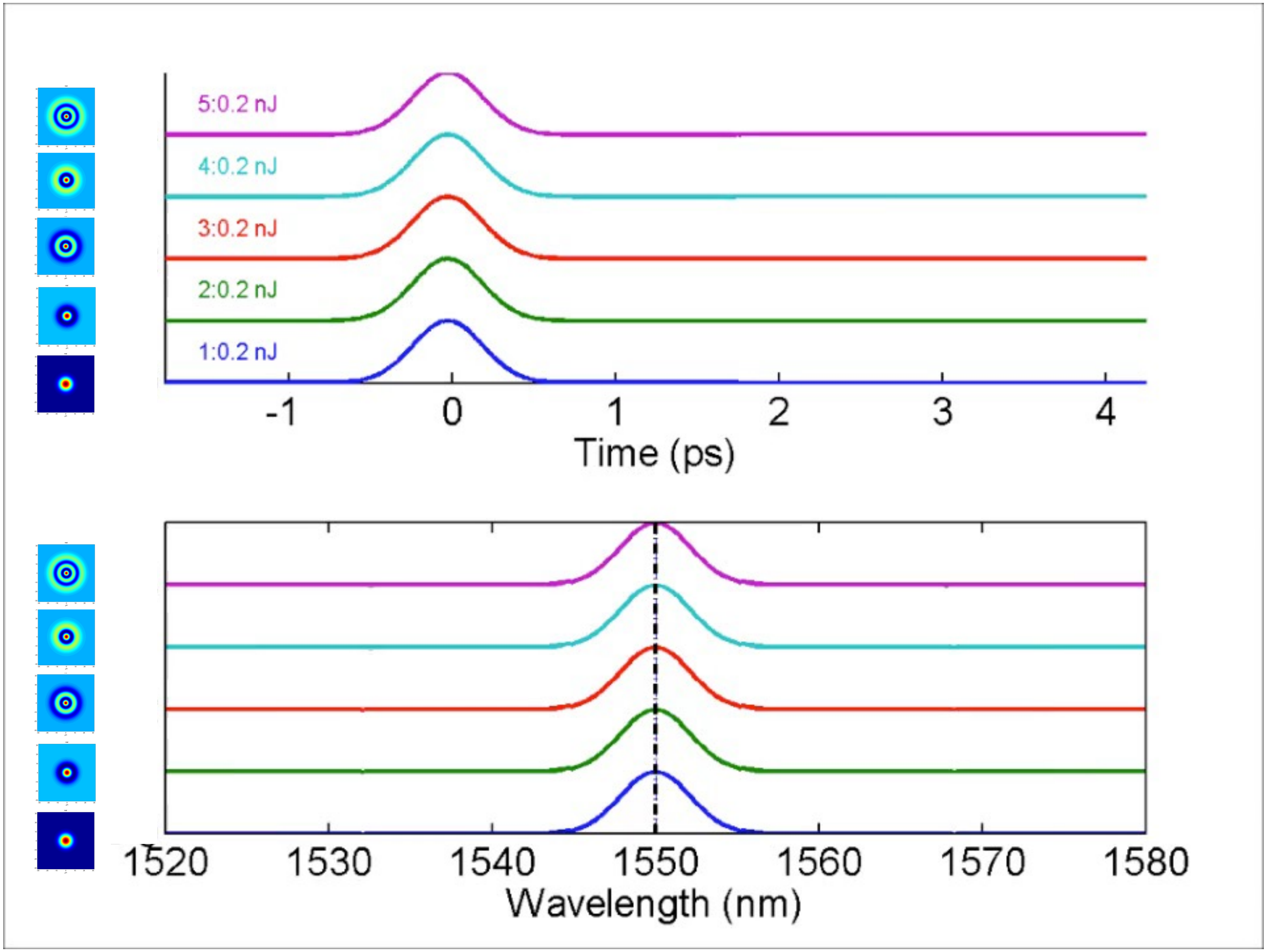
Multimode solitons

(spatiotemporal organization)

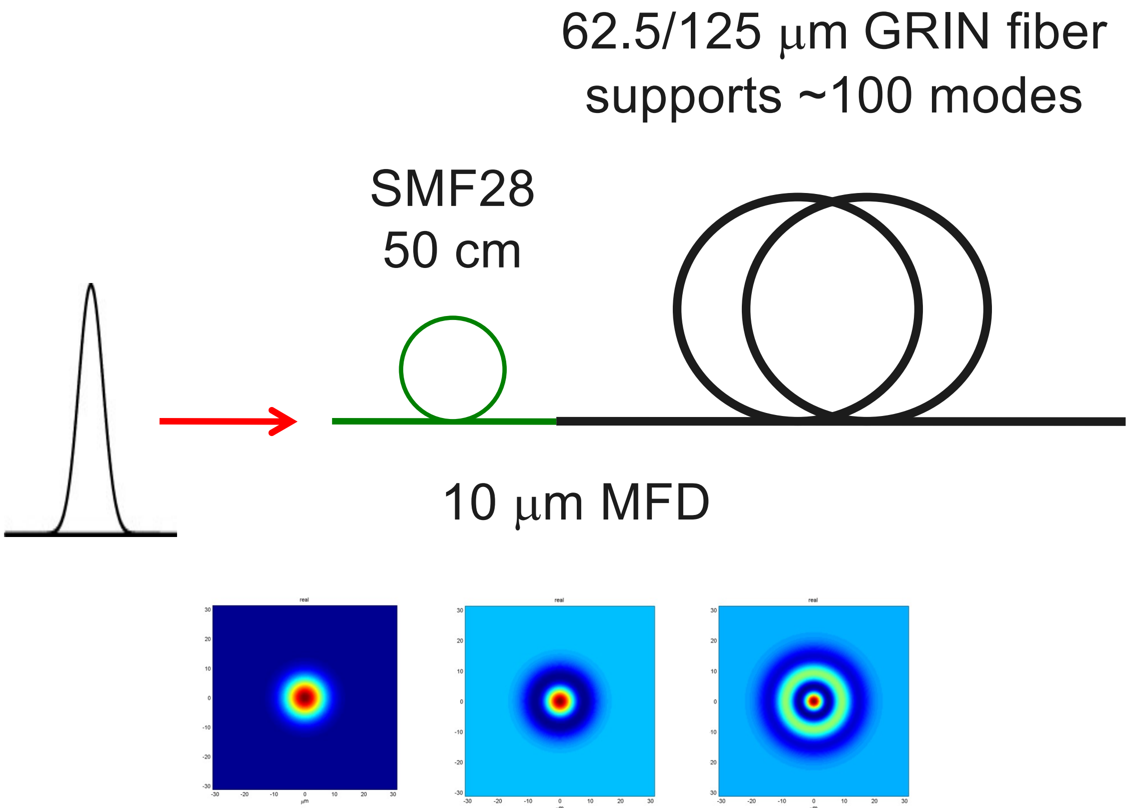
Tme domain: linear propagation



Multimode soliton formation

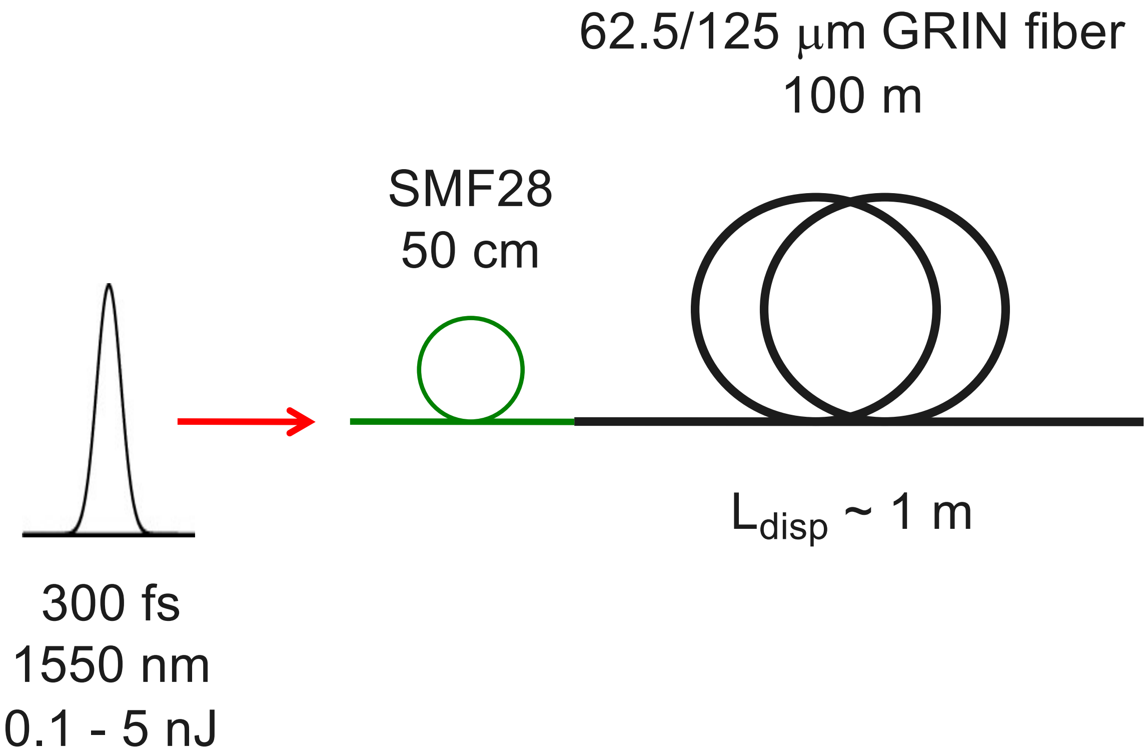


Baby steps: 3 modes



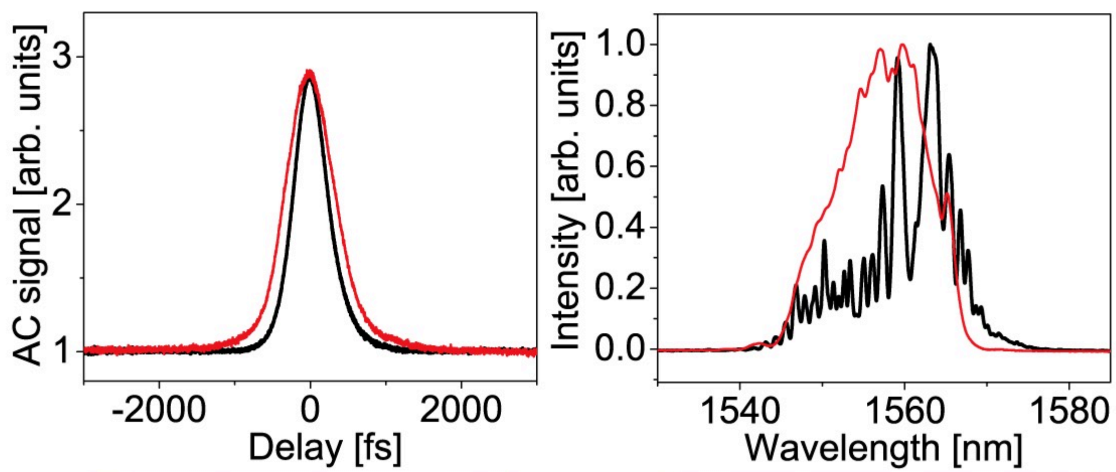
- Excite 3 lowest modes

Experiment

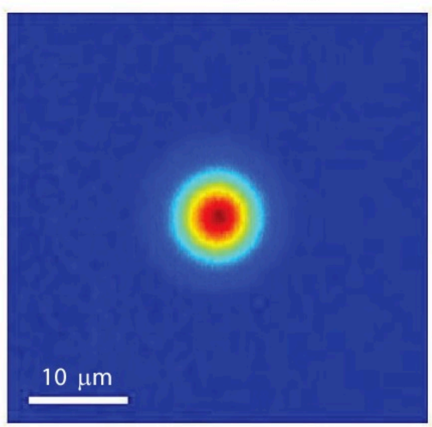

62.5/125 μm GRIN fiber
100 m
SMF28
50 cm
L_{disp} ~ 1 m
300 fs
1550 nm
0.1 - 5 nJ

Experimental results

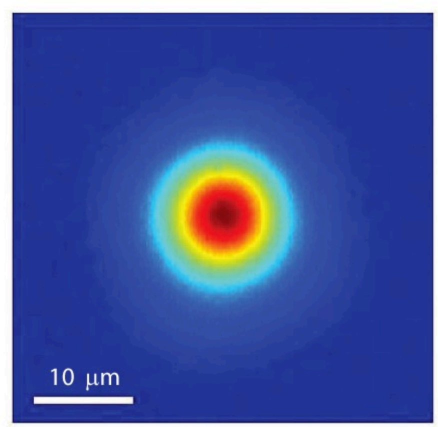
- For E < 0.1 nJ pulse disperses
- 0.5 nJ pulse energy



input
output



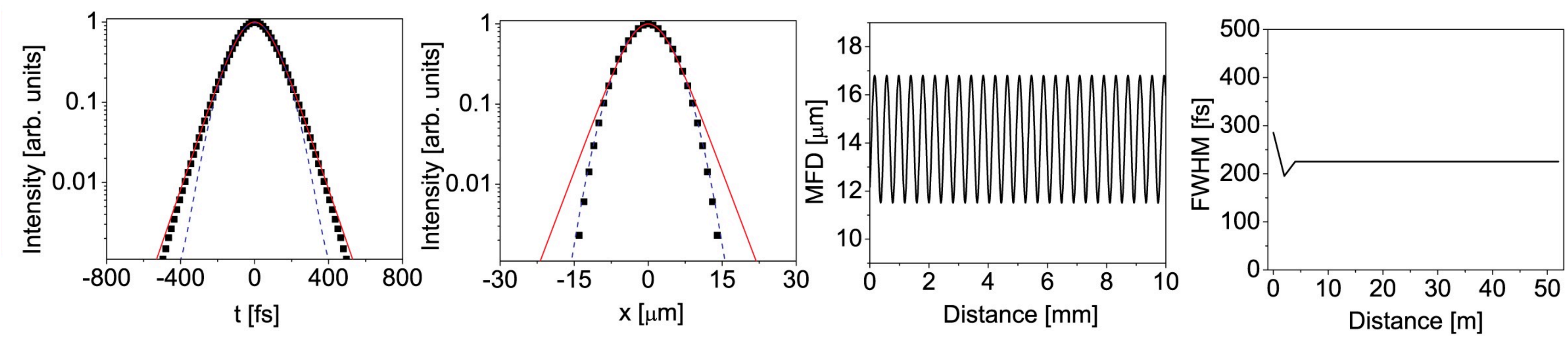
input



output

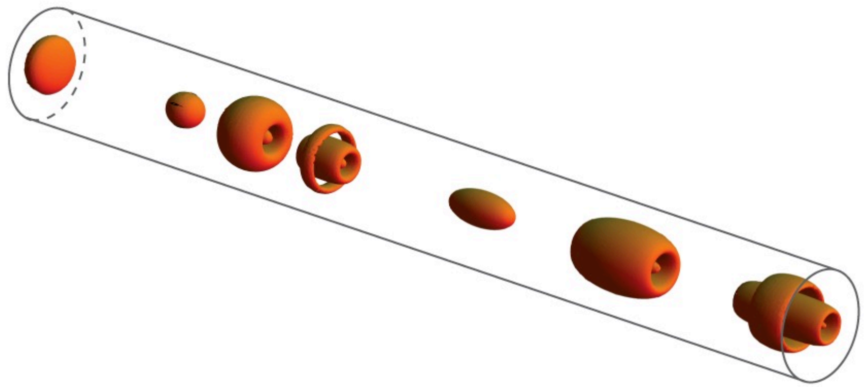
3 modes: theory

- Launch 0.5 nJ / 300 fs



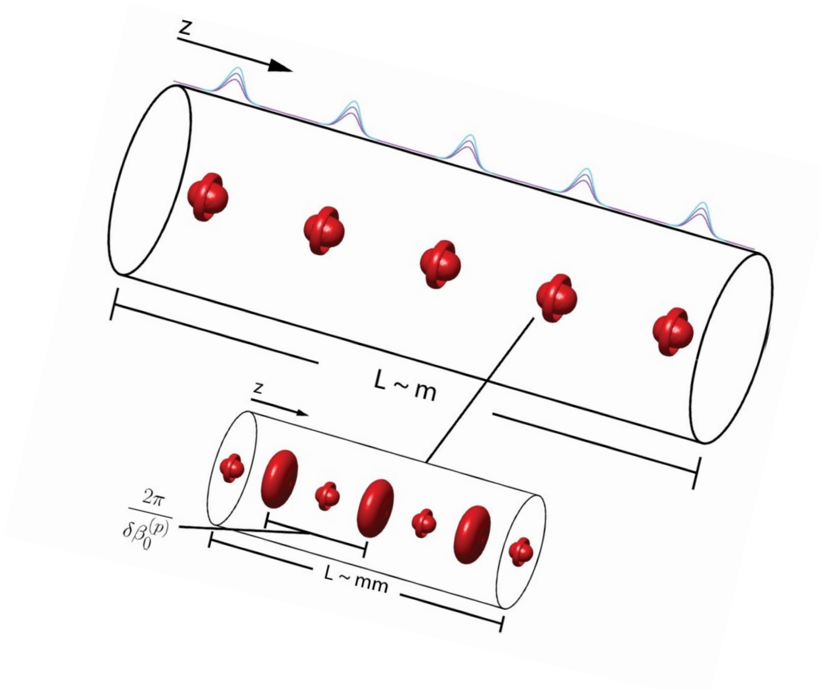
Intuitive picture

Linear propagation



Intuitive picture

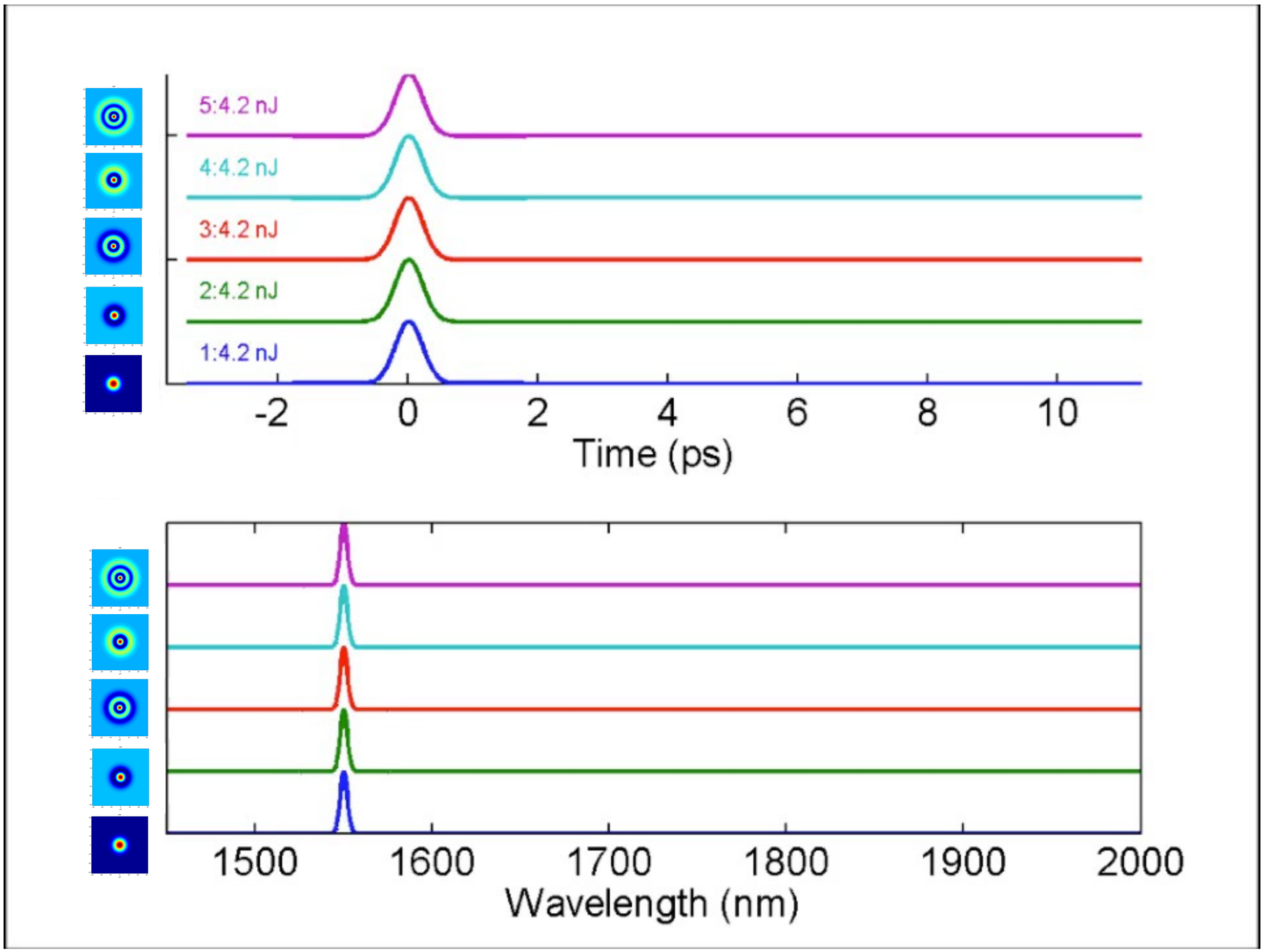
Multimode soliton



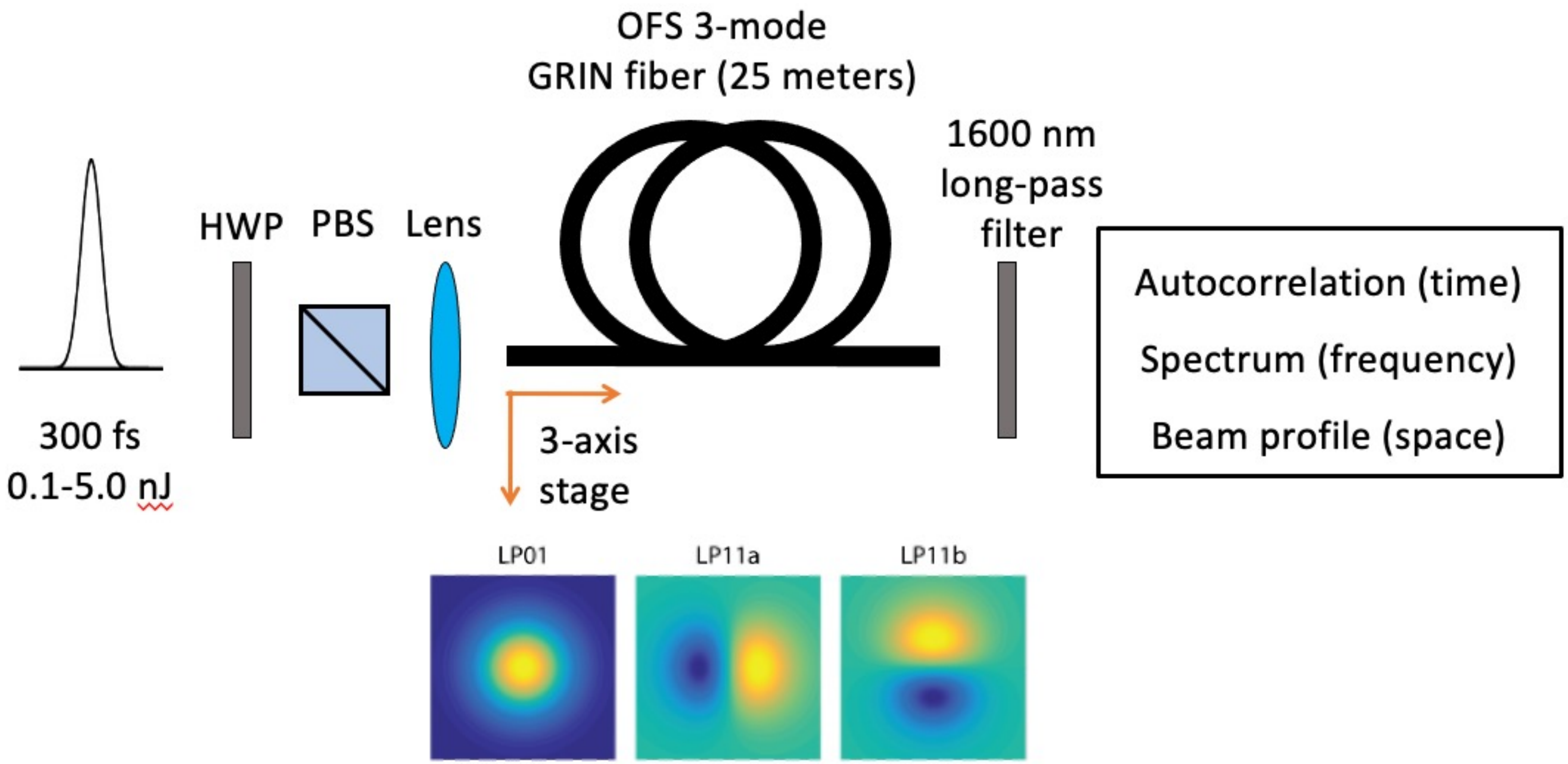
- Solitons with up to 10 modes generated
- Solitons with more modes require greater nonlinear phase / energy

Renninger et al., Nature Commun 2013
Wright et al., Opt Exp 2015

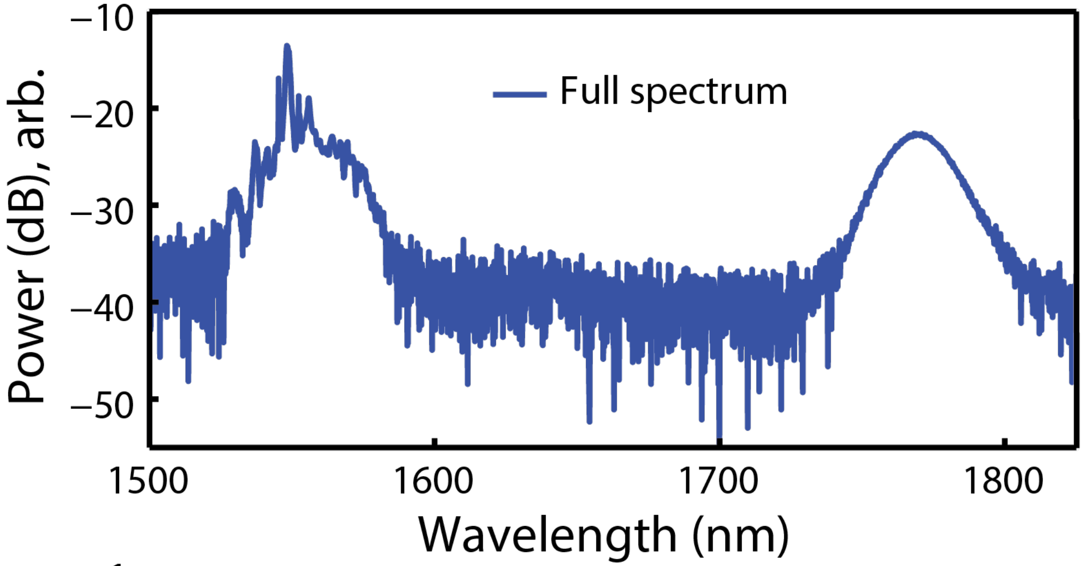
Higher energy: multimode soliton fission

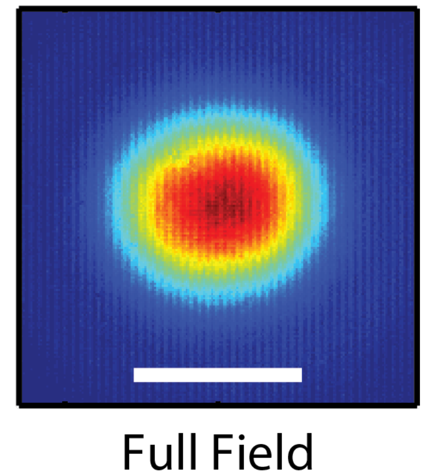


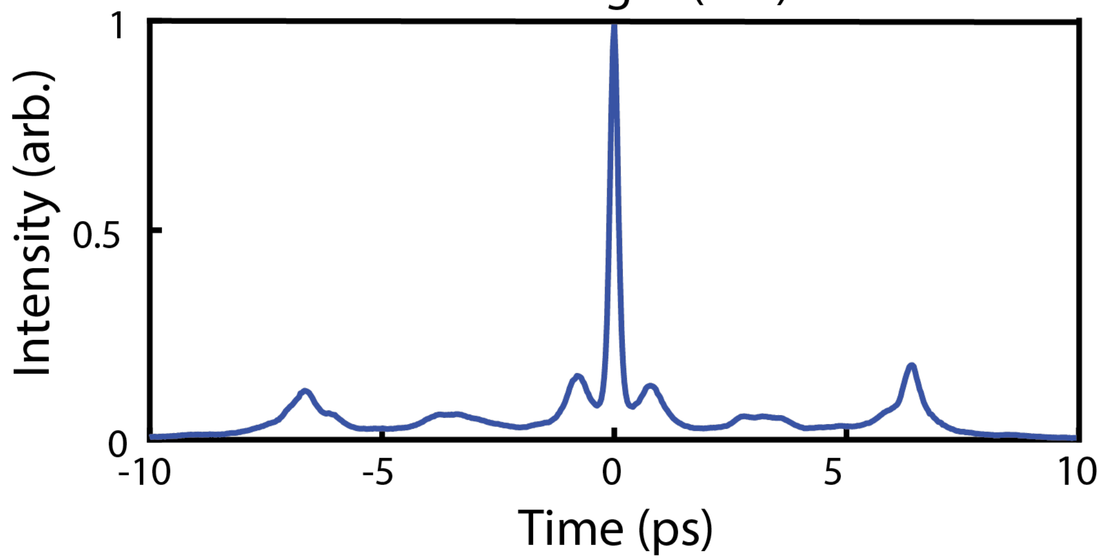
Raman solitons in few-mode fiber



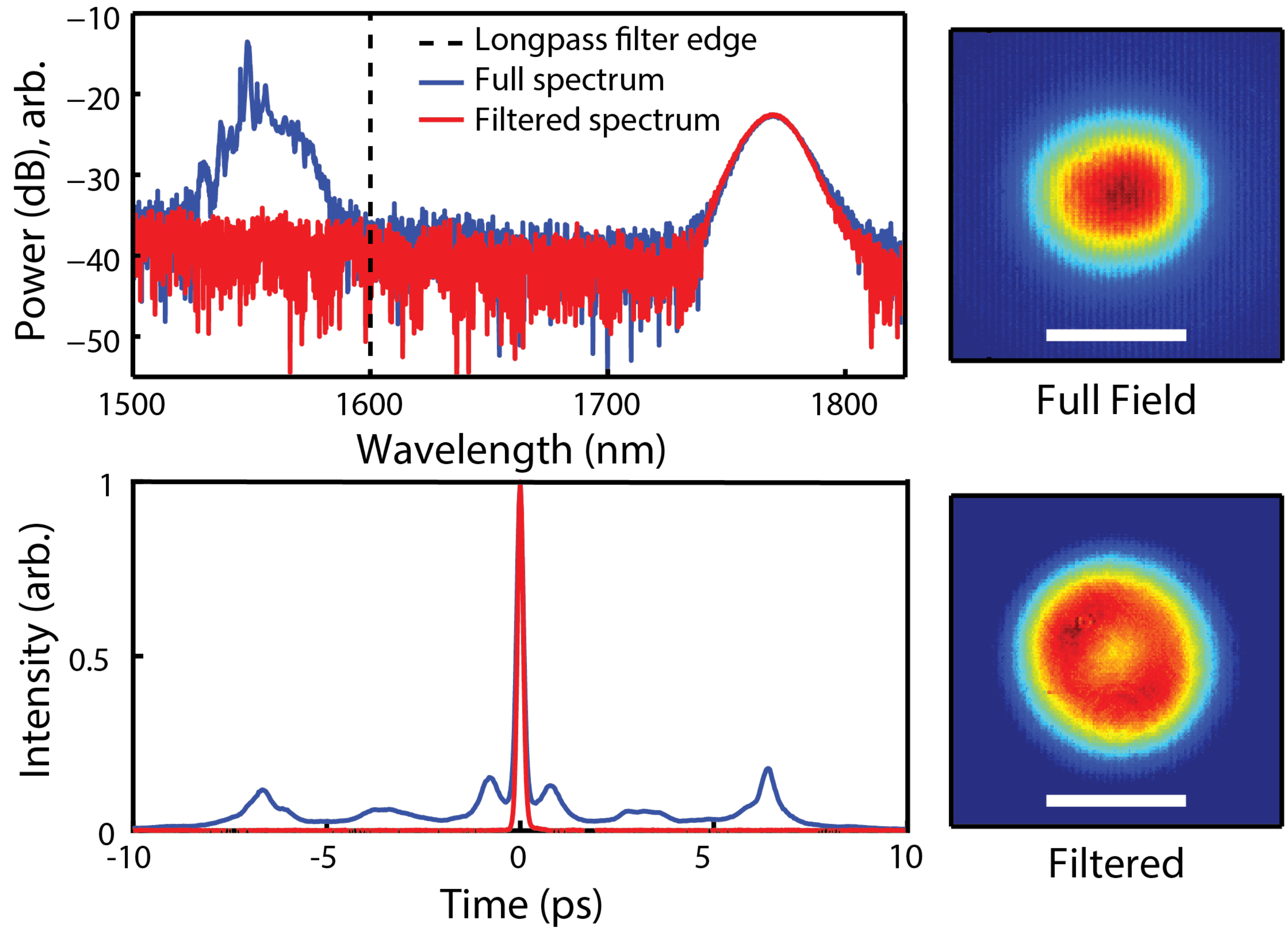
Raman solitons in few-mode fiber





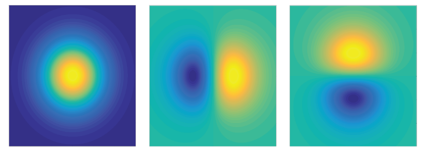


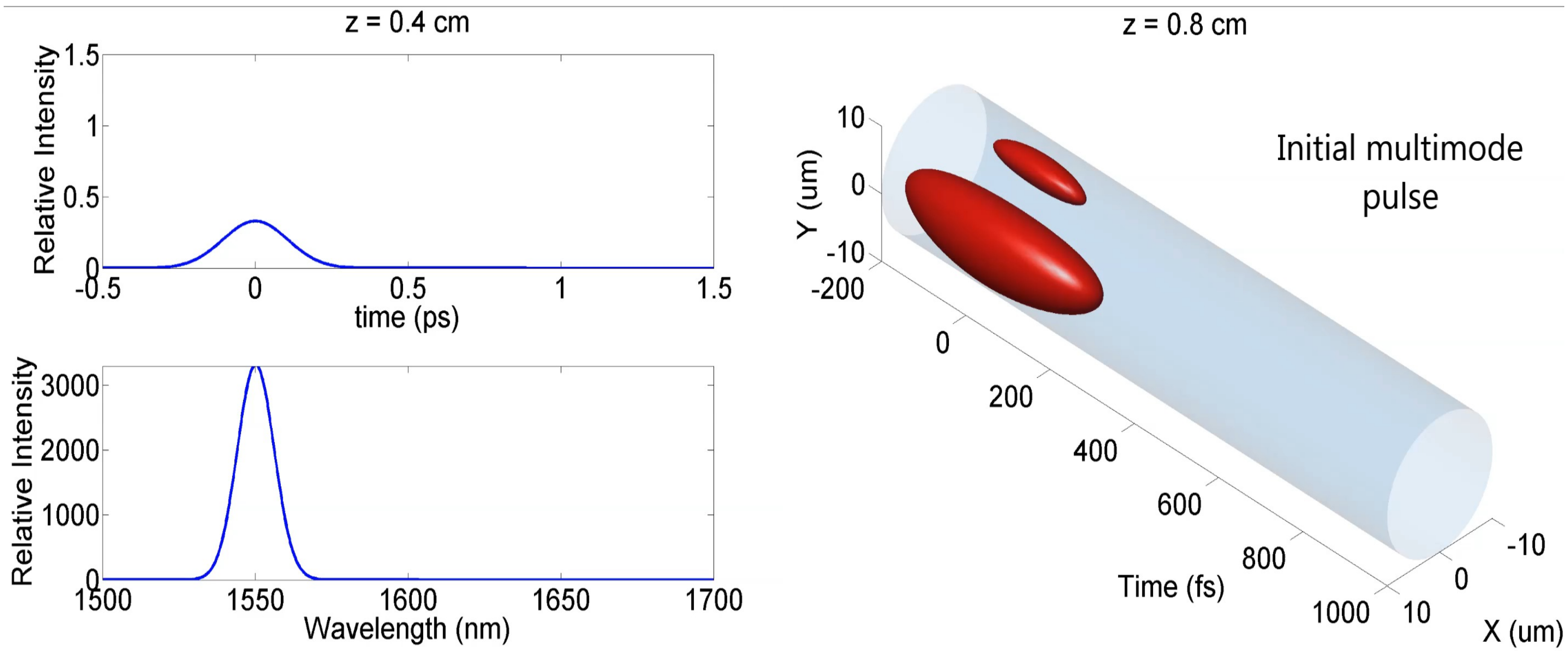
Raman solitons in few-mode fiber



Zhu et al., Opt Lett 2016

Raman solitons in few-mode fiber



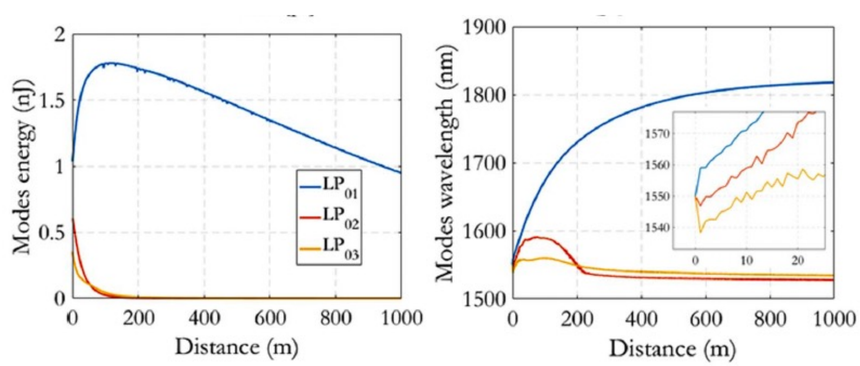


- More modes: *Zitelli et al., Opt Express 28, 20473–20488 (2020)*

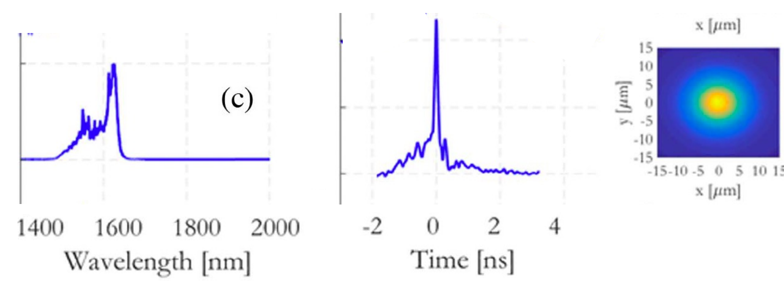
Questions

- Does XPM play a role after soliton formation?

- What is the final state? Single-mode (Raman) soliton?

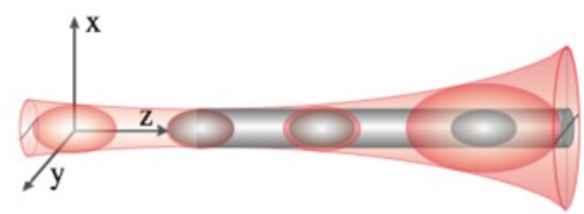


z = 1 km ~ 5600 L_D



Zitelli et al. Photonics Research 2021

- Can “light bullets” form in multimode fiber?



Instabilities

(spatiotemporal organization)

Coupling of waves in GRIN fiber

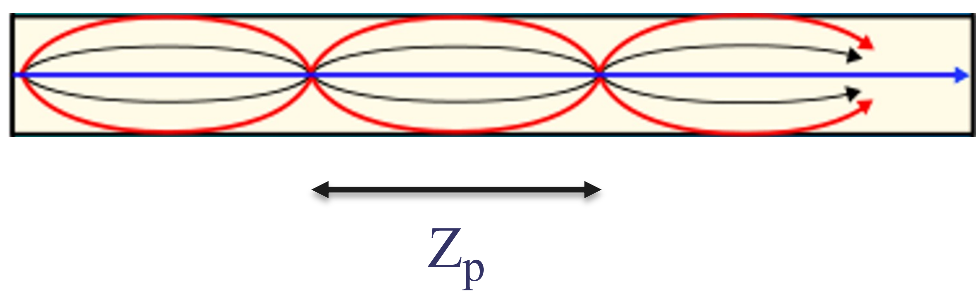
- 4WM can transfer energy between waves

$$2k_{pump} = k_{signal} + k_{idler}$$

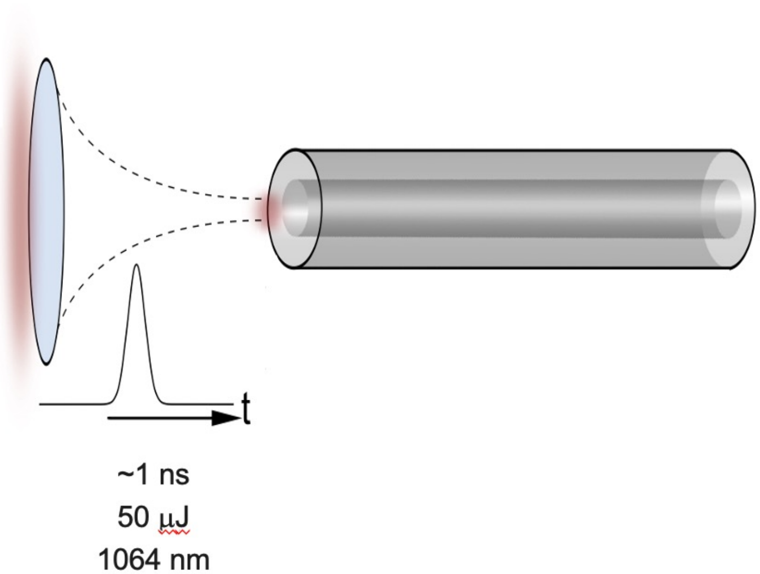
Coupling of waves in GRIN fiber

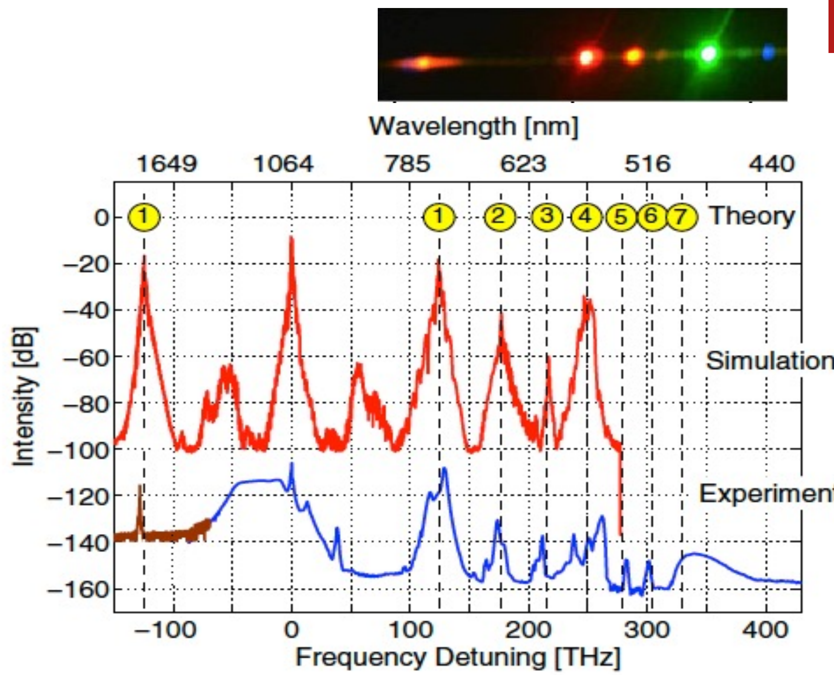
- 4WM can transfer energy between waves

$$2k_{pump} = k_{signal} + k_{idler} + \frac{2\pi m}{z_p}$$



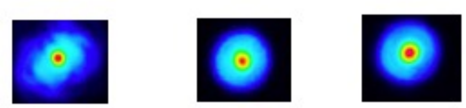
MI in GRIN fiber





- Nanosecond input pulses

$$2k_{pump} = k_{signal} + k_{idler} + \frac{2\pi m}{z_p}$$



- "Geometric parametric instability"
- 300 THz frequency range

Krupa et al., Phys Rev Lett 2016

Coupling of waves in GRIN fiber

- Perturbed solitons can emit dispersive waves

$$k_{sol} = k_{dis}$$

Coupling of waves in GRIN fiber

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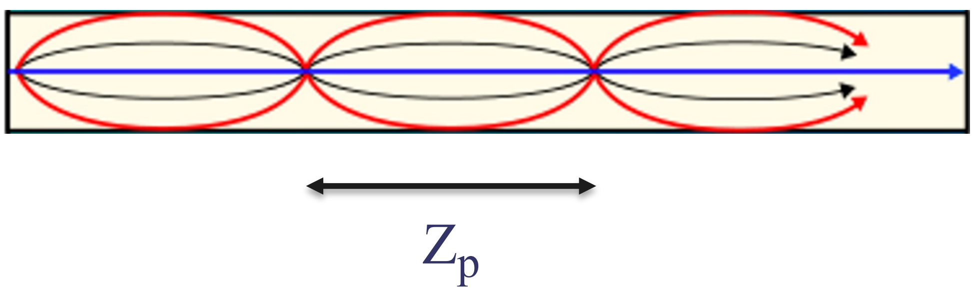
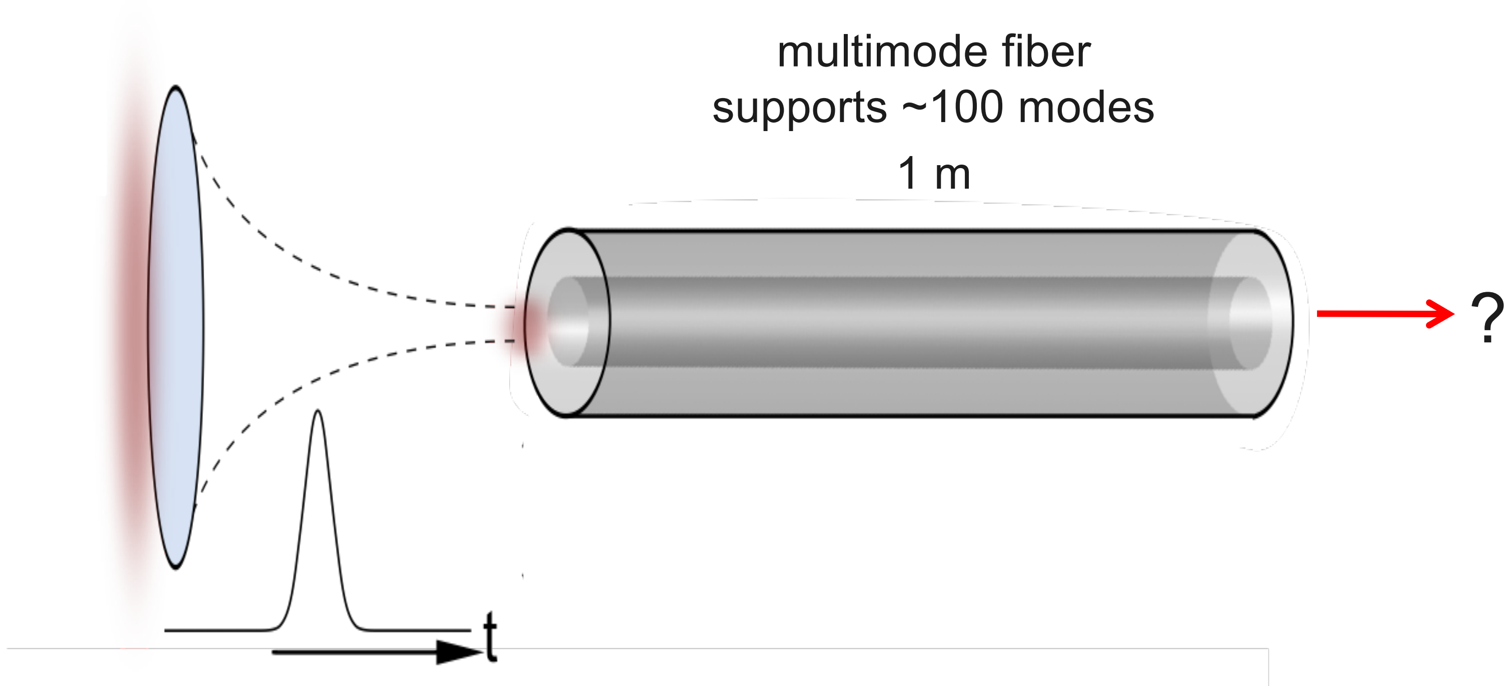
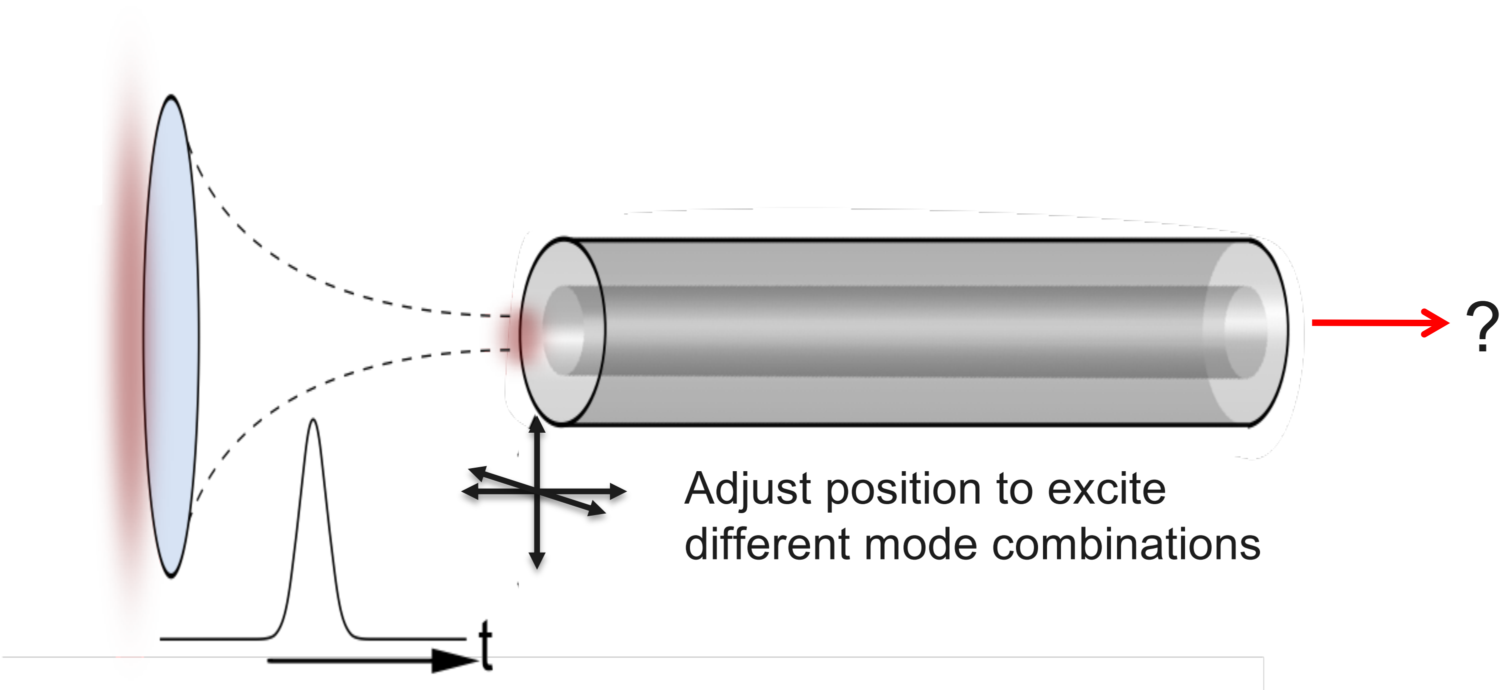


Experiments

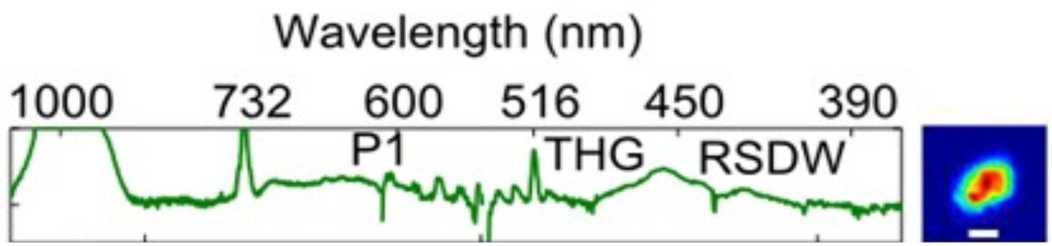


500 fs
energy up to 1 μJ
peak power up to MW
1550 nm

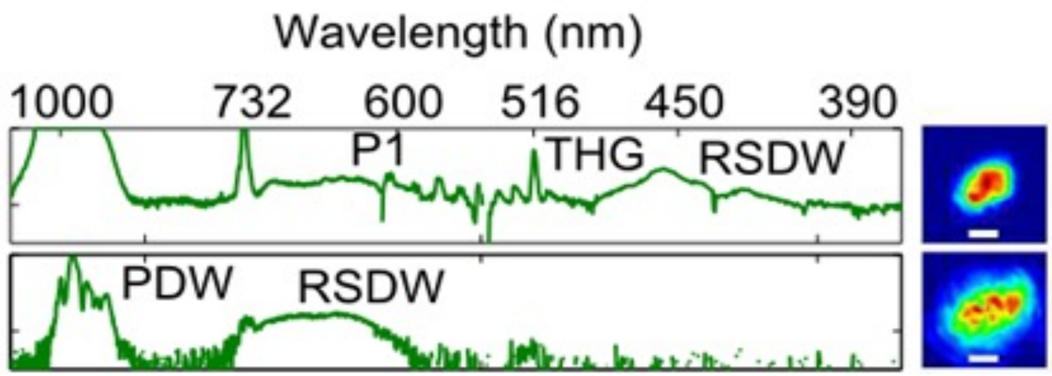
Experiments



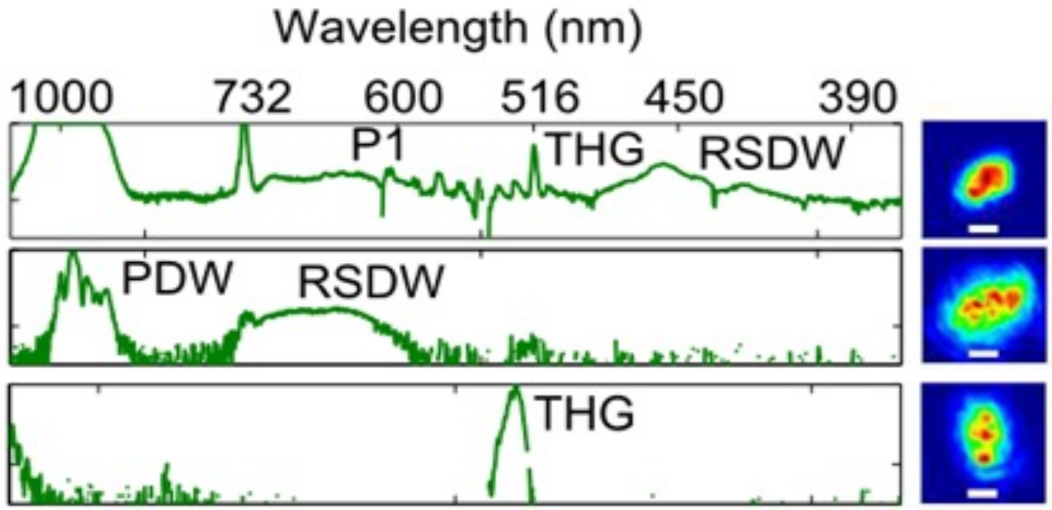
Controllable continuum



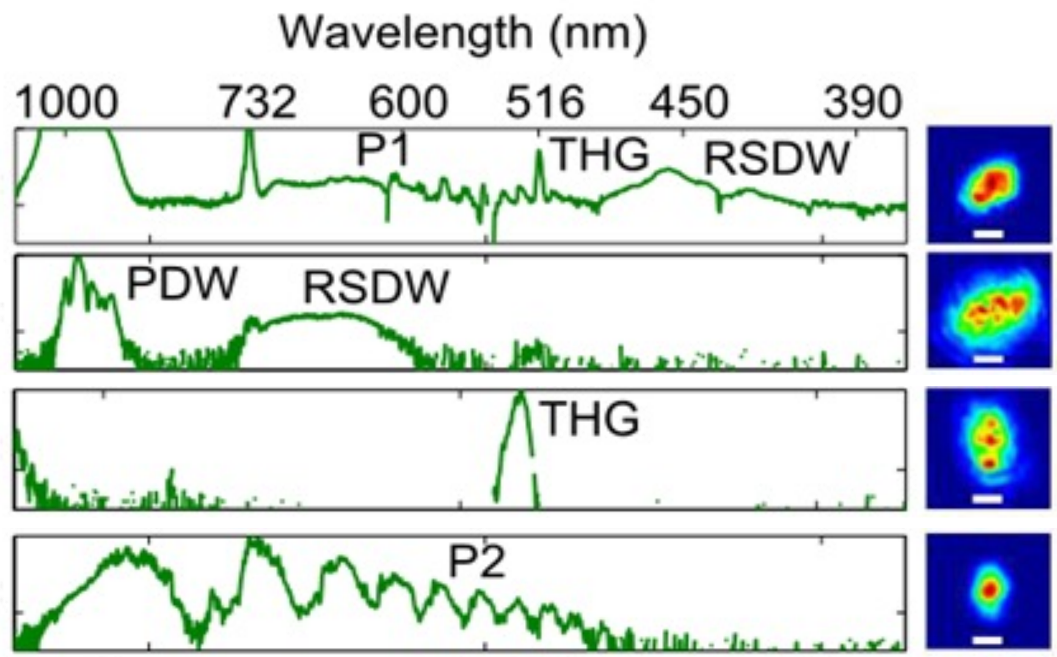
Controllable continuum



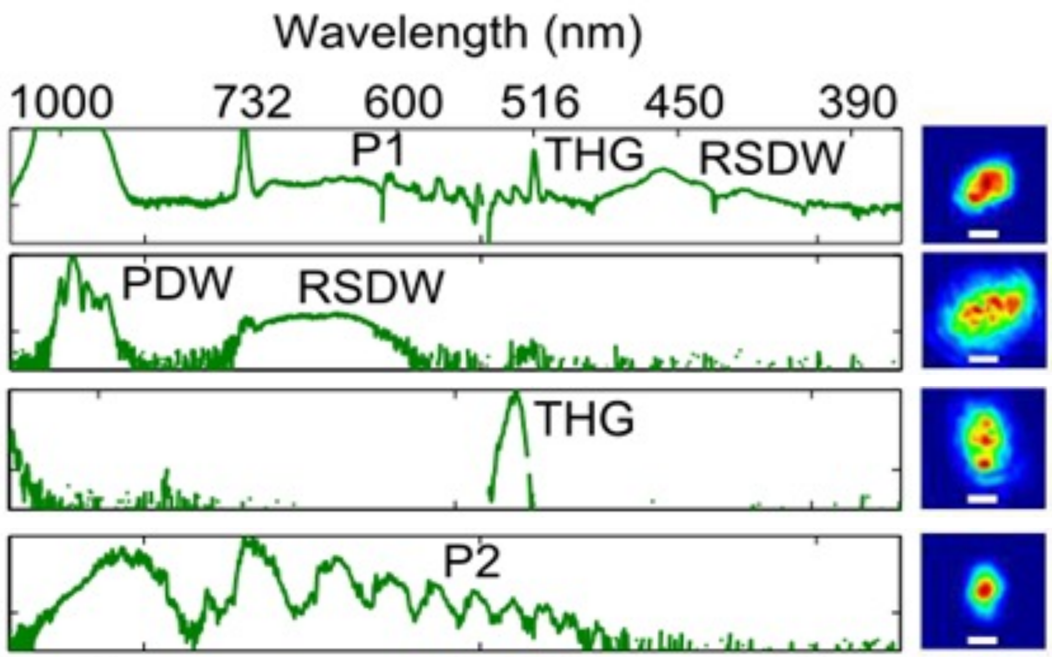
Controllable continuum

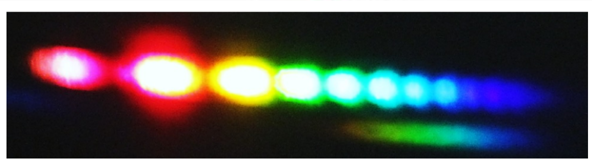


Controllable continuum



Controllable continuum





Perturbation of solitons (1D tutorial)

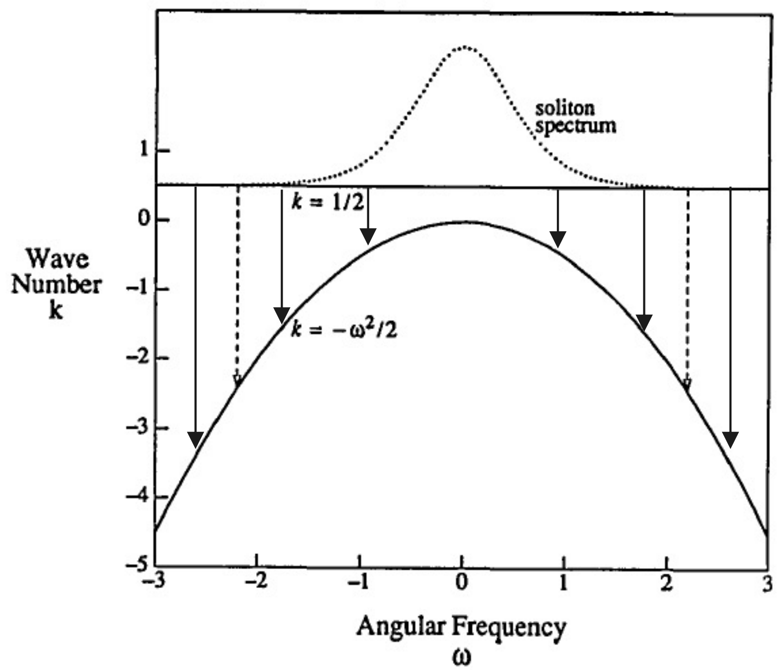
- Perturbed soliton adjusts to reach $A_0\tau_p = \sqrt{\frac{\beta_2}{\gamma}}$

  and radiates dispersive wave

- Periodic perturbation (period = Z_p)
  Resonant energy transfer when wave vectors match

$$k_{sol} - k_{dis} = 2m\pi/Z_p$$

$$\Omega_{res} = \frac{1}{\tau}\sqrt{\frac{8Z_0 m}{Z_p} - 1}$$



Gordon, J Opt Soc Am B 1992

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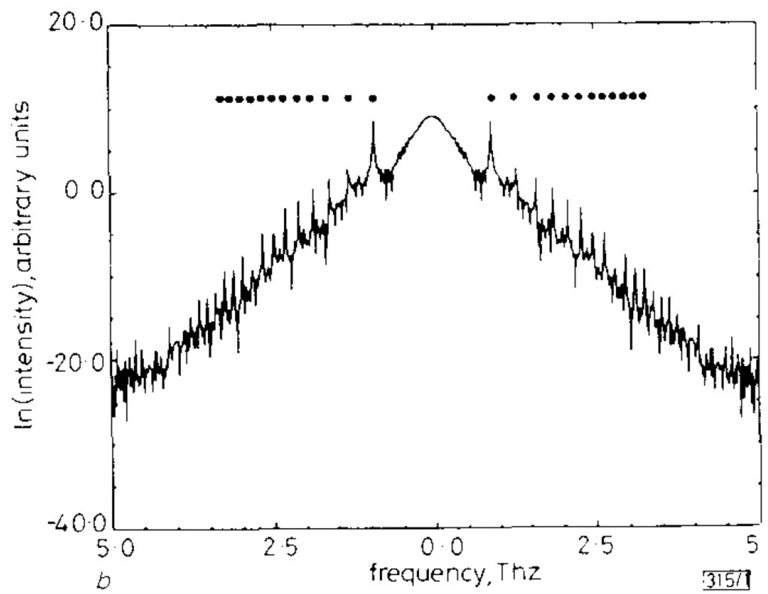
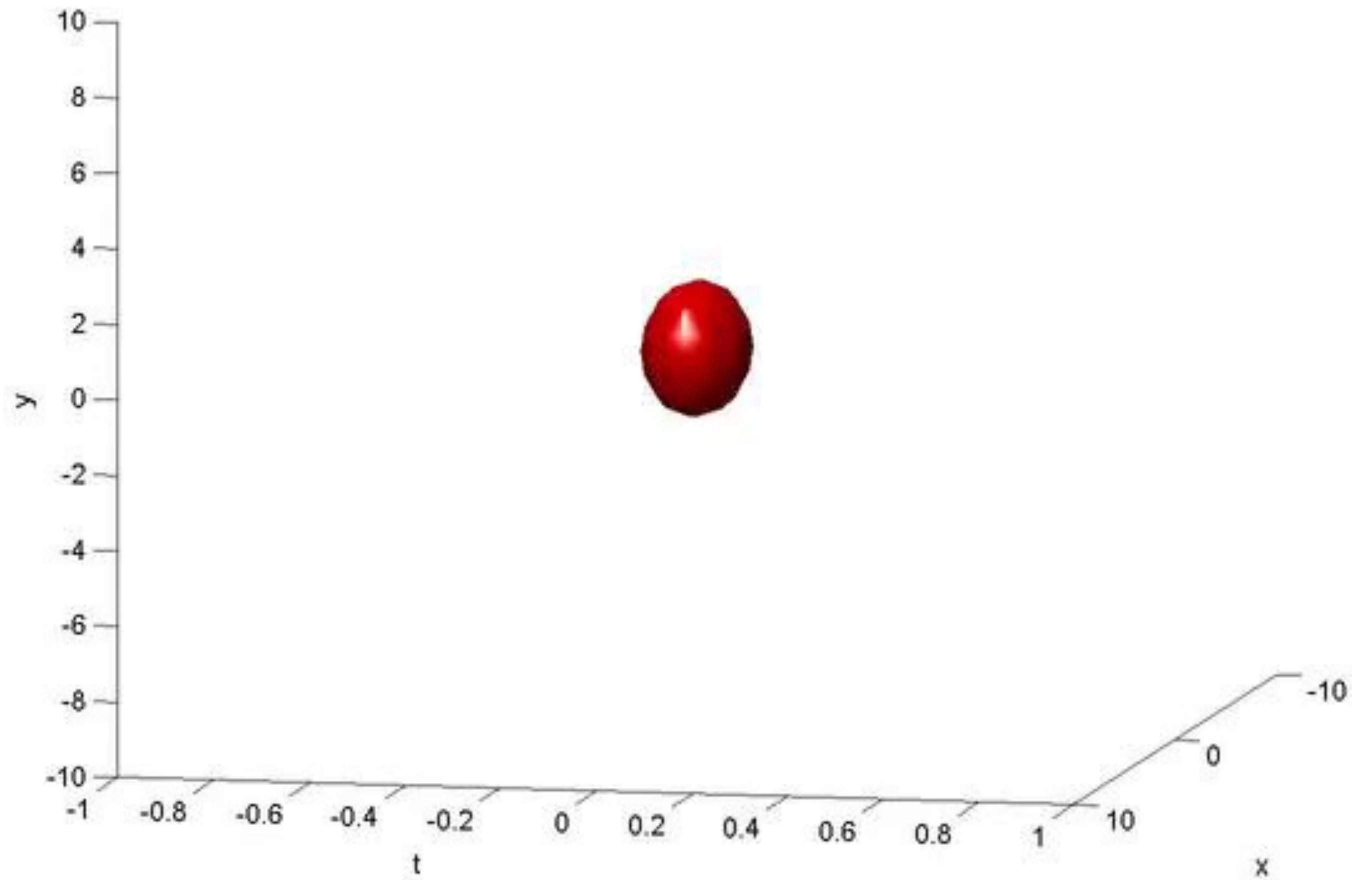
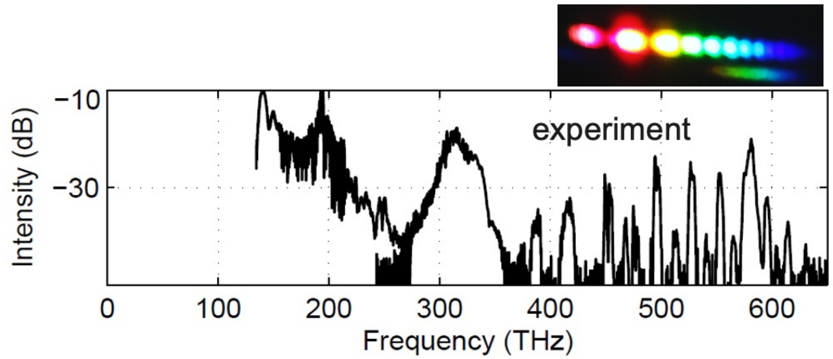


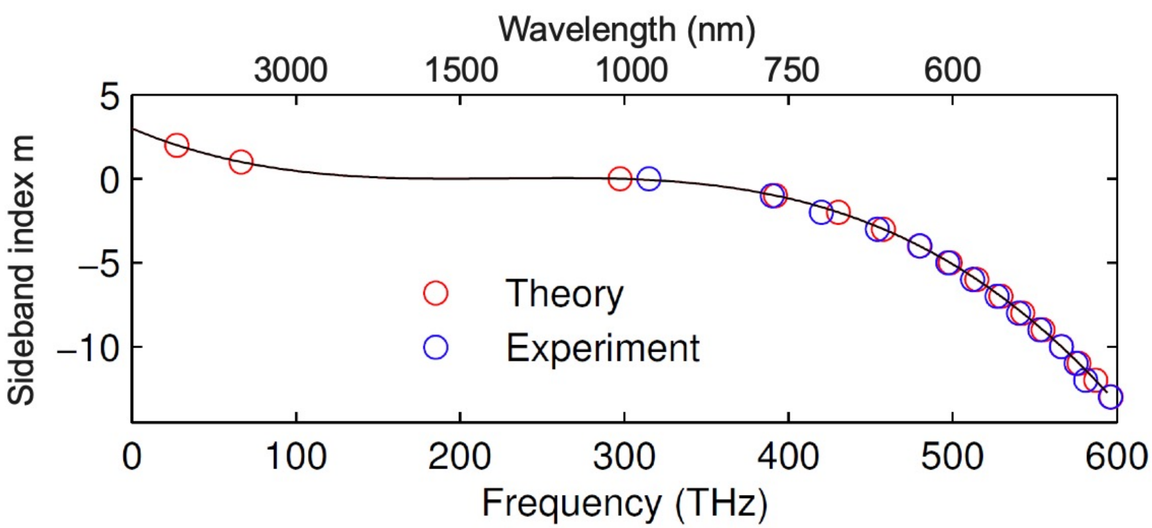
Kelly, Electron Lett 1992

Spatiotemporal profile in GRIN fiber



Theory and experiment





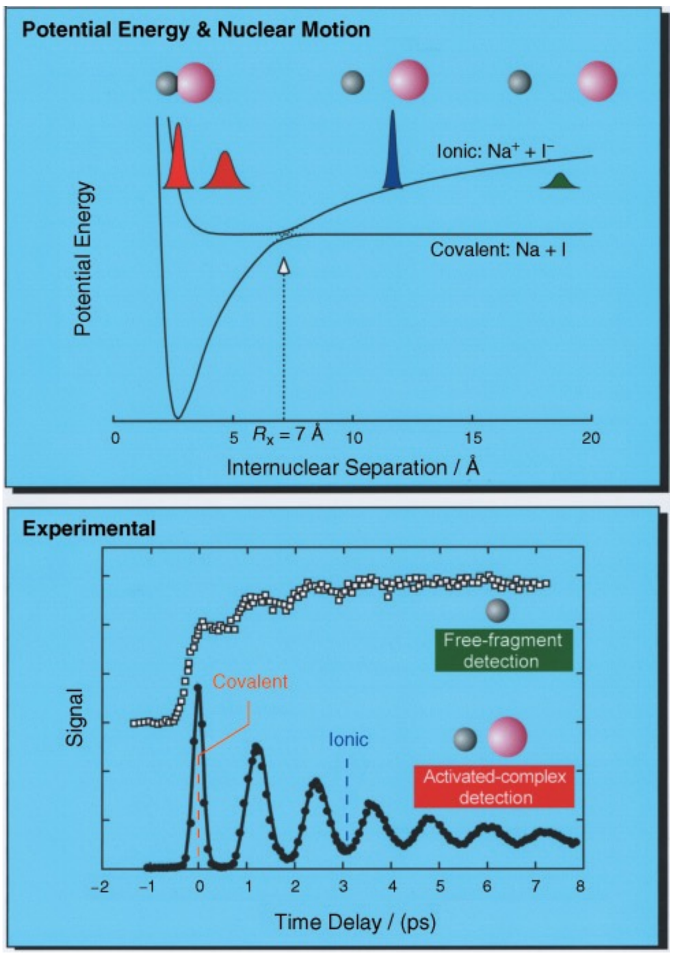
- Simulation, experiment and analytic theory agree well
- Self-imaging perturbs the field
- 300 THz frequency range

Wright, Wabnitz et al., PRL 2015

Spatiotemporal Mode-Locking in Fiber Lasers

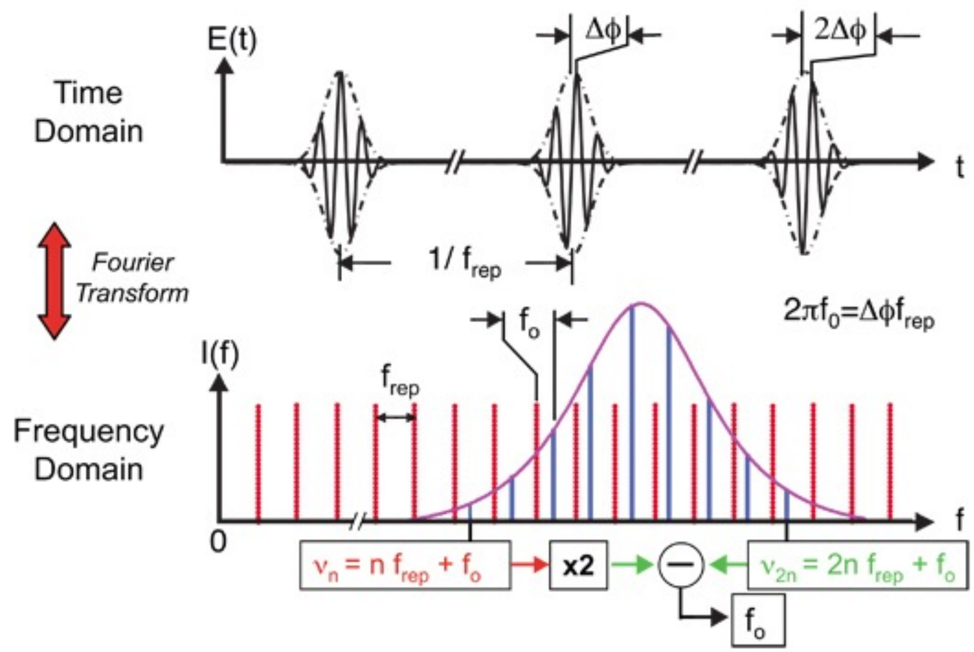
Ultrafast science

Femtochemistry



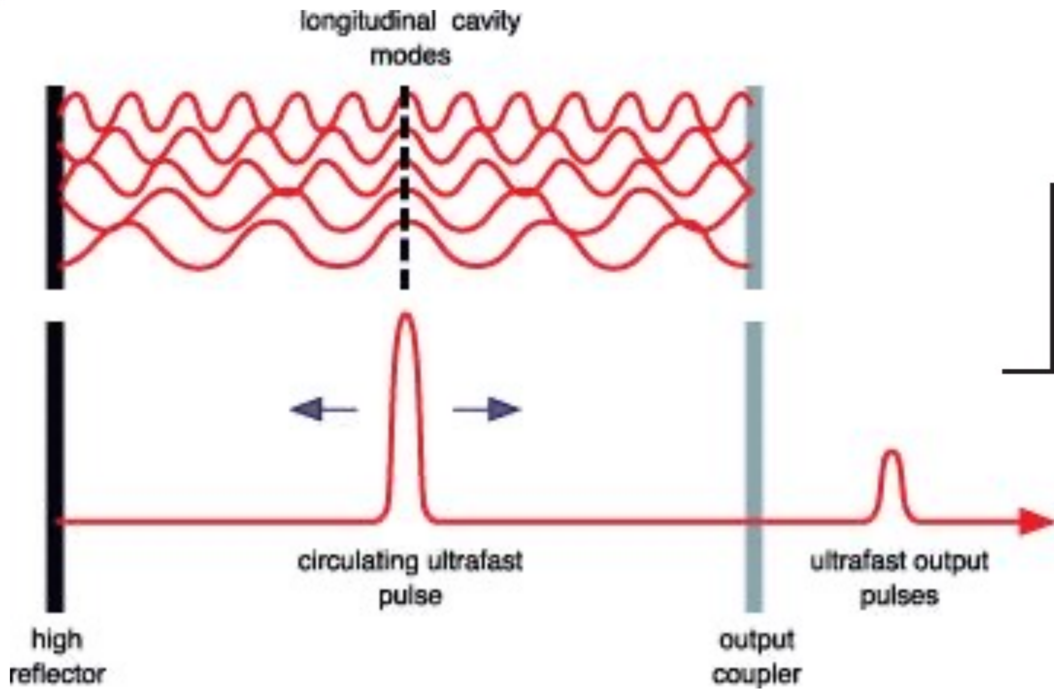
Zewail, J. Chem. Ed. 78, 739 (2001)

Frequency Combs



www.physics.ubc.ca/~djjones

Mode-locking



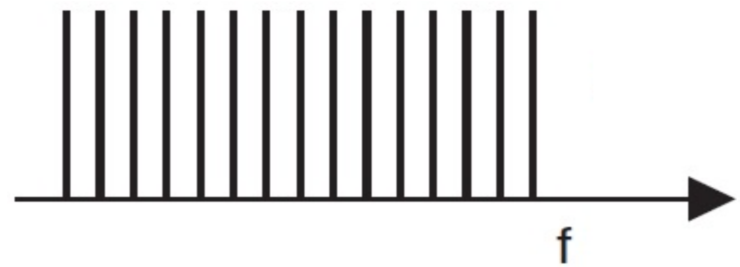
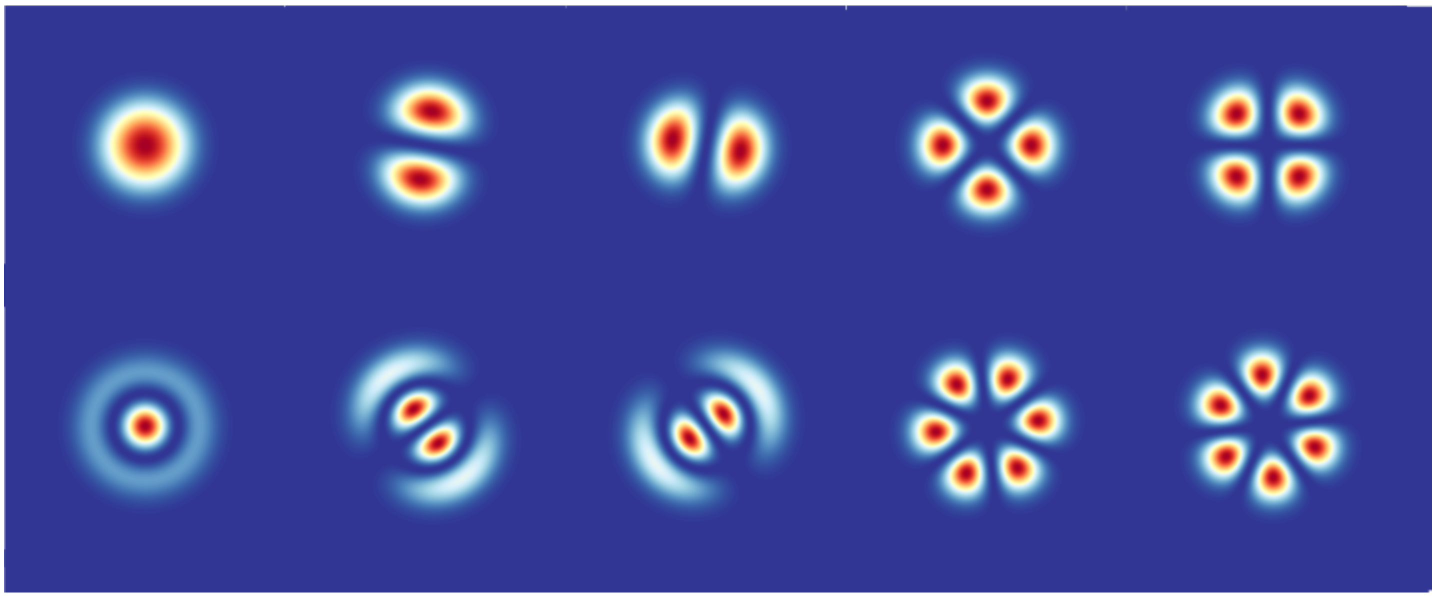


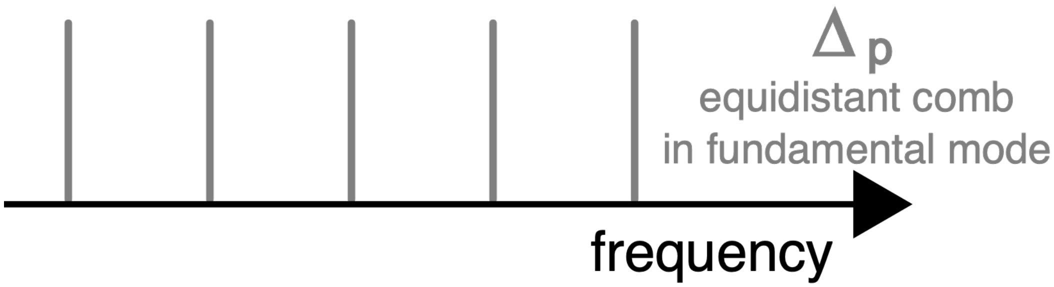
Image credit: University of Oregon Physics

Transverse modes



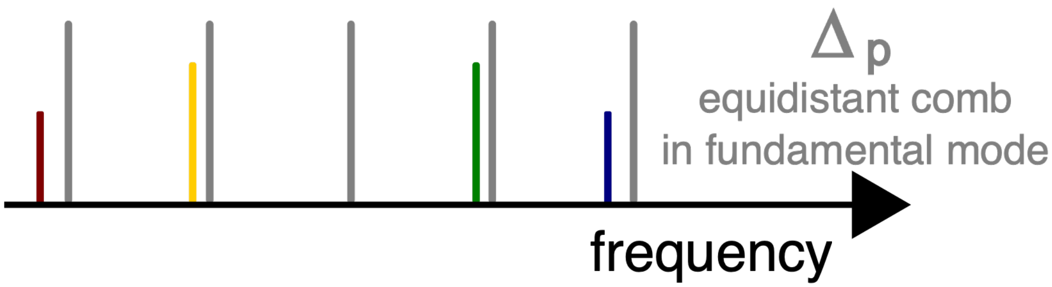
Can we lock transverse *and* longitudinal modes?

Resonant lasing frequencies



Without dispersion, longitudinal modes form a frequency comb

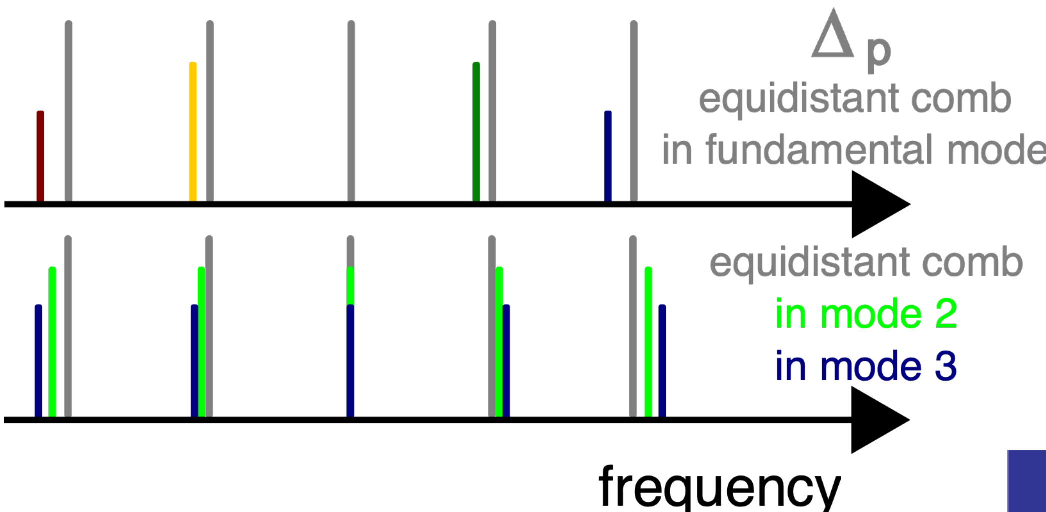
Resonant lasing frequencies



Chromatic dispersion shifts resonances

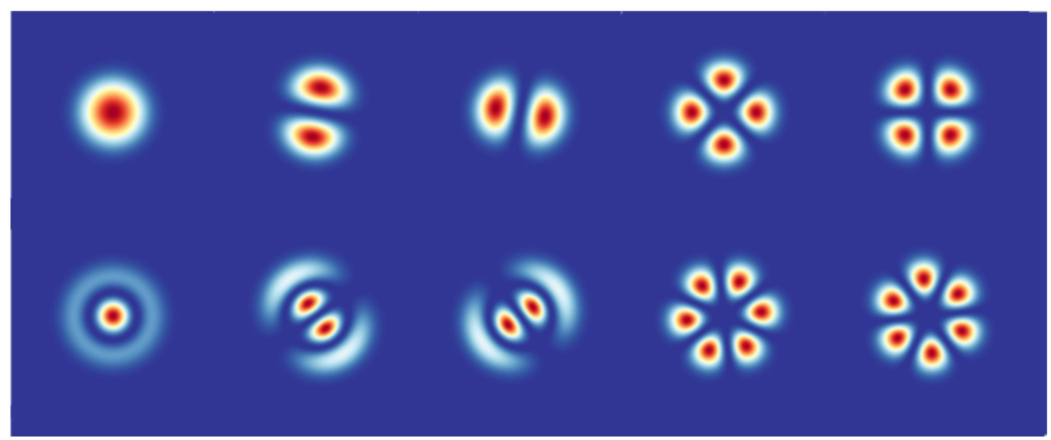
Resonant lasing frequencies

Linear dispersive effects



Chromatic dispersion shifts resonances

Each mode family has different resonances

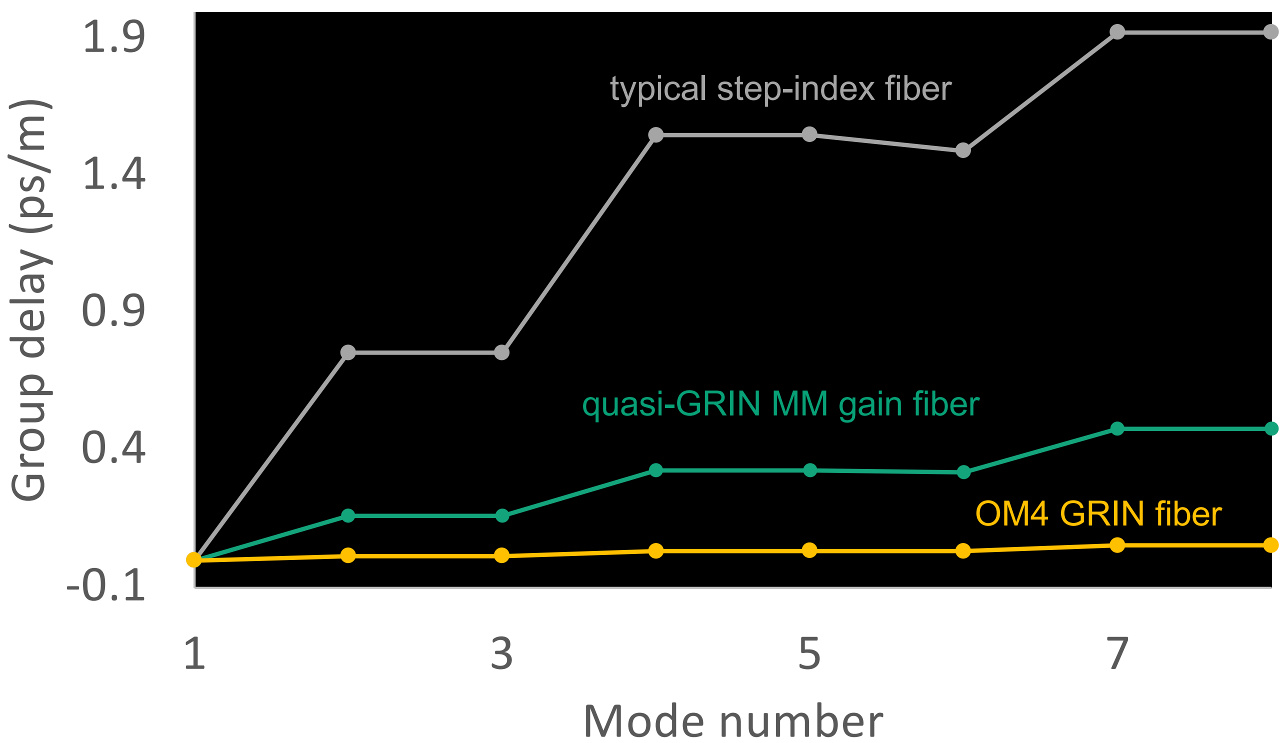


What do we need for 3D mode-locking?

- Low *spatiotemporal* dispersion
- Phase-sensitive nonlinear interactions between *3D* modes
- A *spatiotemporal* saturable absorber

Our approach

- Parabolic-index fiber makes dispersions comparable



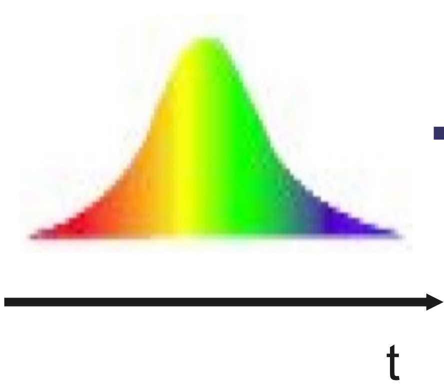
Our approach

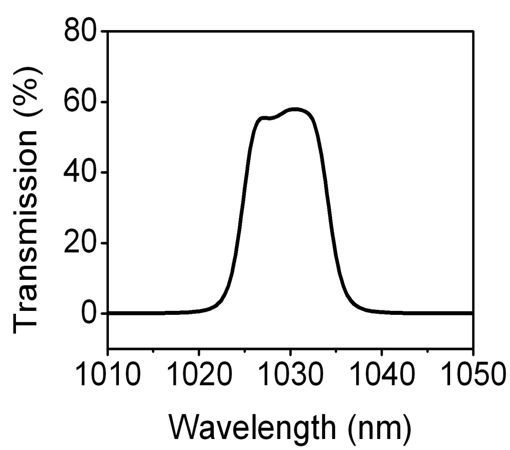
- Parabolic-index fiber makes dispersions comparable

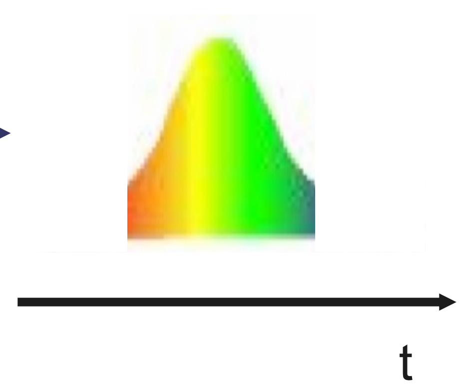
- Temporal profile

  Normal group-velocity dispersion

  Strong spectral filtering



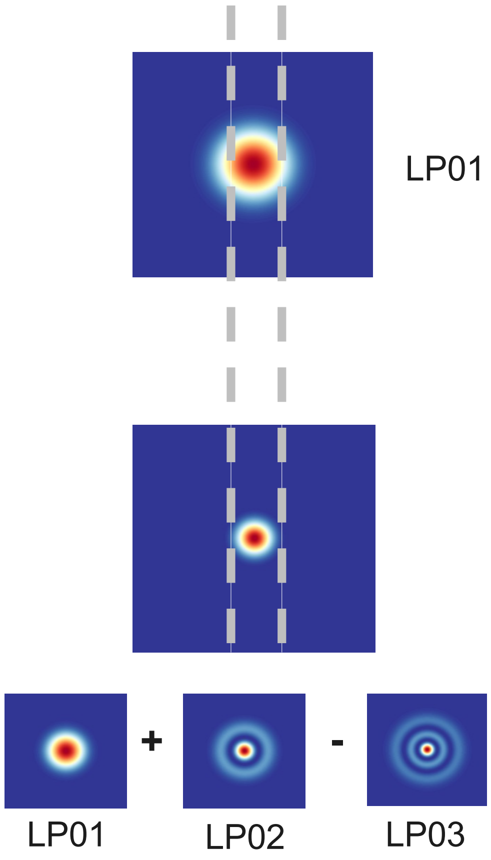




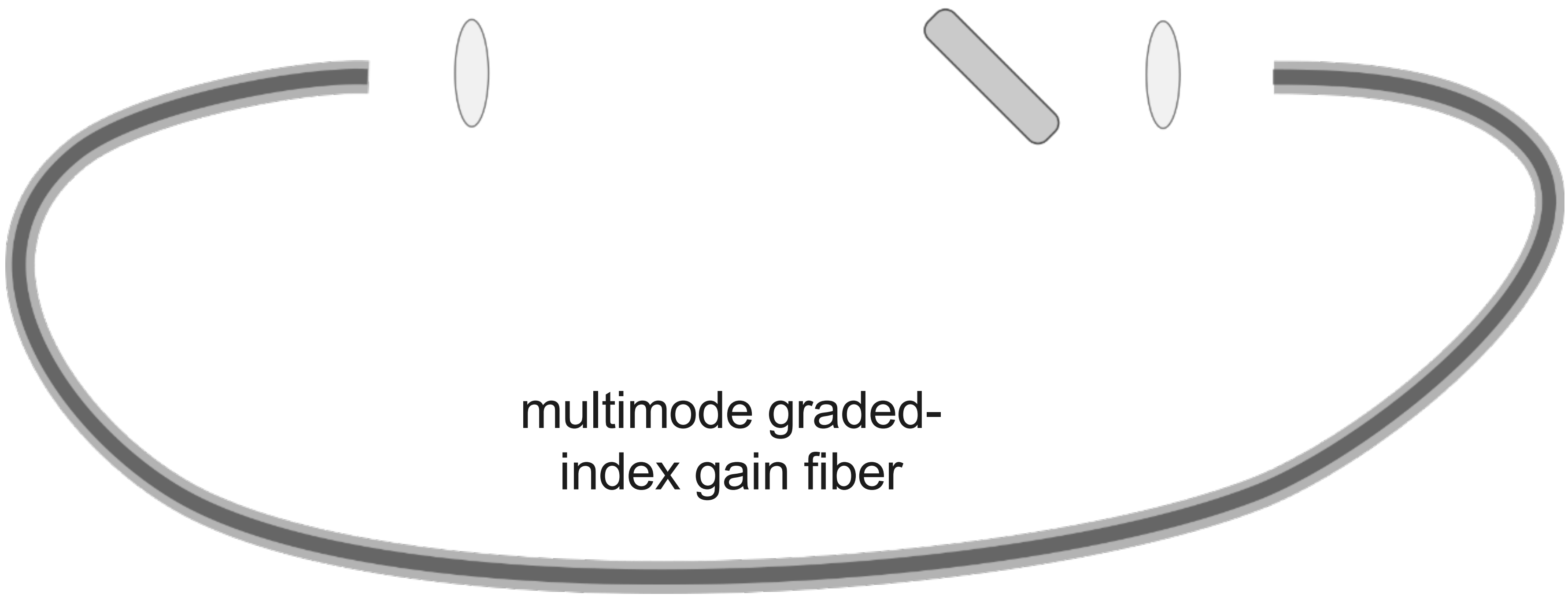
Our approach

- Spatial profile

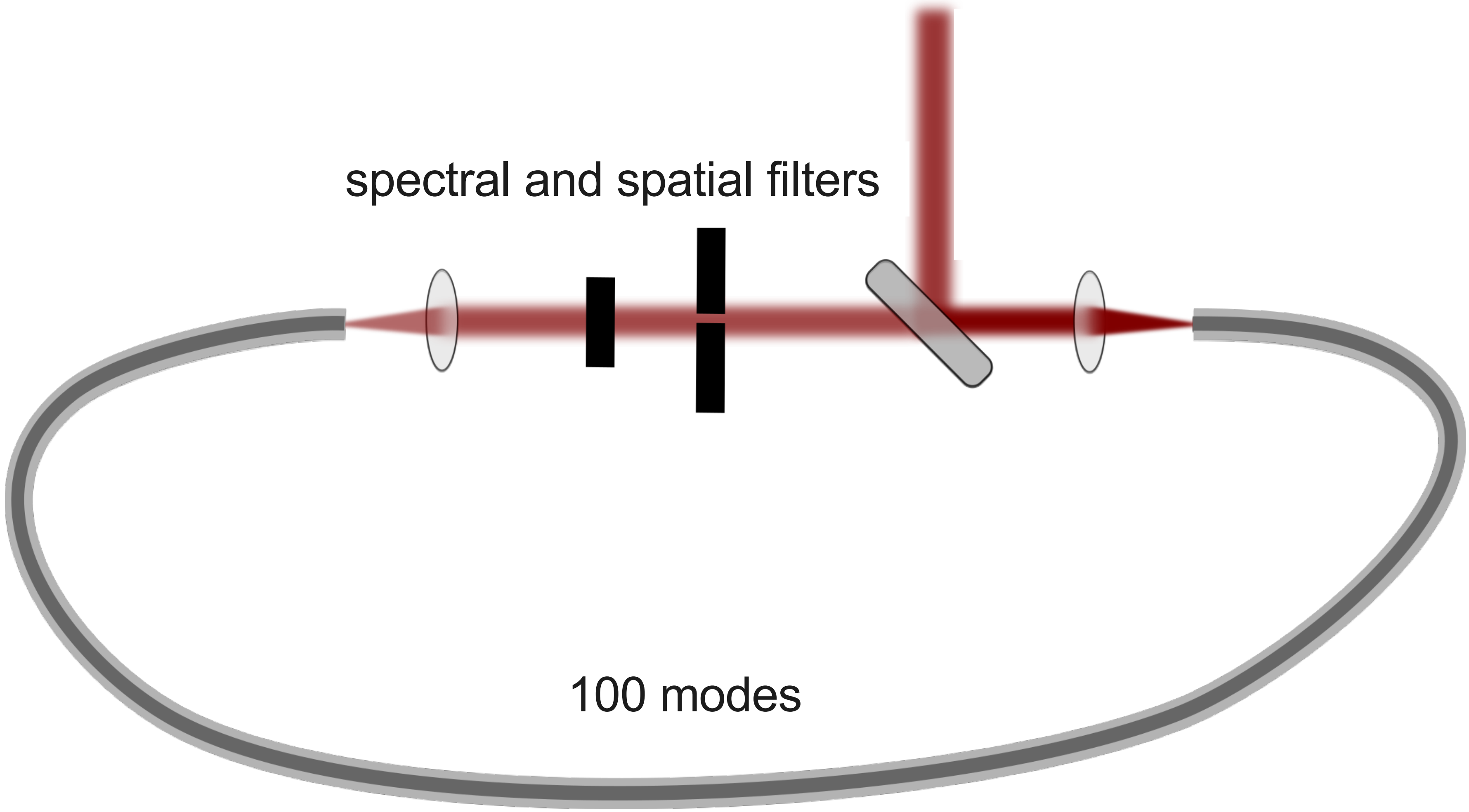
  Strong spatial filtering promotes multiple transverse modes



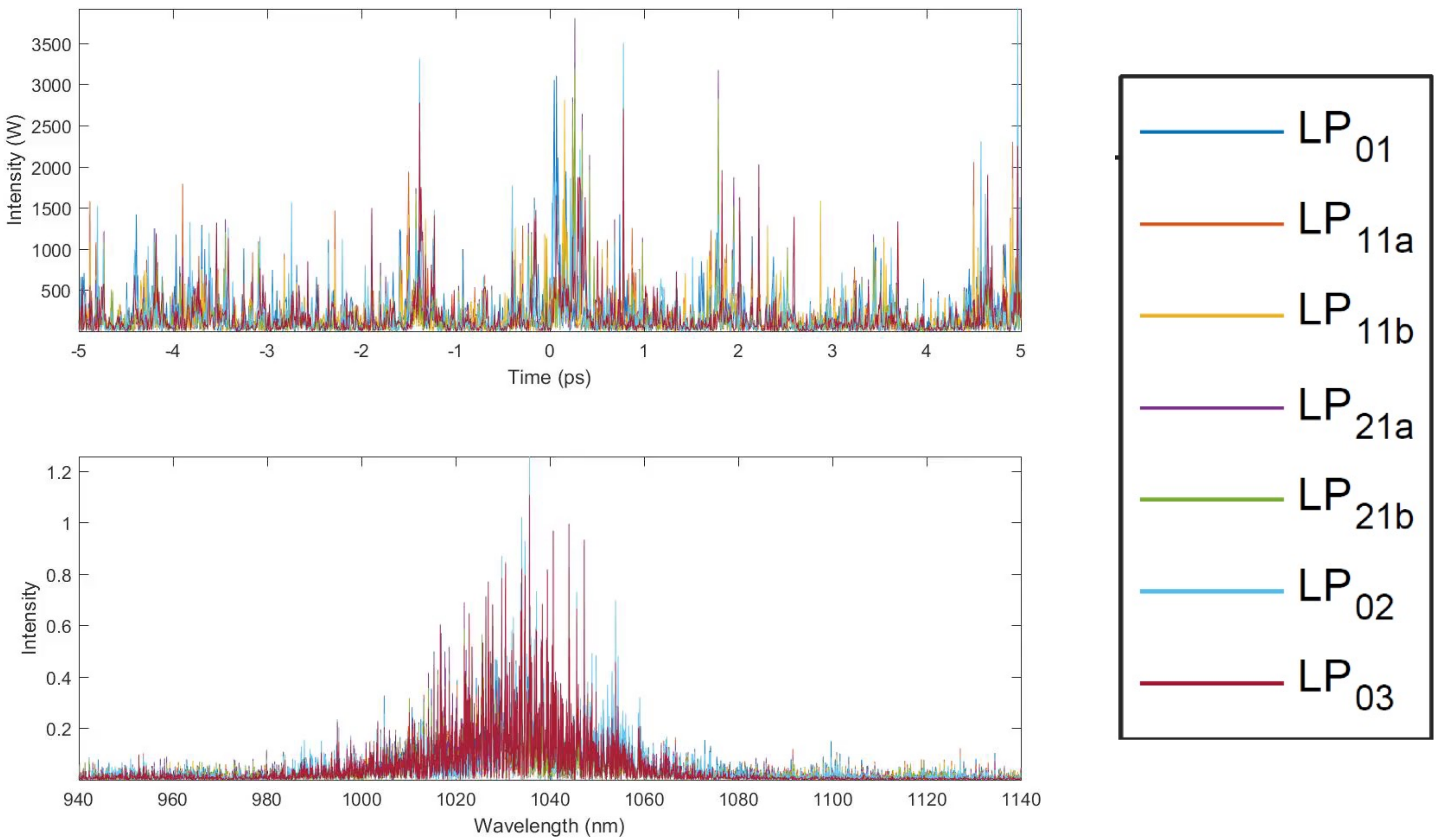
How to do it?



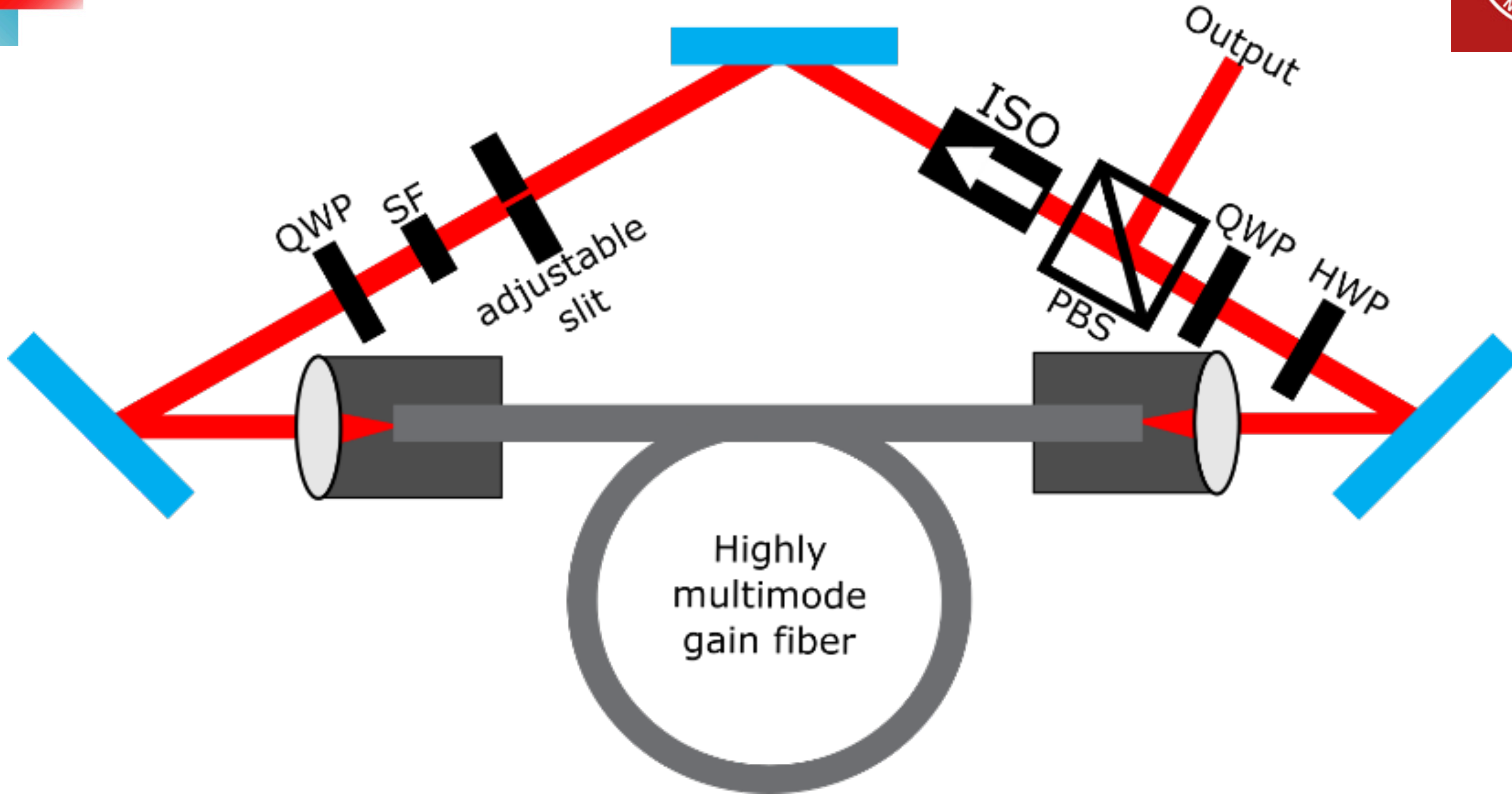
How to do it?

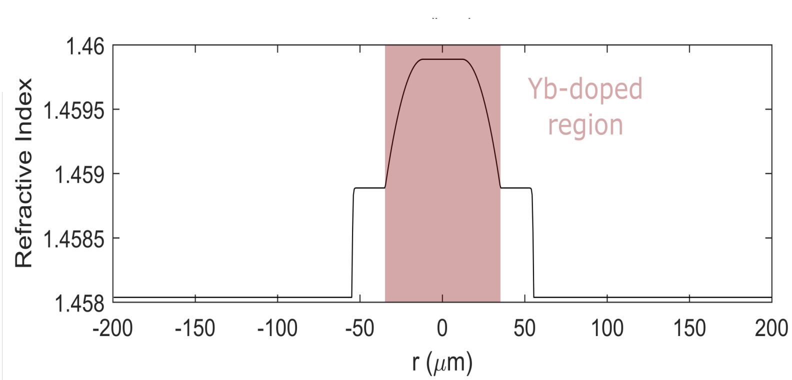


Example simulation



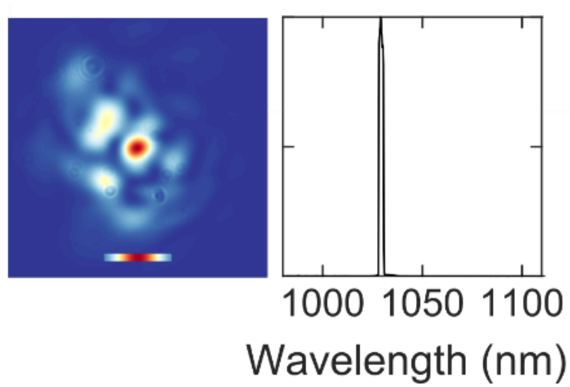
Experiments





- 100 spatial modes x 2 polarizations
- Nonlinear polarization rotation for saturable absorber

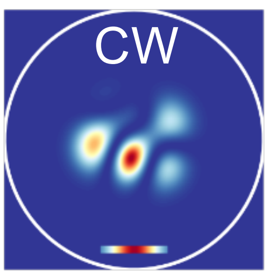
Experiment



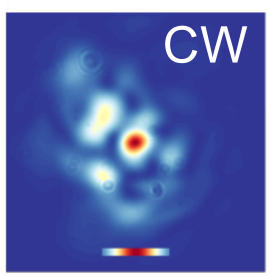
Experiment

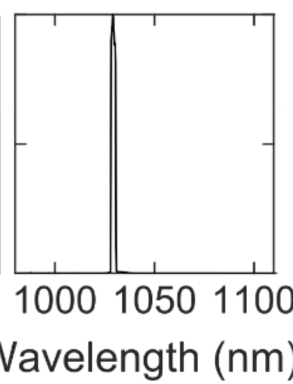
Theory

10 modes



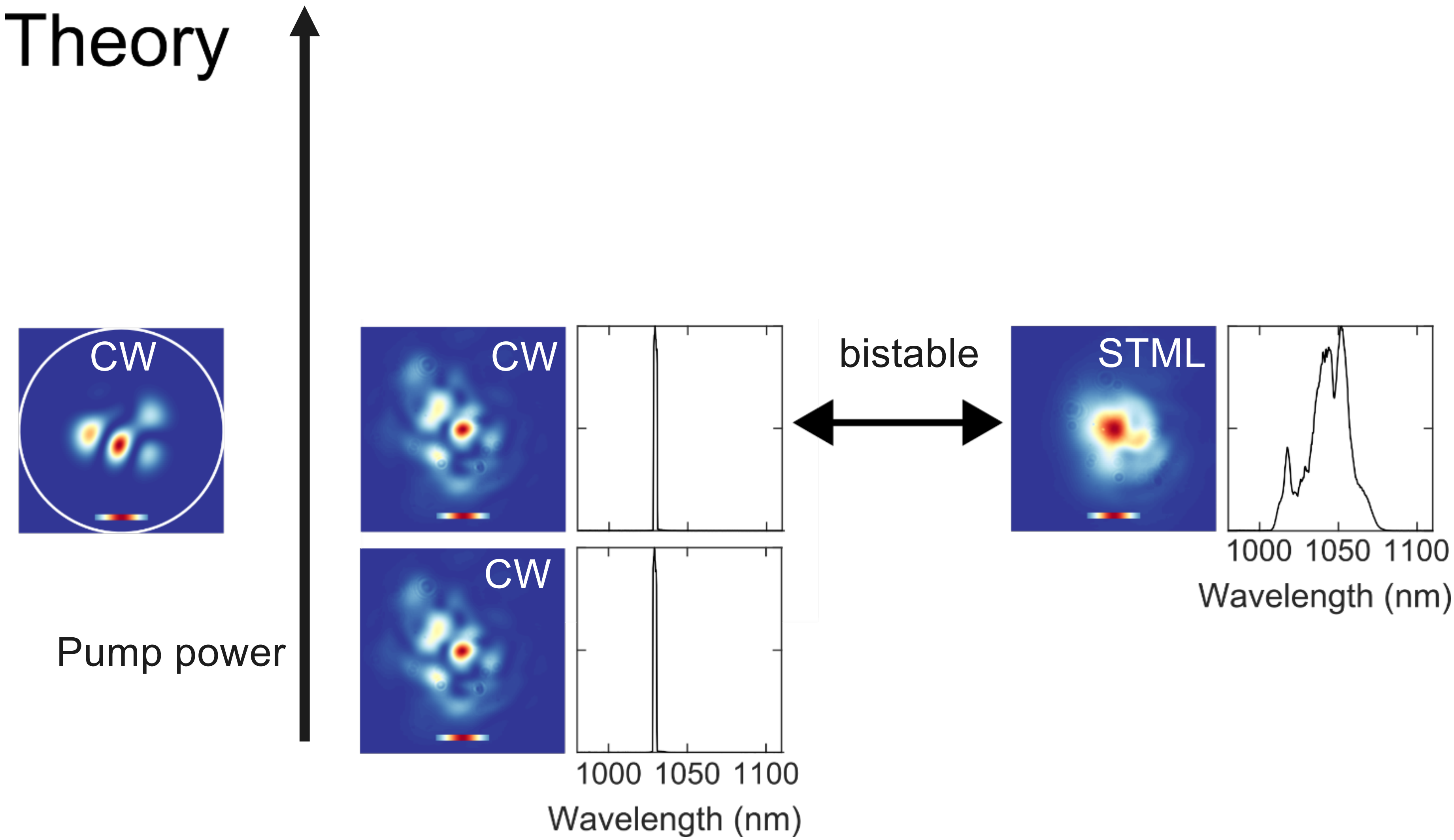
Pump power



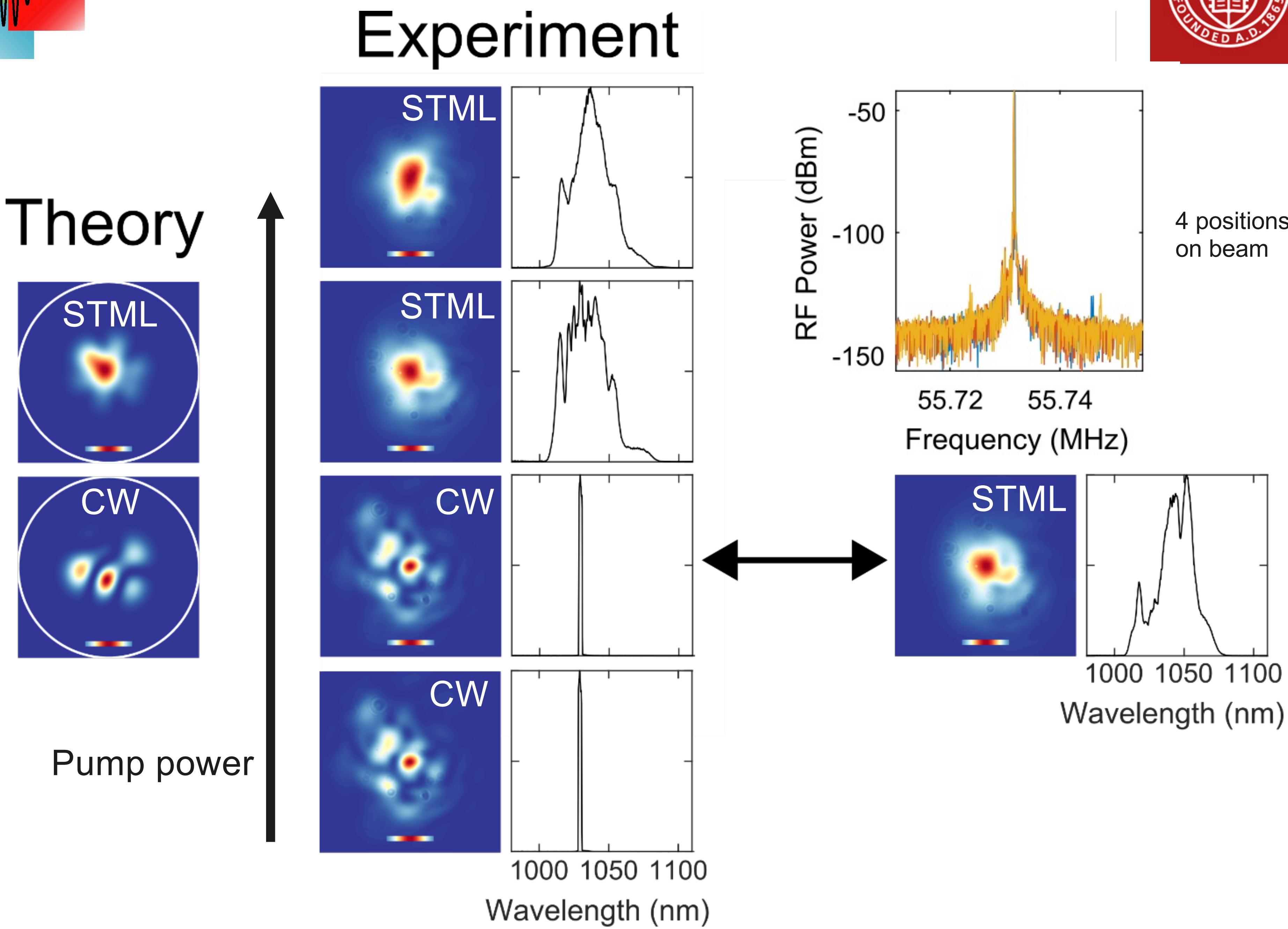


~100 modes

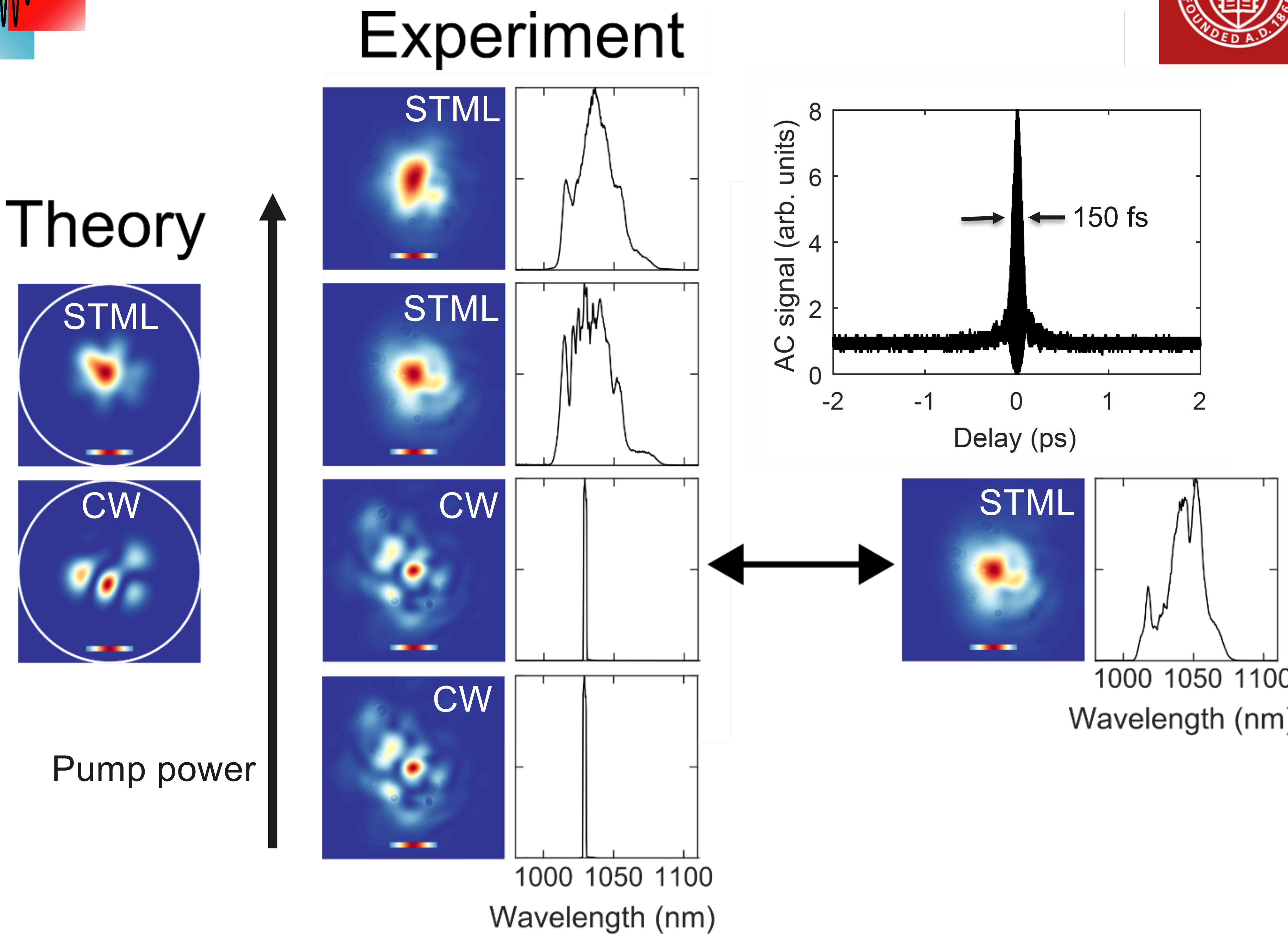
Experiment


Theory
CW
Pump power
CW
CW
1000 1050 1100
Wavelength (nm)
bistable
STML
1000 1050 1100
Wavelength (nm)

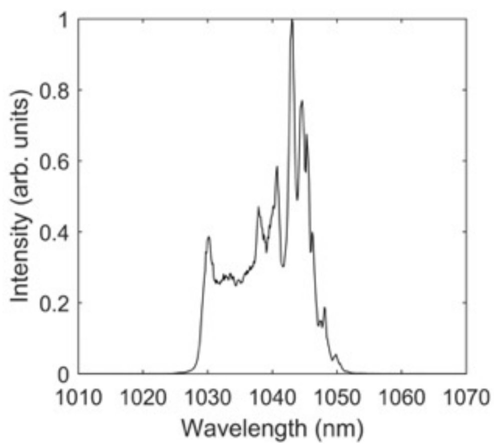
Mode-locked laser

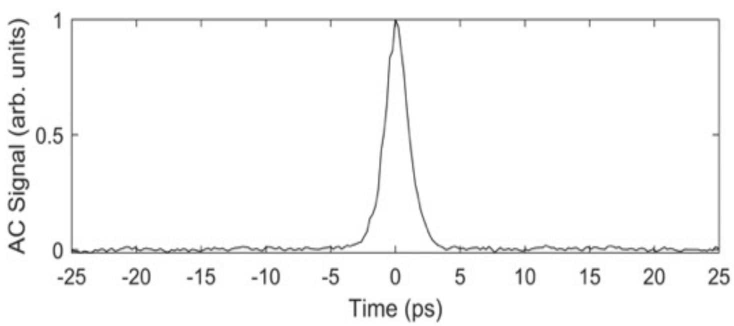


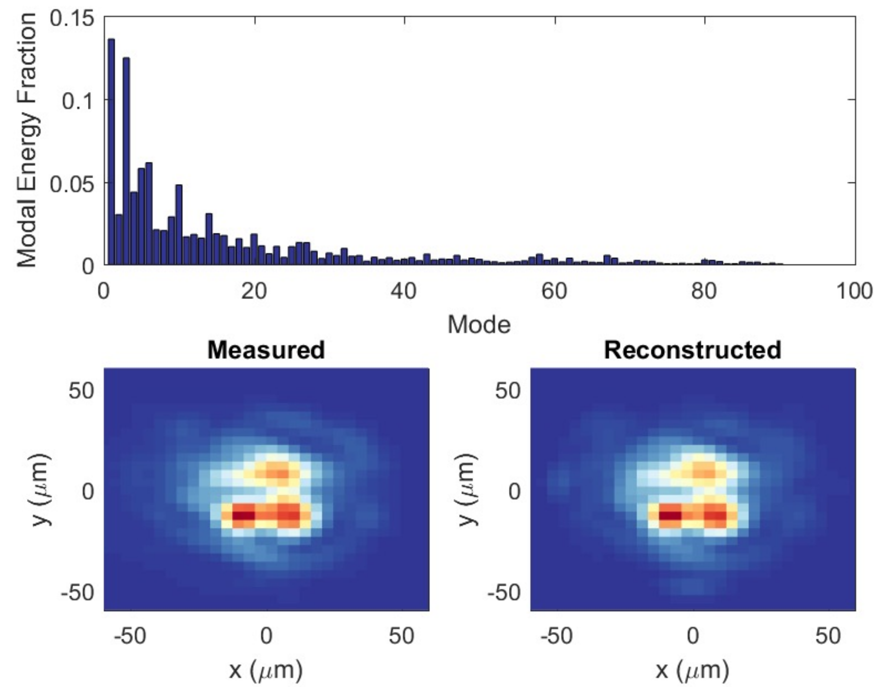
Mode-locked laser



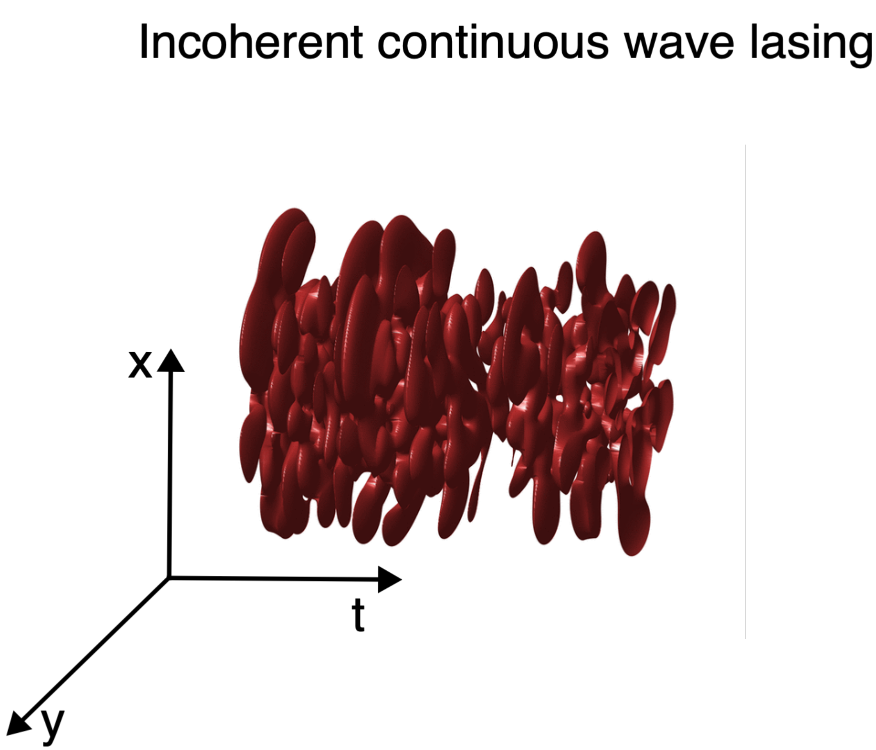
3D characterization

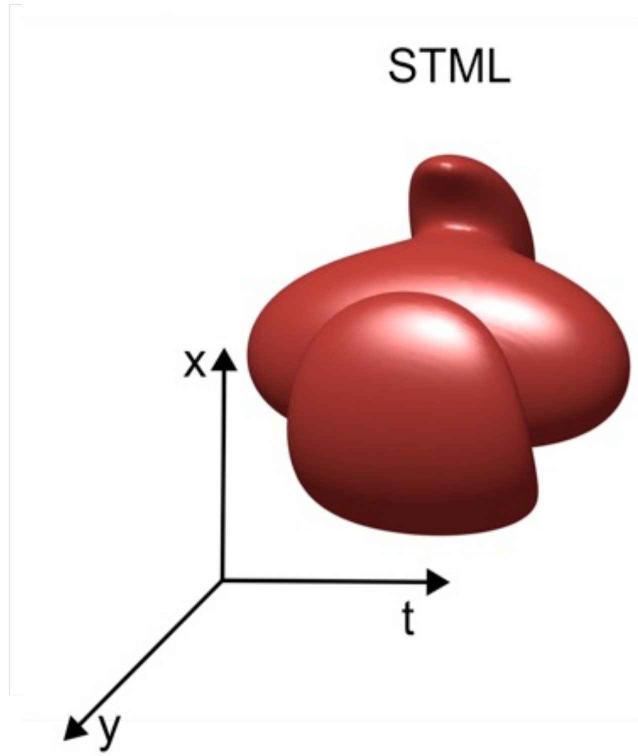






Mode-locking transition





Why is it important?

Why is it important?

- There are many new mode-locked states

- States and phenomena that have no analogs in 1D

  involve up to 10^8 modes

- Theoretical understanding is crude

Why is it important?

- There are many new mode-locked states
- States and phenomena that have no analogs in 1D

  involve up to 10^8 modes
- Theoretical understanding is crude

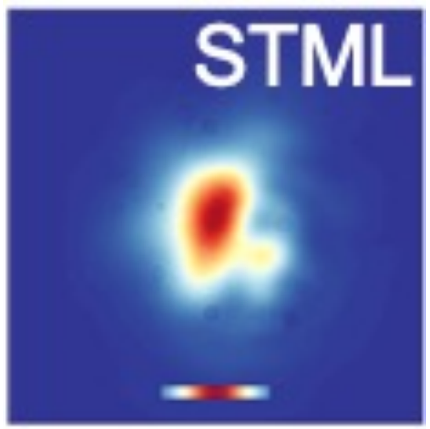
Space-time mode-locking may offer routes to

- 3D shaped ultrafast pulses, pulse sequences
- Higher peak power and intensity than existing lasers

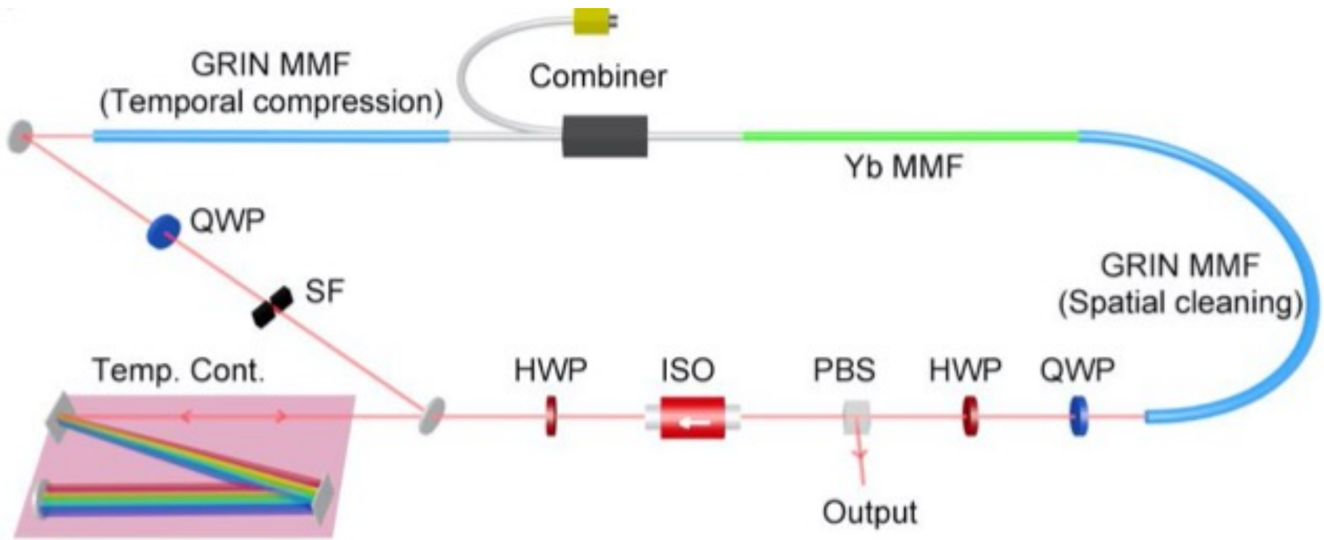
Wright et al., Science 358, 94 (2017)

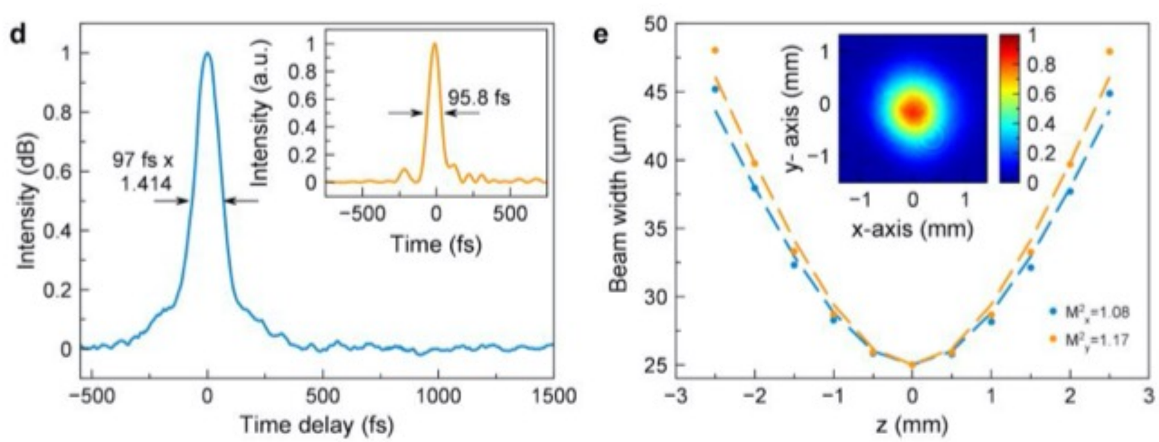
Frequently-asked question:

What's M^2 ?



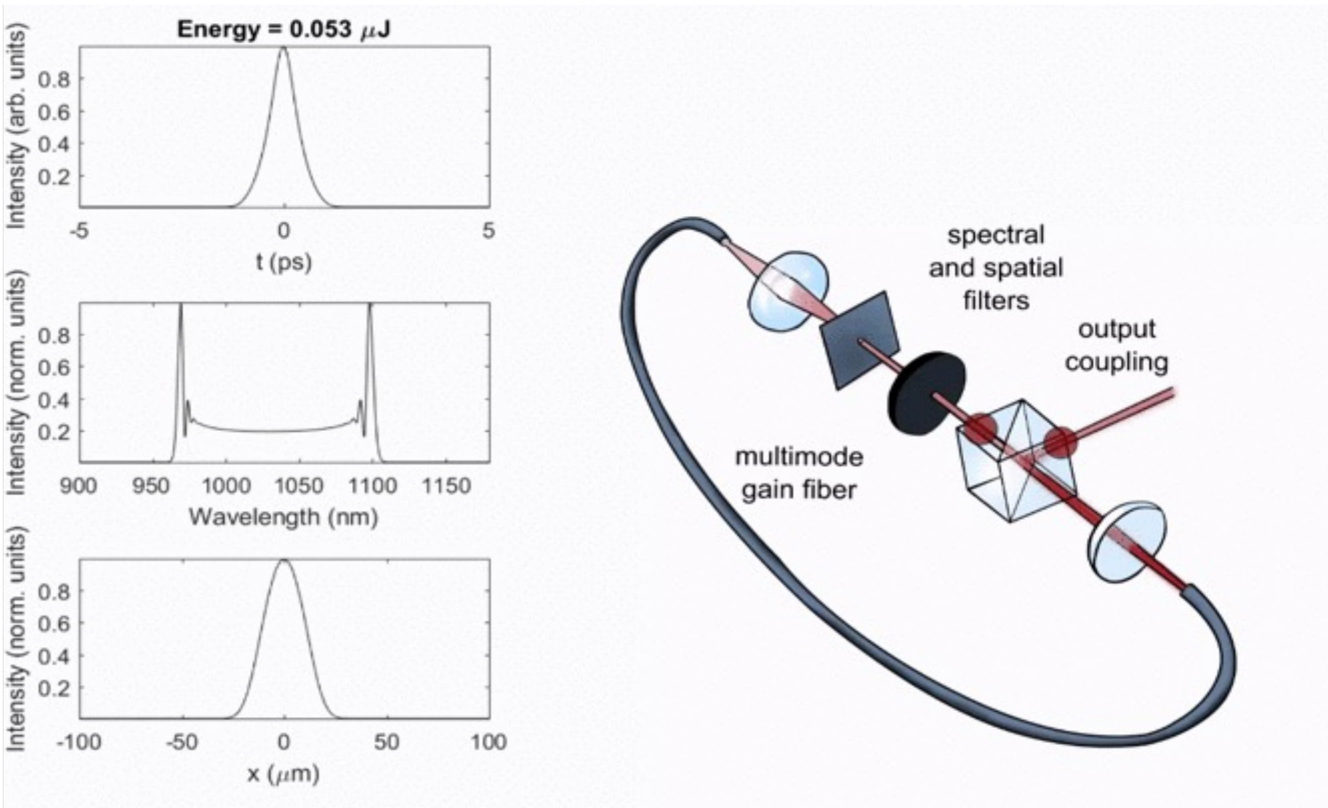
Can we exploit Kerr beam cleaning?





Tegin et al., Advanced Photonics 2, 056005 (2020)

High-energy solutions

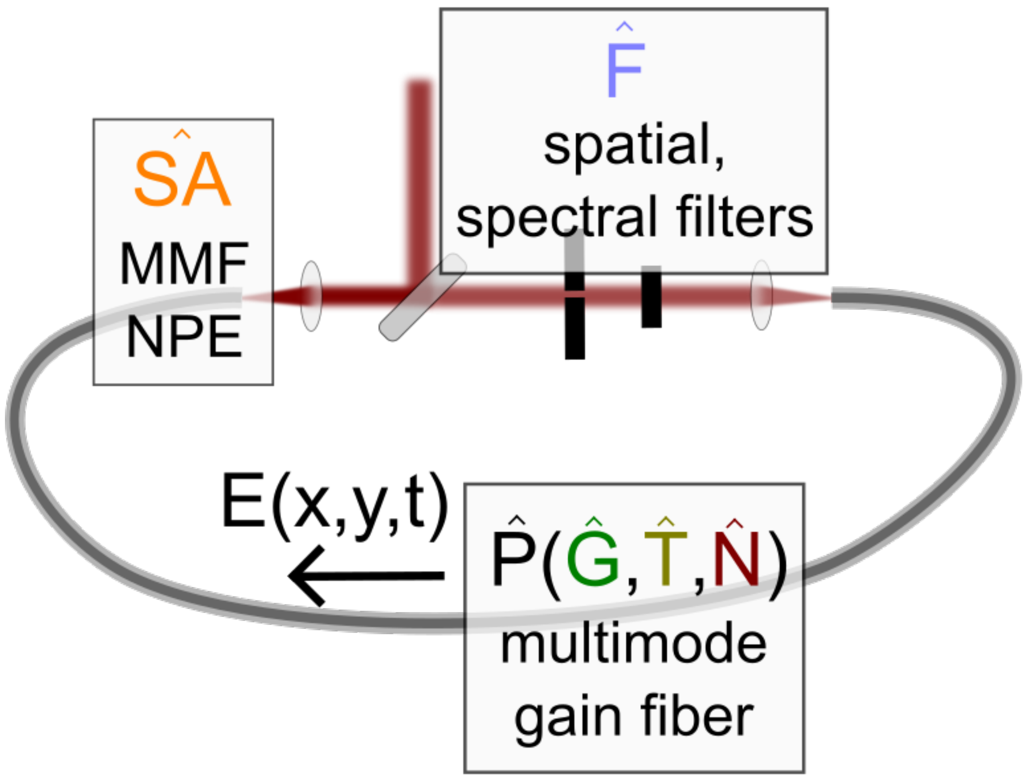


- Find (multimode) states with ~gaussian-beam outputs
- Possible formation of nonlinear mode

Theory of spatiotemporal mode-locking

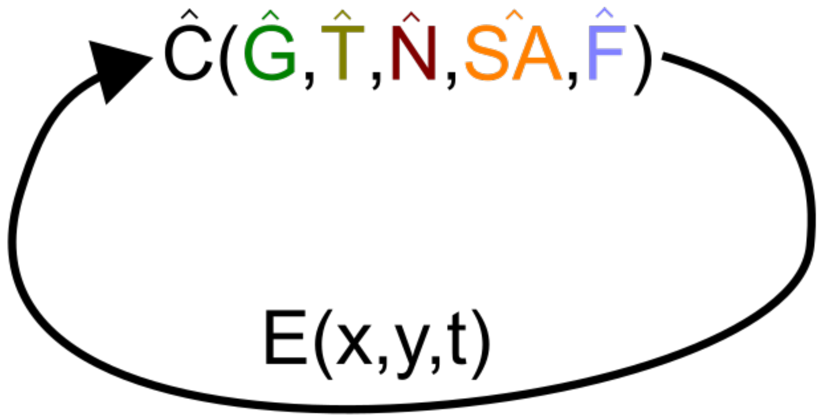
(if time permits)

Modeling: a simple view of an oscillator



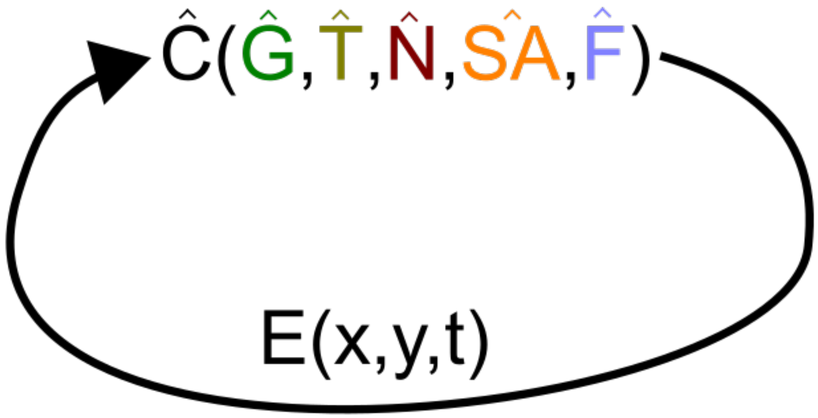
$$\hat{C} \approx \hat{F} \cdot \widehat{SA} \cdot \hat{P}$$

Modeling: a simple view of an oscillator



$$\hat{C}E_i(x,y,t) \rightarrow E_{i+1}(x,y,t)$$

Modeling: a simple view of an oscillator

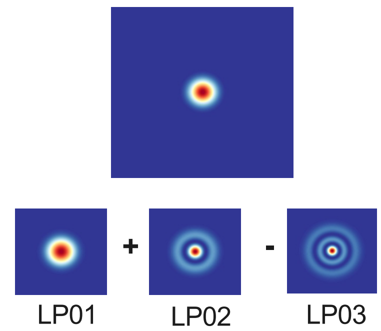


$$\hat{C}E_i(x,y,t) \rightarrow E_{i+1}(x,y,t)$$

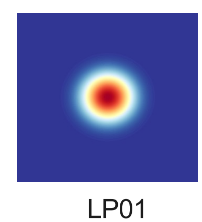
What emerges asymptotically? Why?

Identification of critical effects: attractors

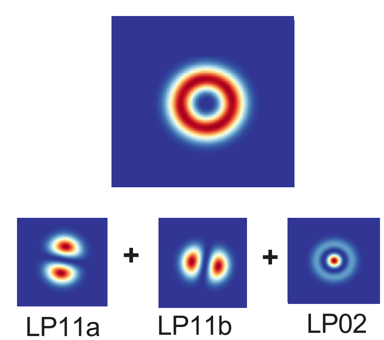
- Attractor for spatial filter



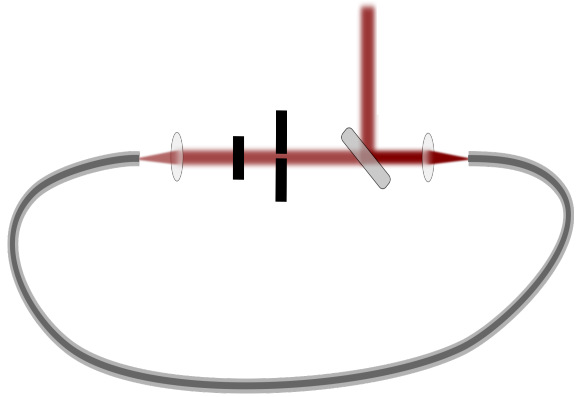
- Attractor for saturable absorber

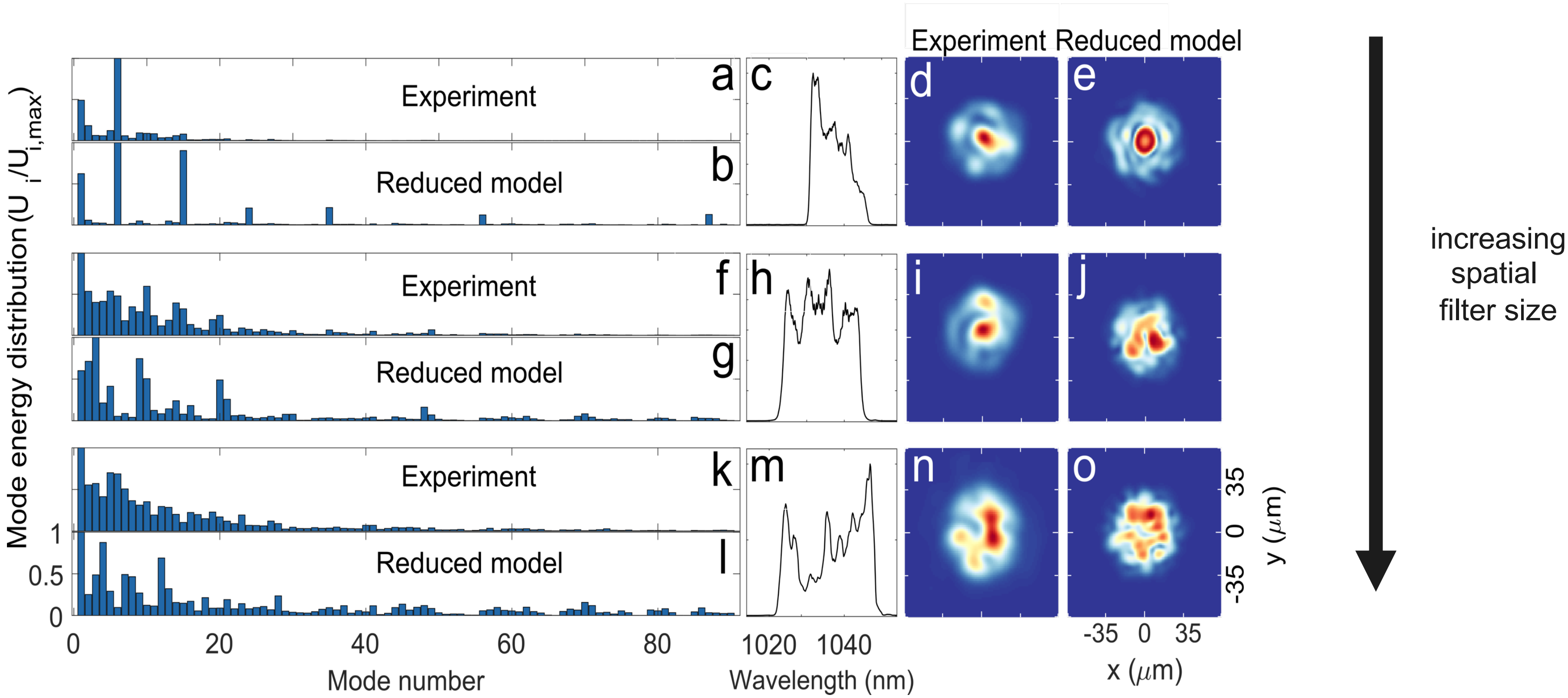


- Attractor for gain extraction



Comparing to experiments

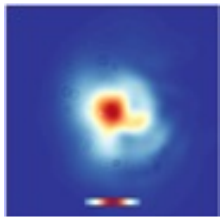




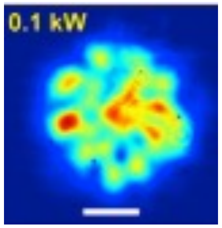
Wright et al., Nature Phys. 16, 565 (2020)

Example of intuition

Saturable absorber favors high intensity => few modes



Gain extraction favors many modes



spatial filter

small → large

few radial LP0N modes

diverse higher-order modes

End of summary of theory

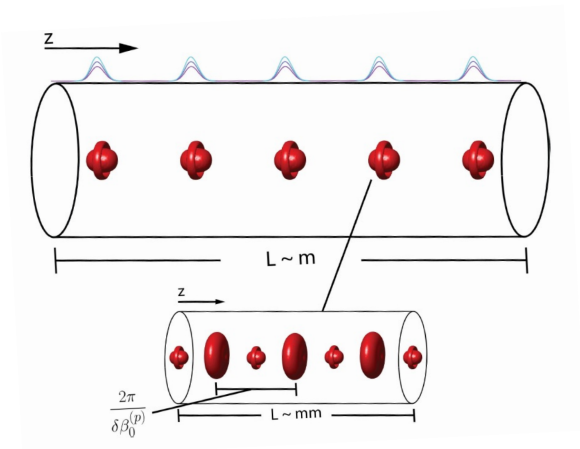
What about…

- anomalous dispersion – multimode soliton laser
- step-index fiber
- multi-core fiber
- solid-state gain media
- Raman or parametric gain
- synchronous pumping to select modes
- …

Interesting directions for MM NLO

The role of disorder in MM nonlinear optics

- Disorder can enhance multimode nonlinear optical effects
- Much interest for telecom



MM solitons

Manakov system

increasing disorder

The role of disorder in MM nonlinear optics

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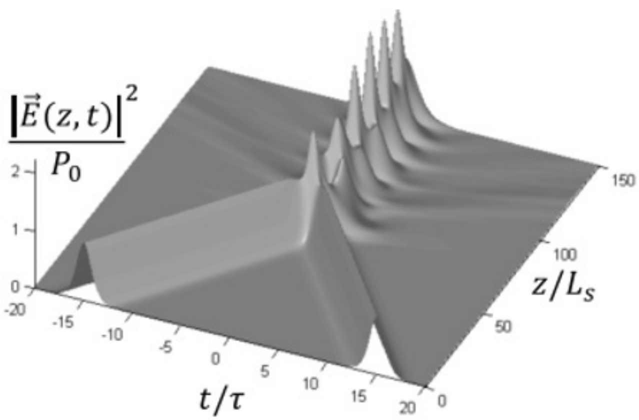

Weak Coupling

MM solitons

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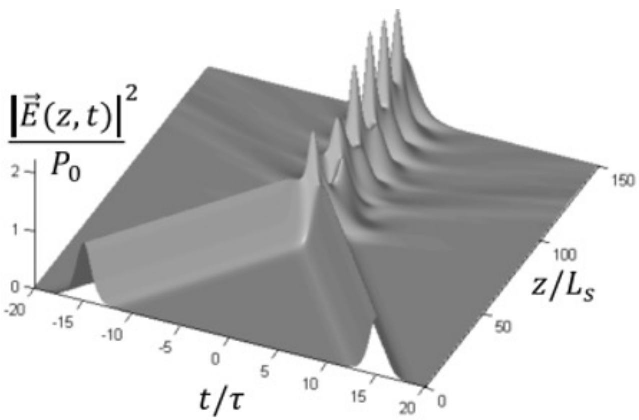
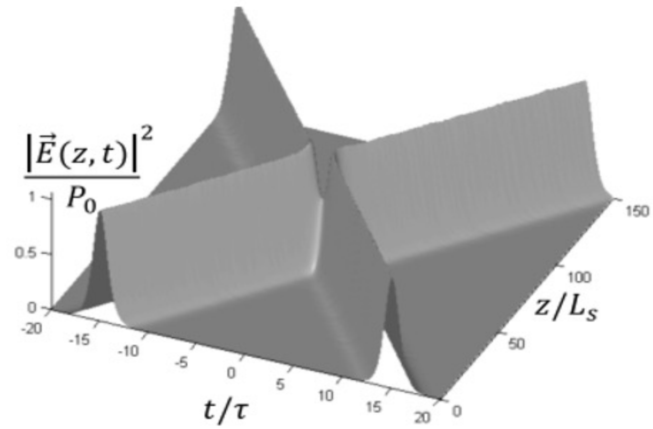


Weak Coupling

MM solitons



Strong Coupling

Manakov system

Mecozzi et al., *Opt. Express* 2012

increasing disorder

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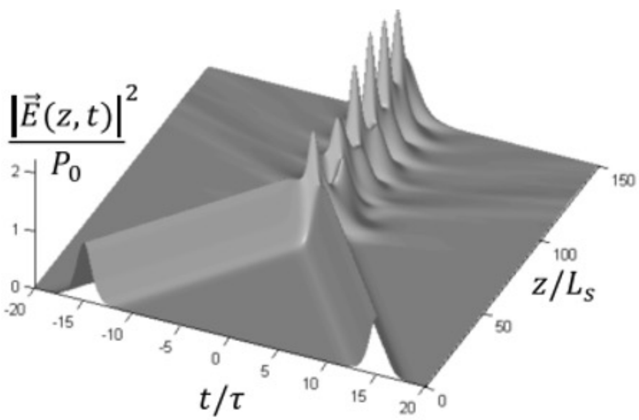
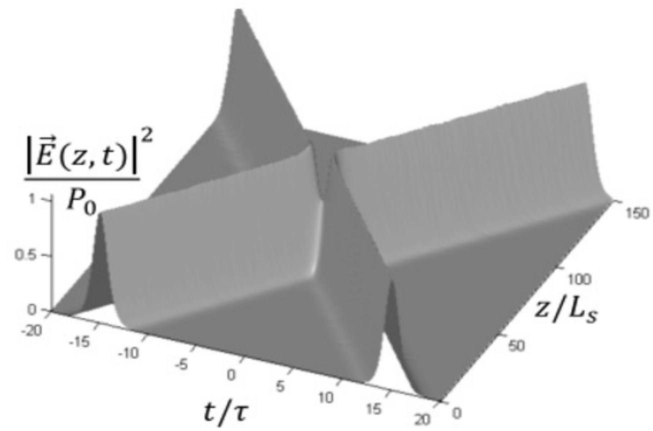


Weak Coupling

?



Strong Coupling

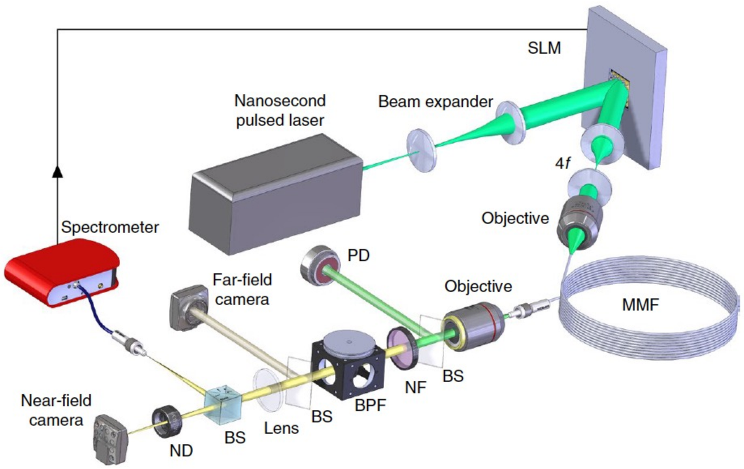
MM solitons

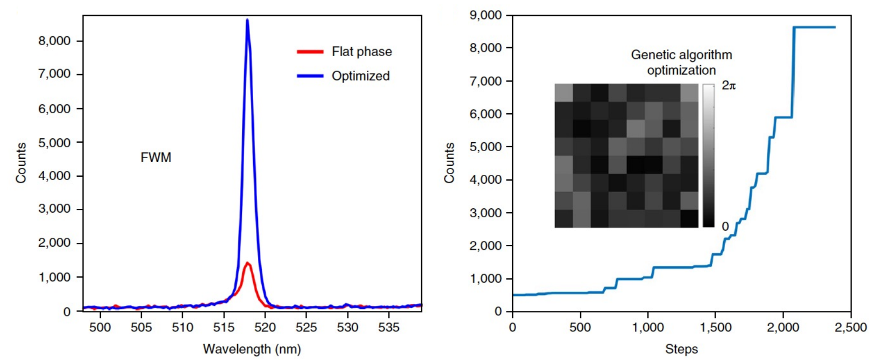
Manakov system

increasing disorder

Wave-front shaping

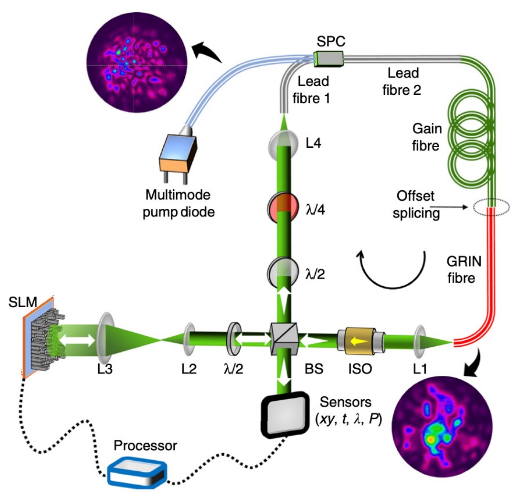
- Control of continuum

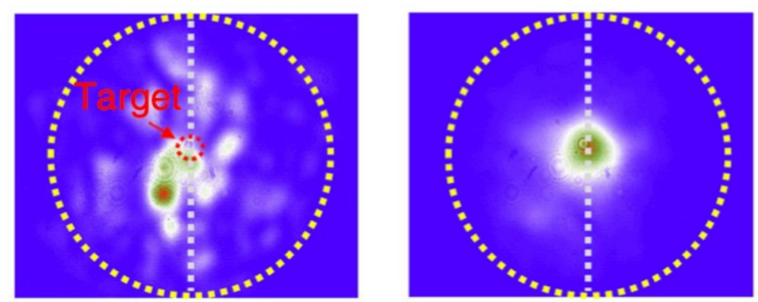




Tzeng et al., Nature Photon 2018

- Control of multimode lasing

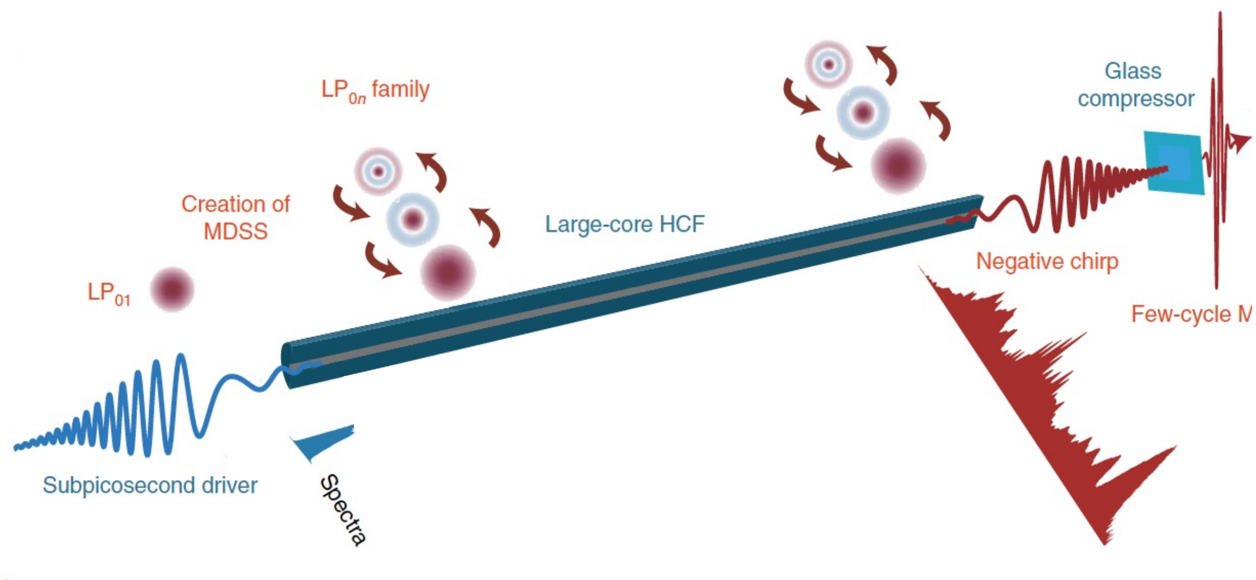




Wei et al., Light Sci Appl 2020

MM propagation in hollow-core fibers

- Isolated examples since 2000
- Multidimensional solitary states



Safaei, Fan et al., Nature Photon 2020

Resources

- Massively-parallel code for solving GMMNLSE (with GPU)

  https://wise.research.engineering.cornell.edu/ or
  https://github.com/wiselabaep/gmmnlse-solver-final

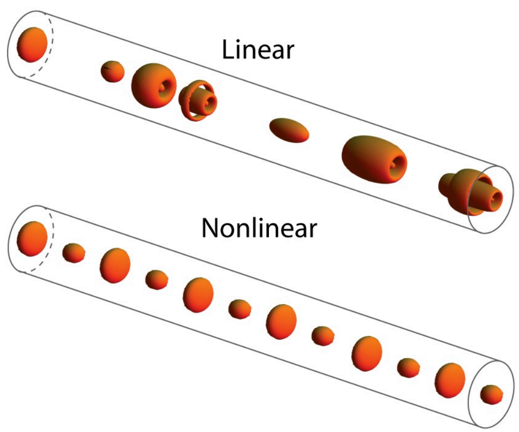
  Wright *et al*., "Multimode Nonlinear Fiber Optics: Massively Parallel Numerical Solver, Tutorial and Outlook," *IEEE J. Select Topics Quantum Electron*. 24, 5100516 (2018)

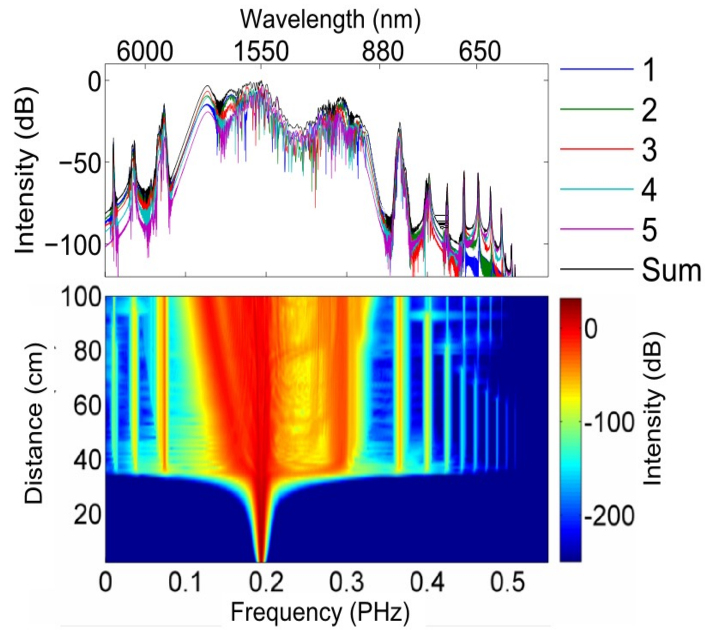
- *Nonlinear Fiber Optics* 6th Ed. (2019) by G. Agrawal, Ch 14

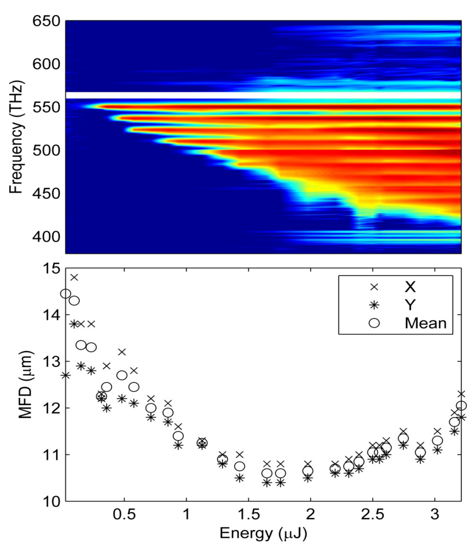
Cornell University Founded A.D. 1865

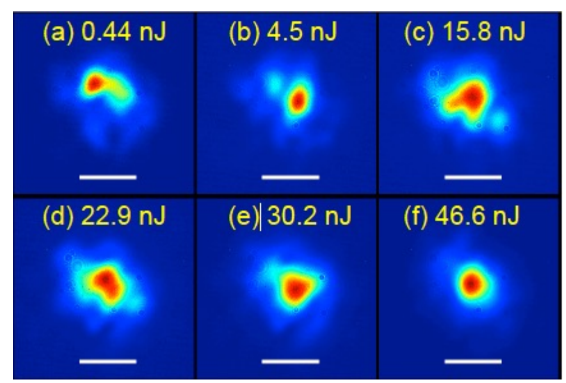
Office of Naval Research
ONR
Science & Technology

NSF


Linear
Nonlinear


Wavelength (nm)
6000
1550
880
650
Intensity (dB)
0
−50
−100
1
2
3
4
5
Sum
Distance (cm)
100
80
60
40
20
Intensity (dB)
0
-100
-200
0
0.1
0.2
0.3
0.4
0.5
Frequency (PHz)


Frequency (THz)
650
600
550
500
450
400
MFD (μm)
15
14
13
12
11
10
X
Y
Mean
0.5
1
1.5
2
2.5
3
Energy (μJ)


(a) 0.44 nJ
(b) 4.5 nJ
(c) 15.8 nJ
(d) 22.9 nJ
(e) 30.2 nJ
(f) 46.6 nJ

Reserve slides

References

Nonlinear Fiber Optics 6th Ed. by G. Agrawal (Ch 14)

Multimode soliton formation, fission

Renninger et al., Nature Commun 2013

Wright et al., Opt Exp 2015

Related: Buch and Agrawal, Opt Lett 2016

Buch and Agrawal, JOSA B 2016

Controllable spatiotemporal nonlinear processes

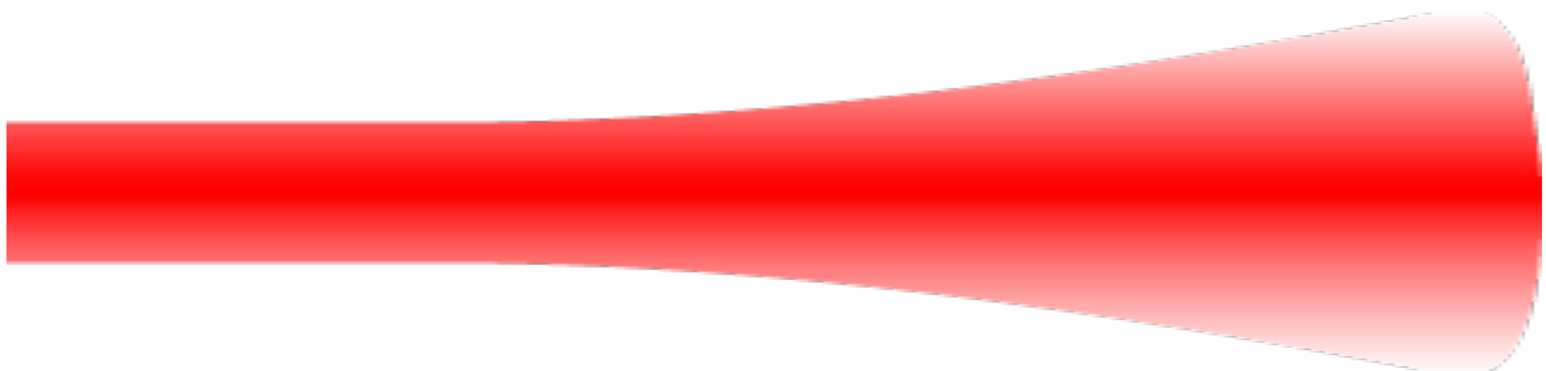
Wright et al., Nature Photon 2015

Spatiotemporal generation of dispersive waves

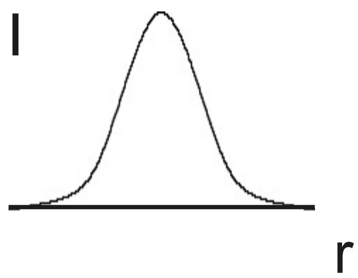
Wright et al., Phys Rev Lett 2015

Space domain: linear wave propagation

- beam spreads owing to diffraction



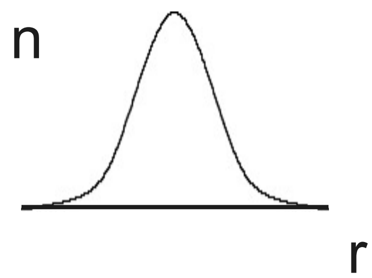
Nonlinear propagation ($\chi^{(3)}$)



$$n = n_0 + n_2 I$$

nonlinear phase shift produces self-focusing

Nonlinear propagation ($\chi^{(3)}$)



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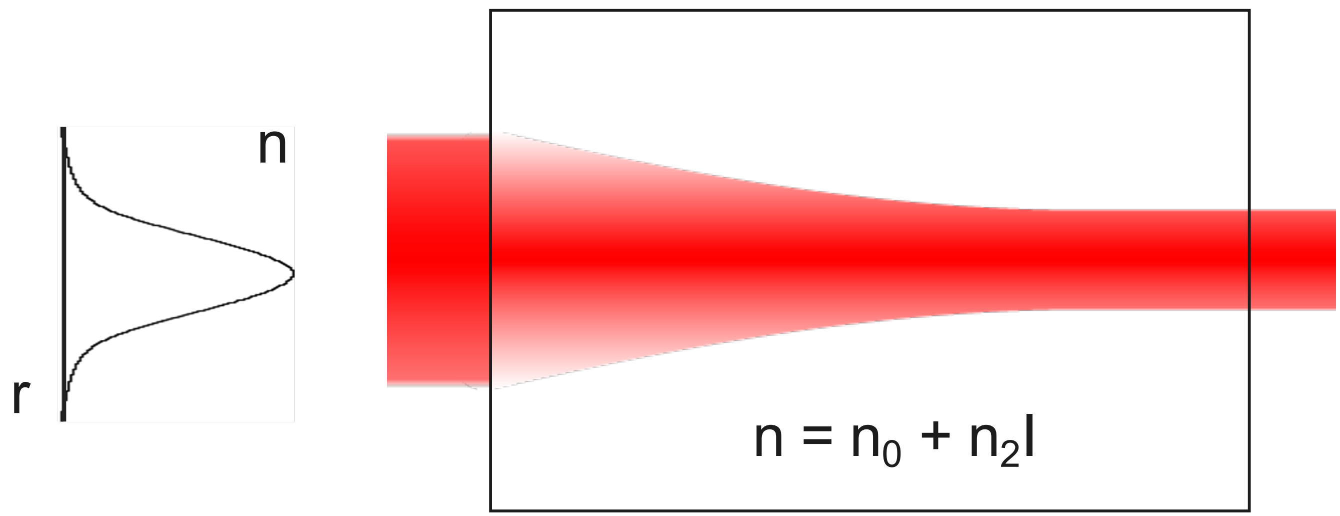
Nonlinear propagation ($\chi^{(3)}$)

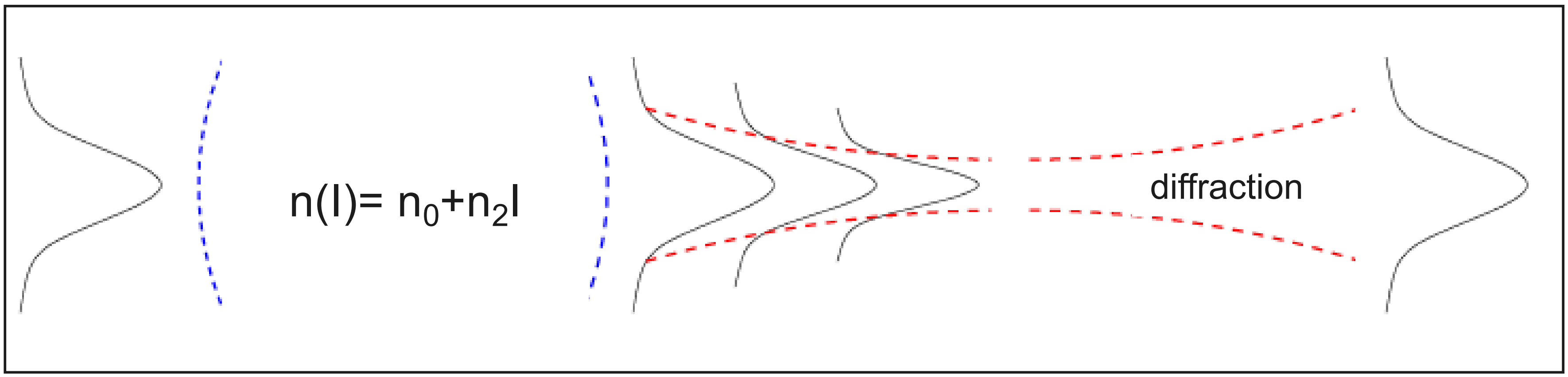


nonlinear phase shift produces self-focusing

Critical power

- diffraction balances self-focusing for

$$P = P_{cr} \sim 5 \text{ MW in glass}$$



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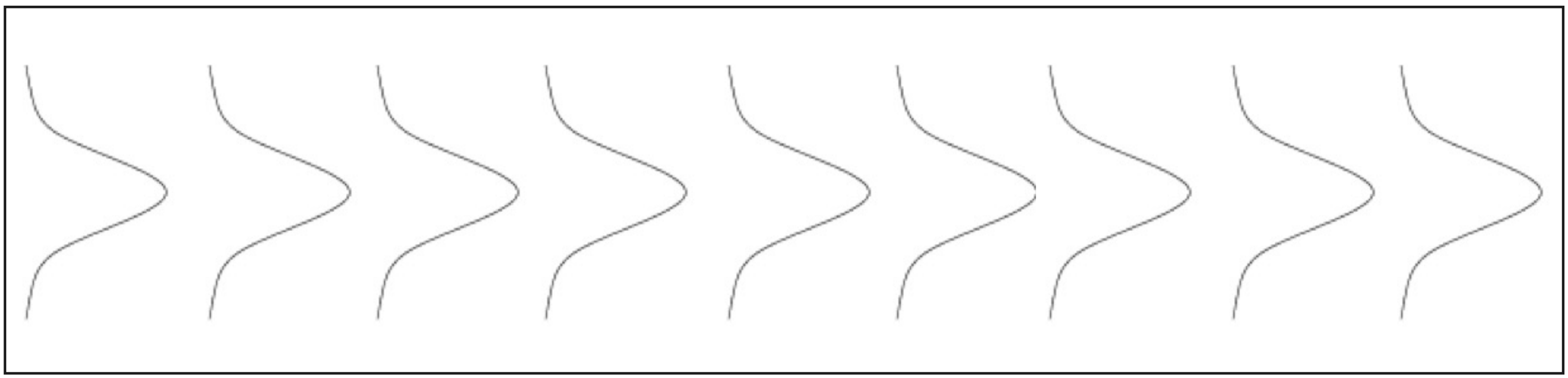
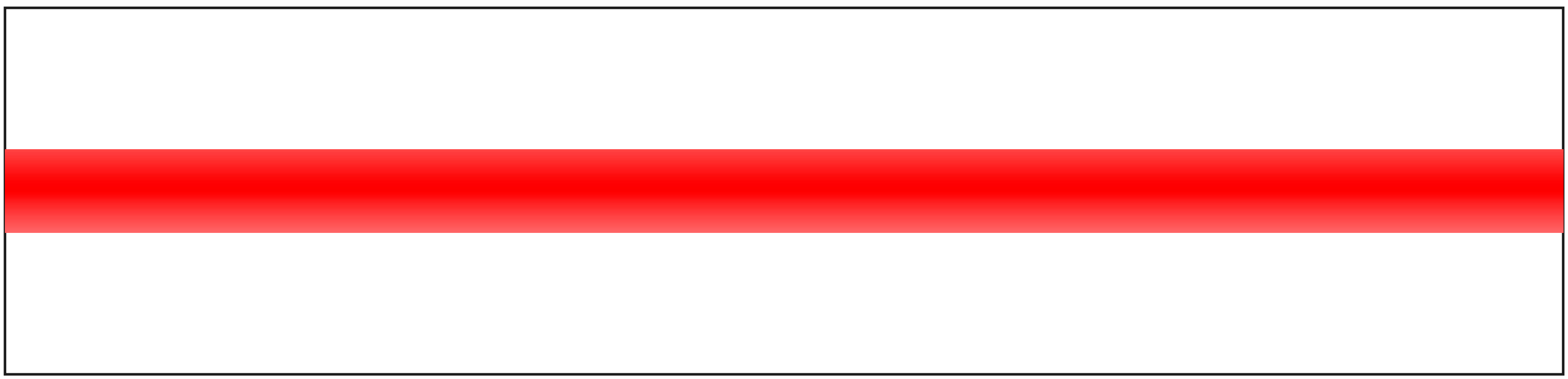


- Spatial soliton

Critical power

- diffraction balances self-focusing for

$$P = P_{cr} \sim 5\text{ MW in glass}$$



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Critical power

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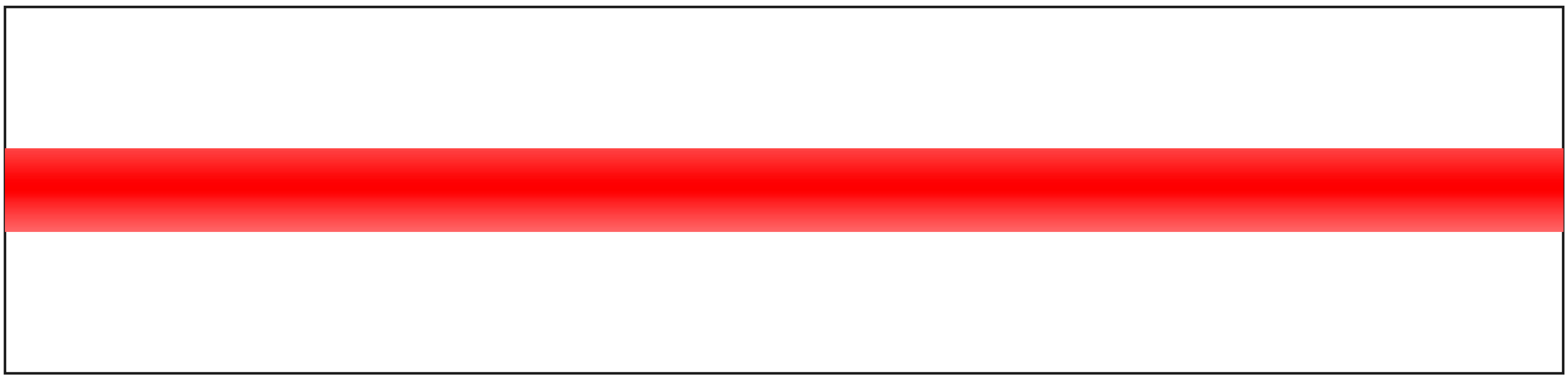
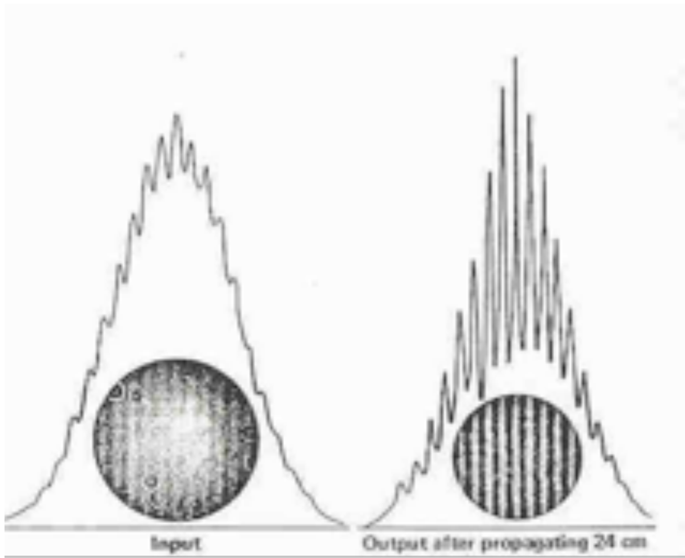
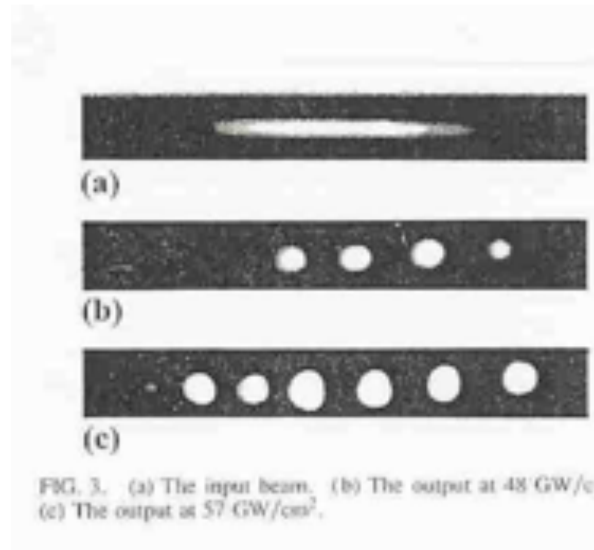
$$P = P_{cr} \sim 5 \text{ MW in glass}$$

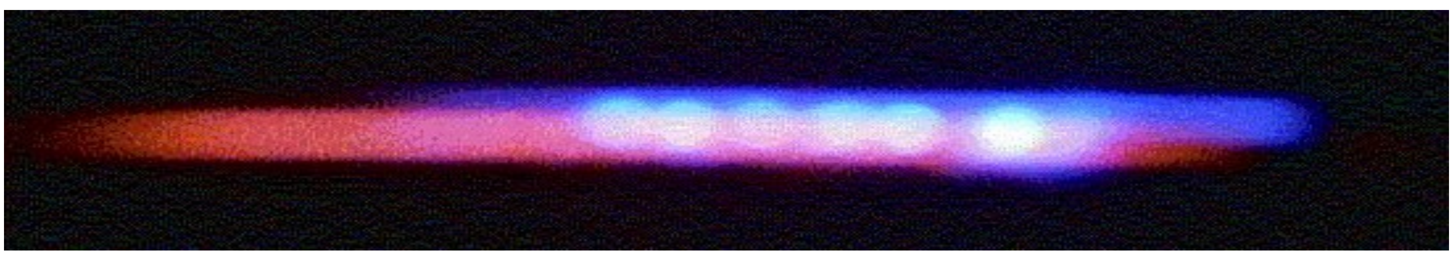


- 2D: balance is unstable in cubic nonlinear media

Modulation instability (spatial)



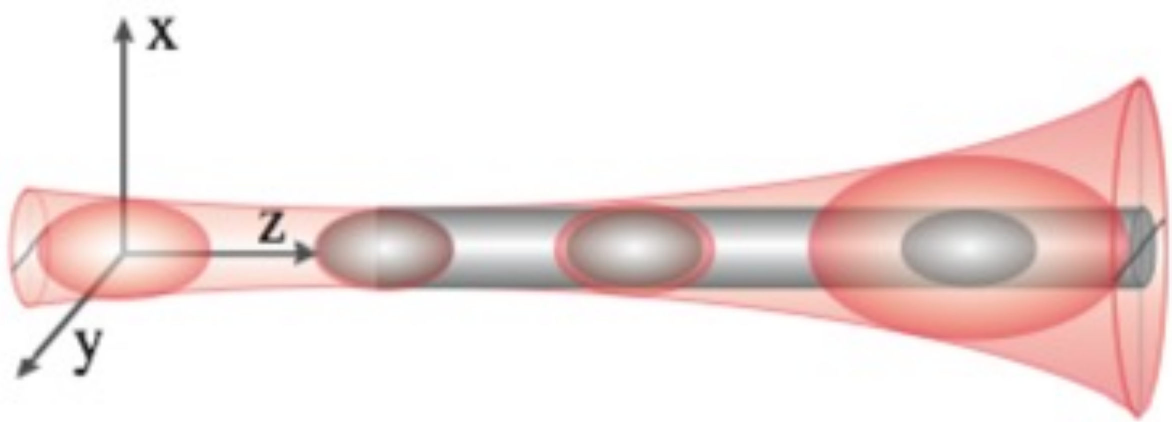




- A beam breaks into its component (spatial) solitons

Spatiotemporal solitons

phase modulation balances dispersion
and
self-focusing balances diffraction



- "light bullet" (Silberberg 1990)
- unstable ($\chi^{(3)}$)

Coupled mode analysis

"GMMNLSE"

$$\partial_z A_p(z,t) = i\left(\beta_0^{(p)} - \Re\left[\beta_0^{(0)}\right]\right)A_p - \left(\beta_1^{(p)} - \Re\left[\beta_1^{(0)}\right]\right)\frac{\partial A_p}{\partial t} + \sum_{m=2}^{3} i^{m+1}\frac{\beta_m}{m!}\partial_t^m A_p$$

modal wavenumber mismatch **modal velocity mismatch** **group velocity dispersion**

$$+i\frac{n_2\omega_o}{c}\left(1+\frac{i}{\omega_o}\partial_t\right)\sum_{l,m,n}\{(1-f_R)S_{plmn}^{k}A_l A_m A_n^* + f_R A_l S_{plmn}^{R}\int_{-\infty}^{t} d\tau A_m(z,t-\tau)A_n^*(z,t-\tau)h_R(\tau)\}$$

shock **Kerr** **Raman**

F. Poletti and P. Horak, J. Opt. Soc. Am. B 25, 1645 (2008)

A. Mafi, J. Lightwave Technol. 30, 2803–2811 (2012)

Single-field model for GRIN fiber

$$\frac{\partial A}{\partial z} = \frac{i}{2k_0}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right) - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - i\frac{k_0\Delta}{R^2}(x^2+y^2)A + i\gamma|A|^2 A$$

diffraction dispersion index profile Kerr

Relation between modes

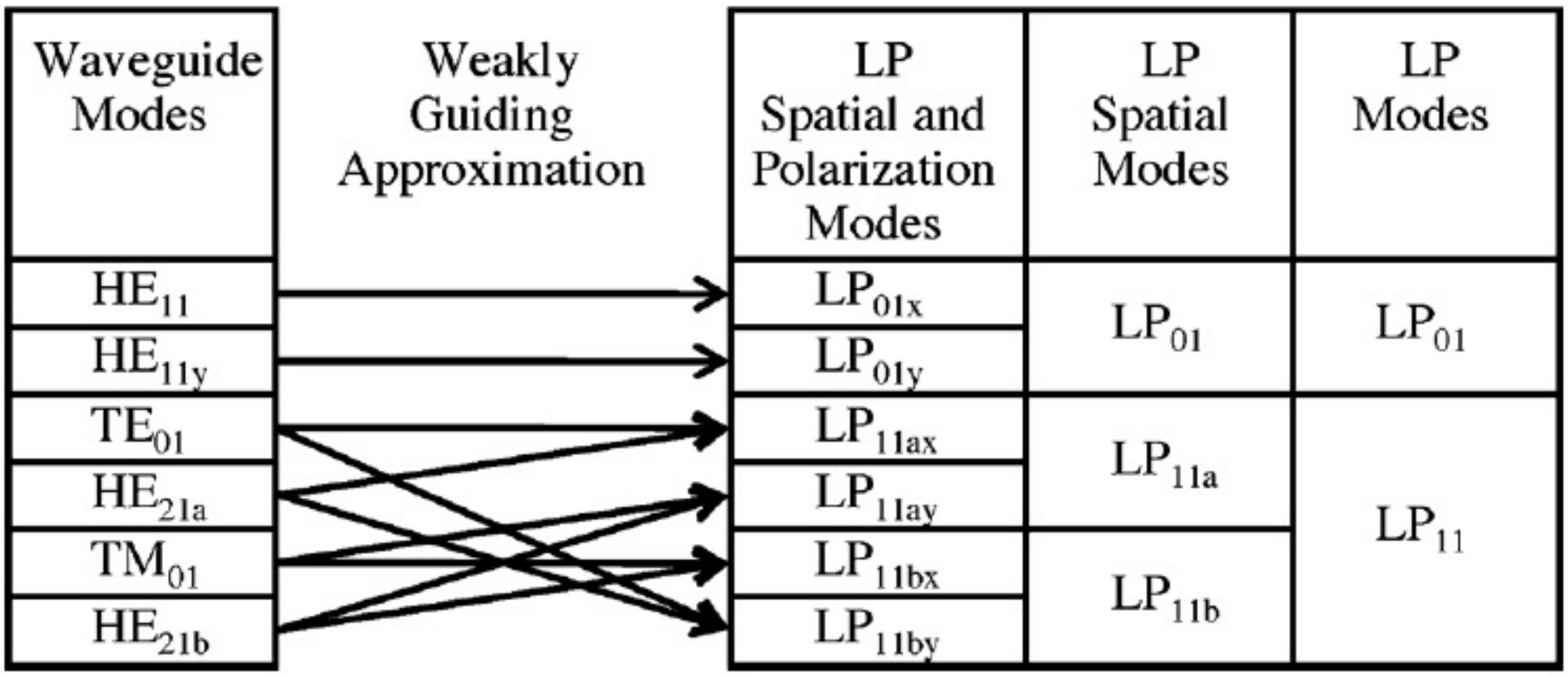
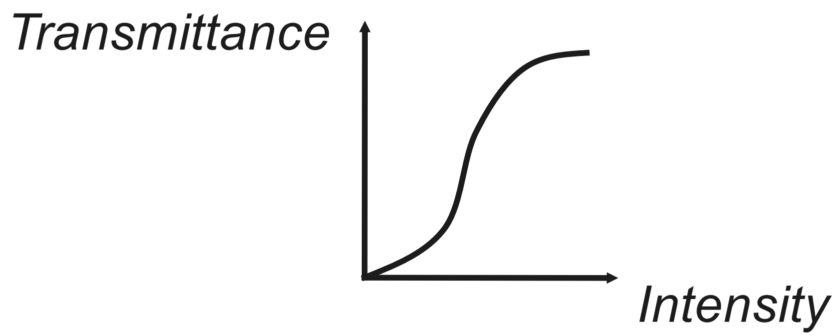
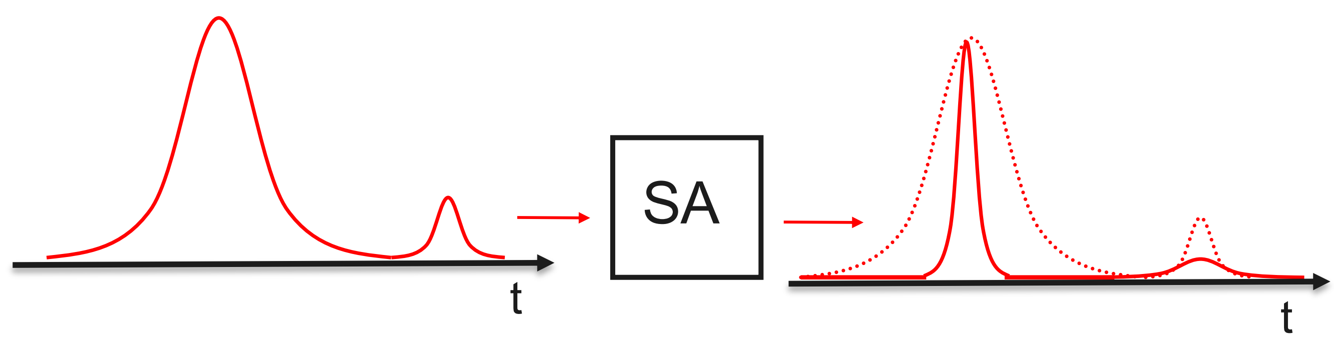


Fig. 1. Relation between the LP modes and the real waveguide modes HE_{11x}, HE_{11y}, TE_{01}, TM_{01}, HE_{21a}, and HE_{21b} of the six-mode FMF.

Ryf *et al., J. Lightwave Tech.* 30, 521 (2012)

Saturable absorber





- Higher intensity experiences lower loss
- SA can be material, nonlinear interference,…
- Self-amplitude modulation